# Design of a Thickened Mill Sands Management System within a Heap Leach Pad

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# ABSTRACT

The La Quinua Gold Mill will process higher-grade ores from Minera Yanacocha mines in Peru and produce the first milled waste, termed "mill sands," at that site. The waste will be thickened and pumped to a mill sands storage facility (MSSF) within an existing heap leach pad. The design required that the leach ore forming the embankments would provide the high level of stability and security required for a major tailings dam. Contrary to usual tailings dam construction, the ore in this facility will be placed in thick, uncompacted lifts to maintain an adequate permeability for leaching. Since the site is seismically active, dynamic stability is critical and has been assessed in detail. Key design elements include: (1) thickening the mill sands to reduce the volume of water in the facility; (2) constructing wider embankments to accommodate leaching and allow for larger structural shells; and, (4) employing rotational deposition to form thin-layered, drained beaches with a slope that will maintain a small surface water pond away from the embankments. This paper presents the design of a unique impoundment for thickened mill sands slurry within an operating heap leach facility. It includes a discussion of the mill sands and leach ore characteristics, as well as the thickening and pumping processes and dynamic stability of the embankments.

# 1 INTRODUCTION

The La Quinua Gold Mill Project will process Yanacocha ores that are deemed unsuitable for heap leaching. The milling process will consist of crushing and grinding followed by gold recovery in a carbon in leach (CIL) circuit with counter current decantation (CCD). The residual material left after removing the gold, termed "mill sands," will be passed through a cyanide washing stage in the CCD circuit and thickened to a slurry solids content of 69.0% by weight before being deposited in an engineered Mill Sands Storage Facility (MSSF). The design has been carried out to accommodate 45 MT of dry mill sands produced at a rate of 5 MT per year for nine years. The period of operation will be from January 2008 (start-up) through December 2016.

The MSSF is a unique design in that it will be located within the limits of the existing La Quinua Heap Leach Pad (HLP) and will use heap leach ore, strategically placed within certain lines and grades, to form the containing embankments. The HLP has been in operation since 2001 and is being expanded in annual stages, which presented the opportunity that future stages could be configured to contain the MSSF. Stage 6 of the HLP, which is currently being developed, will form the embankments on the west and east sides while containment on the north side will be provided by the main body of the HLP that was developed in Stages 1 through 5. Containment on the south side will be provided by the natural topography, which rises to a high point at the southern limit of the MSSF. A general plan of the MSSF and HLP is shown in Figure 1.

# 2 OVERVIEW OF DESIGN

Mill sands thickening will be accomplished with a high rate thickener acting as the final stage of the CCD circuit. The slurry underflow will be transferred to a mill sands tank that will feed transport pumps for delivery to the MSSF. Transportation of the slurry to the MSSF will be through one of two 12 inch diameter,

unlined, steel pipelines in parallel that will be fed by one of two centrifugal pump trains. Under normal operations, one pump train and one pipe will be in use at any time. The second pump train and pipe will allow flushing water to be conveyed to the MSSF to clear the slurry distribution lines around the facility when a switch is made from depositing on one side of the facility to the other.

Mill sands will be deposited into the MSSF from multiple off-takes extending from a pair of distribution pipelines around the east, north, and west sides of the impoundment. The distribution pipes will be installed on a designated bench on the inside face of the leach ore embankments, and the pipes will be raised as the leach ore embankments and the mill sands deposit rise. The mill sands will be transported down onto the beaches via drop bar pipes running down the inside faces of the embankments. The drop bars will prevent the slurry from eroding the faces of the embankments and will allow for controlled distribution and placement of the slurry in the facility. The mill sands will flow at a low velocity over the beaches to enhance liquid/solid separation; and by frequently rotating the points of deposition, a thin layered, drained, and stable deposit will be developed against the embankments.

The beaches will be sloped into the south-central area of the facility so that surface water draining from the mill sands and runoff from precipitation falling on the beaches will be displaced away from the embankments. A small surface water pond will be maintained directly against an underdrain blanket covering the lined base of the facility. The pond will progressively rise as the mill sands fill the basin, but it will remain in continuous contact with the underdrain, which will serve as the principal method for removing the water. A series of decant towers will also be installed in the MSSF to assist in removing water.



