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Relating Atterberg limits to rheology

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Abstract

The combination of rheology and soil mechanics is a relatively rare occurrence. Both fields are generally applied in quite separate circumstances, not usually at the same time. However, the transport and storage of thickened tailings slurries has created a situation in which both of these fields do come together. Whilst a thickened tailings slurry is flowing, it can be considered as a fluid. Rheometric equipment can be used to measure its flow properties, and rheological models and theories can be successfully applied to describe its behaviour. Once the tailings slurry is discharged into a storage facility, it typically flows across a 'beach' of previously deposited tailings, and eventually comes to rest. From this point onwards, the tailings particles are often considered as a soil, and the models and theories of soil mechanics then apply in describing the behaviour of the material.

Tailings engineers often measure both the rheology and plasticity (by the use of the Atterberg limits test) of tailings materials in order to design for the transport and containment of the tailings, but sometimes samples are not available for testing, and documented lab test data for only one of these two aspects may exist. This paper presents new empirical relationships for the estimation of one from the other, based on a data set featuring 26 different tailings samples.

Much of the observed variability in rheology can be accounted for by the plasticity of the material.

1 Introduction

Rheology is the study of flowing materials. Whilst rheology may be theoretically contemplated at an atomic scale, it is more often considered in an empirical context, in which the resistance to flow of a fluid at various temperatures, rates of shear and shear histories is used to define the flow behaviour of the fluid, without the need to consider any of the physical aspects of what is actually happening in the fluid at a microscopic scale. Rheology has primarily been exploited in the laboratory characterisation of the flow behaviour of various fluids, through the use of rheometers (also called viscometers). The data gathered by these methods can be of practical value in predicting pumping requirements for viscous fluids such as slurries. It can also be useful in predicting slumping, sheet flow, channel flow and extrusion behaviours of various fluids, particularly those that exhibit non-Newtonian flow characteristics (varying viscosities at different rates of shear and shear histories).

The question of relating slurry concentration and yield stress has presented itself as a challenge to rheologists in the past. It has long been recognised that factors related to particle size and mineralogy have significant influences on the rheology of soil particle slurries, e.g. Sofrá and Boger (2002). Many rheologists have regarded these variations as material specific and only quantifiable by actual rheology testing. Johnson et al. (2000) and Zhou et al. (2001) experimentally investigated the effects of van der Waal's forces and other forces acting on slurry particles at a microscopic scale, but did not go so far as to provide any methods of predicting slurry rheology or yield stresses. Stickel et al. (2006) presented a complex model for the prediction of yield stress using complex finite element computational analysis of the Brownian forces and charged particle interactive forces acting on individual particles in suspension, but the limits of computational power currently prevent their model from being applied to full scale practical situations, due to the large numbers of particles that must be simulated.

This paper approaches the problem with the exploitation of laboratory tests that measure soil strength, which are available to help quantify the effect of soil particle composition on rheology.

The Atterberg limits are a series of three semi-empirical indices that describe the behaviour of a soil. They consist of the liquid limit (LL), which is defined as the moisture content at which a soil changes behaviour from a plastic to a liquid, the plastic limit (PL), which is defined as the moisture content at which a soil changes behaviour from a plastic solid to a brittle solid, and the shrinkage limit, which is the moisture content at which any further loss of moisture will no longer reduce the volume. When the plastic limit is subtracted from the LL, the plasticity index (PI) is the result. This index defines the magnitude of the moisture content range through which the soil may be described as being a plastic solid.

The Atterberg limits are used by geotechnical engineers to describe the behaviour of fine grained soils. Under the commonly used Unified Soil Classification System (ASTM), the Atterberg limits determine whether a fine grained soil should be considered to be a silt or a clay, and define the relative degree of plasticity of the soil. It should be noted that this classification relates to the engineering properties of the soil. It is expected that this classification will generally reflect the particle size and mineralogy of the soil.