Figure 1 Mill sands management system map of facilities

# **3 DESIGN CRITERIA**

Table 1 outlines key design criteria used for the mill sands pumping, conveyance, distribution pipes, and drop bars.

Table 1	Mill sands pumping,	conveyance, distribution pipes,	, and drop bars criteria
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Parameter	Units	Value
Mill conde colide throughput (decign)	t/hour	620
Min sands sonds throughput (design)	Mtpa	5.0
Design allowance	% of design	20.0

Parameter	Units	Value
Nominal design solids content	% by weight	69.0
Solids content design range	% by weight	67.0 to 71.0
Operating times	Days/year	365
Operating times	Hours/day	24
Operating quailability	Thickening CCD	92%
Operating availability	Pumping	92%
Project life	Years	9

Key design criteria used for the mill sands storage facility are outlined in Table 2.

 Table 2
 Mill sands storage facility criteria

Parameter	Value	
Mill sands material	Non-plastic sandy silt classified as non-plastic silt with sand (ML under the Unified Soil Classification System, i.e. the USCS)	
Planned density of mill sands in storage	Base case	
	• Average dry density 1.50 t/m <sup>3</sup> during years 0-3	
	• Average dry density linearly increasing from 1.50 to 1.65 t/m <sup>3</sup> during years 4-9	
Leach ore material and placement in MSSF embankments	Silty, clayey gravel with sand (GC-GM under the USCS system) divided into three categories based on a "field-called" fines content as follows:	
	• Zones A and C – up to 15% fines	
	• Zone B – up to 20% fines	
	• Zone D – up to 25% fines	
	Hydraulic characteristics of these materials are significantly different, particularly under higher stresses	
	Pore pressures in the embankments will be controlled by placing these materials within certain zone boundaries	
	Materials will be placed in end-dumped lifts, each 16 m high, in accordance with current procedures on the HLP	
Design earthquake	Maximum Design Earthquake (MDE) is equal to the Maximum Credible Earthquake (MCE) for the site (Note: This does not apply to areas of the HLP that are remote from and have no influence on the performance of the MSSF)	

Parameter	Value	
	MCE is generated from an intraplate subduction zone event below and east of the site	
	Magnitude is $M_w = 8.0$	
	Distance and depth from the site are 90 and 100 km, respectively	
	Resulting peak horizontal ground acceleration at the site is 0.41 g in the free field	
Minimum acceptable factors of safety for slope stability	Static under steady state seepage conditions, including the case with the embankments under leach – minimum $FOS = 1.5$	
	Post-cyclic static with strain softened or liquefied zones – minimum FOS = 1.1	
Maximum acceptable earthquake-induced deformations	Maximum vertical on the crest = no loss of mill sands containment	
	Maximum horizontal on the crest = $500 \text{ cm} \cong 5\%$ of embankment height)	
	Maximum horizontal on the liner $= 20$ cm	
	No flow slide	

# 4 MILL SANDS CHARACTERISTICS AND DESIGN OF THICKENING, PUMPING AND PIPELINE SYSTEM

The mill sands solids have a specific gravity of 2.80 and a particle size distribution as shown in Figure 2. Tests on flocculated, thickened mill sands, which characterise the expected slurry to be pumped to the MSSF, provided the following results:

- No measurable slump was observed below a solids content of 74.0%.
- Flocculated slurry will form a paste at an approximate solids content of 80.0%. The transition from conventional tailings to high-density slurry occurs at a solids content of 69.6%. The yield stress at this solids content is 6.0 pascals (Pa).
- The flocculated mill sands have a Bingham plastic yield stress of 4.3 Pa and a Bingham plastic viscosity of 0.044 pascal seconds (PaS) at the design solids content of 69.0%.



#### Figure 2 Mill sands gradation

The relationship between yield stress and slurry solids content is shown in Figure 3.



Figure 3 Mill sands slurry yield stress distribution with and without gypsum

The mill sands slurry were found to have a Miller number of 199 with an attrition number of -7. This placed the mill sands in a high abrasion wear category with a high rate of change of the abrasion as a result of particle attrition. The high abrasiveness is attributed to the large proportion of sharp, angular, quartz particles from the hard rock ore feed.

The absence of clays results in the mill sands having good flocculation, settling, and compaction characteristics. Dynamic sedimentation tests also highlighted the importance of setting the thickener feed dilution to a solids content between 15.0% and 20.0%. These tests indicated that thickener overflow clarity problems may occur at a slurry pH below 10.

Flocculant selection tests found Ciba M10 to be the optimum flocculant with a dosage rate of 11 g/t. Based on the design throughput rate, the thickener has been sized at 32 m in diameter.

Small amounts of gypsum may be added into the process in certain years. To investigate the effect of this on the slurry, a series of tests were carried out with gypsum added at dosage rates of 0.5 and 10.0% by weight. The gypsum was found to have an insignificant effect on the flocculant demand because the material is dominated by quartz and contains no smectite minerals. Similar to the dosage rate determined in the test work without gypsum, the optimum flocculant dosage rate fell between 11 and 12 g/t. With gypsum, the slurry showed an increase in yield stress compared to slurry without gypsum, and this is shown in Figure 3. The plastic viscosity was found to be identical for slurry with and without gypsum at the equivalent solids content. The rheology of the slurry with 0.5 and 10.0% gypsum addition was found to be similar.

The effect of the gypsum on pumping is that the system could operate in laminar flow at times, which would result in the development of a settled bed. So that it will not stall, the system has been designed with sufficient pressure available to maintain the required flow rate above the settled bed in this scenario.

The mill sands pumps will consist of one operating and one standby mainline centrifugal pump train with each pump train comprising four variable speed Warman International 8/6 AH pumps fitted with 260 kw (350 horsepower) motors.

The concentration of mill sands pumped to the MSSF will be measured. If necessary, dilution water will be introduced into the mill sands tank and/or suction piping between the tank and the pump station to ensure that the slurry concentration does not exceed the design values.

The mill sands pipelines will consist of 12 inch diameter, unlined steel pipe. The pipes will continue to a point above the northeast corner of the MSSF where they will feed the slurry into either the eastern or western distribution pipes around the facility. This bifurcation point will be provided with a valved crossover that will allow the slurry in either main line pipe to enter either distribution pipe while still allowing flushing of the second line pipe and distribution pipe with water to occur.

The pipelines will be flushed with water prior to any shutdown of the pump station. Emergency power will be provided so that flushing can be completed in the event of a power failure. An Emergency Drainage Facility has been designed at the pipeline's low point to receive the mill sands slurry from either pipeline should it be necessary to drain the pipeline without flushing.

A transient analysis conducted on the pumping system has resulted in the inclusion of a number of combination vacuum breaker and air release valves. These will prevent negative pressures and pressure spikes from developing in the pipe caused by column closure in the line.

Control of the mill sands pumping system consists of three key components:

- 1. The final CCD thickener will produce mill sands slurry at a solids content slightly higher than the mill sands pumping system design concentration. The solids content in the mill sands storage tank will be controlled to the specified value by monitoring the thickener underflow concentration and adding the appropriate quantity of dilution water. The slurry density at the pump station discharge will also be monitored to provide a secondary measurement. Water will be injected into the pump suction if the solids content is too high.
- 2. The mill sands are in the form of slurry that will settle in the pipe if the velocity becomes too slow. The system has, therefore, been designed to operate above a specified minimum flow rate.
- 3. The pumping system will be designed to operate continuously to minimise the flushing requirements while maintaining the minimum flow rate. Should the thickener production be too low to meet the minimum flow rate criterion, water can be added to the holding tank to maintain continuous pumping to avoid flushing the pipeline.

## 5 LEACH ORE CHARACTERISTICS AND DYNAMIC STABILITY ANALYSES OF EMBANKMENTS

### 5.1 Leach Ore Embankment Materials

Combining the heap leach and mill sands facilities into a single management unit presented the challenge of ensuring that the leach ore embankments would provide the high level of stability and security required for a

large mining waste dam. For dynamic (earthquake) stability consideration, the principal issue was that all potentially liquefiable zones of leach ore would remain well confined within shell zones that would not liquefy. Two factors adopted in the design of the embankments to address this were:

- Providing embankments with widths much larger than normal for dams in order to accommodate ore leaching and thereby allowing for larger shell zones.
- Having the ability to separate the leach ore into designated coarser and finer materials and placing the coarser, more freely-draining materials into outer shells in the embankments to produce large, well-drained, unsaturated, and stable structural zones that may strain soften, but not liquefy, under earthquake loading.

The coarser and finer ore materials have been categorised into three types or zones as follows:

- Zones A and C material with up to 15% fines that will largely comprise higher quality run-of-mine, or ROM, ore and represents the material that will remain well drained and unsaturated.
- Zone B material with up to 20% fines.
- Zone D material with up to 25% fines.

Zone B material will be placed in the center of the embankments and will be confined behind large zones of A and C material forming the shell zones. Zone D material will be limited to the upper two lifts in the embankment.

### 5.2 Design Earthquake

The maximum design earthquake (MDE) used for the project is the maximum credible earthquake (MCE) for the site. A seismic risk analysis accounting for both probabilistic and deterministically-derived events indicated the MCE to consist of a deep sub-Andean intraplate event of a magnitude M=8 estimated to produce a peak horizontal ground acceleration (PHGA) at the site of 0.41 g.

For dynamic modelling, three earthquake time history records were considered. This included two timehistory records (horizontal components) from the 1970 Chimbote, Peru interplate subduction zone event. Since those records were not recorded on bedrock and were scaled significantly to achieve the 0.41 g PHGA, a time history from the 2001 El Salvador interplate subduction earthquake that was recorded on bedrock and required significantly less scaling was also included. The computer program, ProSHAKE (EduPro, 2001), was then used to determine the cyclic stress ratio (CSR) profiles through the embankments that the three earthquake records would be expected to impart. The results gave similar profiles; therefore, the 2001 El Salvador time-history was adopted for subsequent liquefaction and dynamic deformation analyses of the embankment. Details of this earthquake are as follows:

- Historic event: El Salvador Earthquake (January 13, 2001).
- Station: Santa Tecia (180 degree horizontal component).
- Magnitude:  $M_w=7.7$ .
- Maximum acceleration: PHGA=0.60 g.
- Epicentral distance: 98 km.
- Focal depth: 60 km.

### 5.3 Liquefaction Considerations

The liquefaction resistance of the three types of leach ore was assessed by carrying out large diameter cyclic triaxial tests on samples of each ore type that were prepared to an initial density of approximately 1.60 t/m<sup>3</sup> and consolidated to an effective confining stress of approximately 200 kilopascals (kPa) (or approximately 11 m depth of burial). Cyclic loading was completed at a frequency of 1.0 Hertz under cyclic stress ratios (CSRs) ranging from 0.10 to 0.26, a range that includes the predicted CSRs from the ProSHAKE analyses. The results of the testing are depicted in Figure 4, which is a plot of cyclic resistance ratio (CRR) versus the

number of cycles to initiate liquefaction for each of the three ore types. The CRR can be considered as the CSR at the onset of liquefaction.



#### Figure 4 Leach ore cyclic resistance ratio

A magnitude M=7.5 event is expected to generate on the order of 15 cycles of significant shaking, which based on Figure 4 indicates that the CRR of the Zones A and C, Zone B, and Zone D leach ore varies over a narrow range of 0.15 to 0.17 at an effective confining stress of 200 kPa. Since these data were generated from cyclic triaxial shear testing rather than cyclic direct simple shear testing, the results must be corrected to simple shear conditions to better represent field behavior during earthquake shaking. Ishihara (1993) presented an appropriate correction factor that when adopted reset the resultant lower limit CRR for the leach ore under simple shear conditions to 0.09. Magnitude scaling factors from the NCEER 1996 Liquefaction Workshop (Youd and Idriss, 1997) were then used to calculate the lower limit CRR under an M=8 event, at a confining stress of 200 kPa, which became 0.08.

The CSR values that are predicted to be induced by the MCE, as evaluated by ProSHAKE, are shown in a vertical profile through the embankment in Figure 5.



Figure 5 Leach ore liquefaction assessment

Assuming that the CRR derived from the cyclic testing discussed above does not vary significantly with depth, it can be seen that the CSR values induced by the earthquake exceed the CRR values of the leach ore thereby indicating that the leach ore, if saturated, is potentially liquefiable under the influence of the MCE.

In recognition of the potential for the saturated ore to liquefy, the design focused on providing large buttressing shells of Zones A and C material that would be at low saturation levels and where liquefaction would not occur.

The moisture retention characteristics of the Zones A and C leach ore indicate that, even when it is under leach, the ore in the unsaturated region will be at moisture contents equivalent to 60 to 70% saturation. Liquefaction of materials at that level of saturation is not expected and would be unprecedented.

### 5.4 Dynamic Modelling of the Embankment

#### 5.4.1 Analyses

The following embankment slope stability modelling was carried out:

- Static and post-earthquake (material softened or liquefied) analyses using the two-dimensional, limit equilibrium modelling software, SLOPE/W (GEO-SLOPE, 2004a).
- Analysis of the potential deformations and extent of liquefaction predicted to be caused by the MCE using the geomechanical finite difference-modelling program, FLAC (Itasca, 2001).

The results of the static slope stability analyses are not presented in detail in this paper.

# 5.4.2 Definition of zones of saturation that represent potentially liquefiable zones in the embankment

A typical result of the finite element seepage modelling carried out to define the zones of potential saturation in the embankments is illustrated in Figure 6. Flow vectors and contours of total head that define the flow regime (gravity dominated with flow largely vertically downward with limited lateral spreading) are presented for the embankment under active leaching.



#### Figure 6 Section B crest elevation 3660 seepage analysis results

The saturated zone is shown to be well contained within the center of the embankment in the Zone B material while the Zone C material at the base of the embankment and the Zone A material forming the outer shell remain freely drained and unsaturated.

Pore pressure data from the seepage files were exported into the static and dynamic stability analyses and included the limits of saturated and unsaturated ore together with the pore pressure values at each finite element node.

#### 5.4.3 Dynamic modelling by FLAC

The dynamic analysis comprised three stages. In the first stage, a computer model representing the geometry of the cross section, including the configuration of the various materials, was developed. The section is depicted in Figure 7.



Figure 7 Section B crest elevation 3660 geomechanical (FLAC) model

Material densities and artificially high modulus and cohesion values were assigned, and the model was analysed to establish in situ stresses under static loading conditions (self weight loads due to gravity alone).

Since the embankment contains a reasonably complex pore pressure regime, including non-hydrostatic pressures within the leach ore for modelling the MSSF, the saturated materials were assigned a constant average pore pressure, which compared well with the results of the seepage modelling. This was incorporated into the model using a series of custom commands in the FISH programming language (that is inherent within the FLAC software).

In the second stage of the analysis, real material properties were assigned to the various zones in the cross section, including the static shear strength parameters for the different zones of materials and the soil-liner interface above the foundation. Small strain shear moduli values ( $G_{max}$ ) as a function of the confining stress were also input for each zone. Selection of these values is discussed in the following section. Since the analyses are not particularly sensitive to the bulk modulus and the analysis converges more efficiently if this is done, bulk modulus values were set equal to the shear modulus.

Results of the static model after the second stage, but before the application of the earthquake, showed the development of several shear bands through the section that closely parallel the potential critical slip surfaces identified during static limit equilibrium slope stability analyses (not reported in this paper). These bands are illustrated in Figure 8.



Figure 8 Section B crest elevation stress state of model zones

This finding confirmed that a high level of consistency has been achieved between the static and dynamic models.

In the third stage of the analysis, the MCE was applied by entering the scaled acceleration time-history together with a liquefaction triggering subroutine developed for FLAC at the University of British Columbia (Beatty and Byrne, 1999). In that subroutine, a real-time analysis is performed where triggering of

liquefaction, or cyclic failure, is evaluated in each finite difference zone by counting the number of cycles of shear stress experienced by that zone until a threshold number is reached (calculated from the cyclic resistance ratio, i.e., the CRR). Strength and stiffness values are then re-assigned to the appropriate post-liquefied values at the moment of liquefaction. The post-liquefied, shear strength parameters used in the dynamic model were the same as those used in the post-earthquake limit equilibrium analyses.

#### 5.4.4 Dynamic material properties

The dynamic stiffness characteristics of the various materials were estimated from shear wave velocity measurements made in the leach ore during previous investigations of the HLP as well as from the material classification data and published typical values for similar materials. Published information on the dynamic properties of soils is provided by Hardin and Drnevich (1972), Ishihara (1982), and Seed et al. (1986).

The shear modulus for each material was calculated using an estimated shear modulus factor,  $K_{2,max}$ , for the material and the following equation:

$$G_{max} = 22 (P_a) K_{2,max} (\sigma'_m/P_a)^{0.5}$$
  
where:  $\sigma'_m$  = mean effective confining stress (1)  
 $P_a$  = atmospheric pressure

Shear modulus reduction factors and material damping during the earthquake are related to the levels of shear strain developed. For the FLAC analyses, the average dynamic shear modulus (defined by the reduction ratio  $G/G_{max}$ ) and dynamic (Rayleigh) damping for each soil unit were established from the results of the ProSHAKE analyses.

Liquefied leach ore was assigned an undrained strength-to-effective stress ratio  $(S_u/p')$  equal to 0.10. Experience dictates that this is a conservative (low) value for a material that contains significant gravel and sand percentages. Non-liquefied leach ore was conservatively strain softened by reducing its strength to two-thirds of the original value.

### 5.4.5 Results of dynamic analyses

The results of the FLAC analyses, consisting of contours of total deformation, are shown in Figure 9.





The predicted permanent deformations at critical locations in the embankment, including along the underlying liner system and the crest, are small. Maximum permanent deformations along the liner are predicted to be on the order of 15 cm. This will not result in significant damage to the liner or any loss of containment of process fluids or impounded mill sands slurry. Maximum vertical deformations on the crest are predicted to be less than 25 cm, and the facility will have sufficient freeboard at all times to accommodate well in excess of this.

In conclusion, the FLAC model shows that the unsaturated outer shells of the leach ore embankments provide sufficient strength and stiffness, even after some strain softening, to confine the inner liquefied materials and keep the overall deformations to a minimum.

# 6 CONCLUSIONS

The La Quinua MSSF is a unique design in that it will be located within the limits of the La Quinua HLP and will use heap leach ore, strategically placed within certain lines and grades, to form the containing embankments.

The MSSF has been designed to provide safe, secure, and cost efficient storage for the mill sands, and its embankments have been designed to withstand safely the dynamic loading from the MCE at the site. While the potential for liquefaction or strain softening of some saturated or near saturated zones in the MSSF embankments and mill sands is predicted to occur under the influence of the MCE, modelling has shown that the strategic placement of Zones A and C, B, and D materials into the embankments, in accordance with the material specifications and zone boundaries in the design, will keep any liquefied materials well confined and supported. A dynamic FLAC analysis of the potential deformations of the embankments confirms that the potential deformations will be minimal. Post-earthquake stability analyses using cyclic strain softened and liquefied strengths (not reported herein) also showed that overall stability of the embankments will be maintained after the MCE.

These findings will be confirmed by a comprehensive geotechnical investigation and follow-up slope stability and dynamic deformation analyses of the actual materials placed in the MSSF within the first two to three years of mill operations.

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