The Atterberg limits were originally proposed in 1913 by Atterberg, a Swedish chemist. Atterberg was studying the plastic behaviour of clayey soils at the time, and created two simple tests for measuring the LL and PL for a soil. He also defined the PI at the same time. His test method for measuring the LL of a soil was refined in 1932 by Casagrande, who invented a simple mechanical device called the 'Casagrande cup', which effectively standardised the test method by enabling repeatable results to be obtained (AS 1289.3.1.1–2009). It is noted that it is also possible to measure the LL with a cone penetration apparatus (Atkinson and Bransby, 1978). This test method has also been standardised (AS 1289.3.3.2–2009). Comparisons of the two methods have shown that they give essentially the same results.

All of the results reported in this paper were obtained using the Casagrande method of testing for the LL, and this is therefore of key interest in this paper.

A substantial amount of research into the test has found that a soil sample at the LL exhibits a shear strength of approximately 1,700 Pa (Pandian and Nagaraj, 1990), though others have proposed values as low as 700 Pa (Atkinson and Bransby, 1978) and as high as 2,500 Pa (Pandian and Nagaraj, 1990). For the purpose of this work, 1,700 Pa has been adopted as the shear strength at LL.

This definition for the LL can be exploited for relating the rheological behaviour of slurries to the Atterberg limits. This paper presents new empirical relationships between rheological parameters and Atterberg limits data for tailings slurries.

2 Data set

The ATC Williams database of test results on tailings has been reviewed for cases where data on each of rheology, plasticity (Atterberg limits) and particle size distributions (PSD) are available. A total of 27 samples meeting these requirements were identified, and this forms the basis for the analysis undertaken in this work. These 27 samples cover a range of mineral types, including tailings from hard and soft rock mining, soils and precipitates.

The Atterberg limits specific gravity (SG) and PSD testing for all samples was carried out in compliance with the test methods described in AS1289 (though it is noted that ASTM and other equivalent international standards exist). The specific test methods used are noted at the end of this paper. The PSD testing was done using sieves and a hydrometer. It is noted that in some cases it has been necessary to extrapolate PSD curves in order to estimate the d_{10} particle size. The rheometry was carried out using a Thermo-Haake VT550 rotational viscometer with MV2 bob and cup measurement system. Particular care was taken to avoid particle settlement in the slurry samples. The preparation and mixing of rheology samples was also of prime importance, particularly in ensuring that each sample was pre-sheared prior to testing. The test method used (after sample preparation and mixing) is to pre-shear the samples for a period of 300 sec, then obtain the rheogram by a ramp-down/ramp-up sequence in the shear rate. The test (including the remixing of the sample) is repeated to confirm a consistent result. The reported results are the best fit of the two individual determinations, with additional tests undertaken if the first two do not compare well. The number of concentrations tested for each sample varied from 1 to 5. A total of 103 rheograms were measured for the 27 samples.

The Herschel–Bulkley rheological model was applied to produce a fit curve to each of the rheograms.

The lab results for the 27 samples are summarised in Table 1:

Table 1 Summary of ATC Williams tailings testing data

Tailings Sample Description	Atterberg Limits Data			SG	Particle Size Data				Rheology (Herschel–Bulkley Fit Parameters)			C_w
	LL	PL	PI		d_{90}	d_{50}	d_{10}	% < 20	τ_y	K	n	w/w
	%	%	%		µm	µm	µm	%	Pa	Pa.s^n		(%)
Coal 1	42	29	13	2.0	310	25	0.8	47	2.122	0.252	0.484	38.1
									3.815	0.107	0.680	40.9
									6.399	0.140	0.701	43.9
									8.628	0.151	0.714	46.0
Coal 2	40	24	16	2.0	450	59	0.7	38	10.425	0.043	0.961	48.3
									18.090	0.041	1.000	52.7
Coal 3	48	20	28	2.0	120	2.1	0.4	75	4.135	0.460	0.472	35.0
Coal 4	49	21	28	2.0	100	1.1	0.2	78	6.195	0.589	0.462	35.1
Coal 5	53	23	30	2.0	270	2.5	0.2	70	4.745	0.955	0.413	35.1
									23.420	2.737	0.425	44.3
									34.255	3.415	0.435	47.6
									48.535	6.409	0.400	50.4
Coal 6	40	30	10	2.0	300	23	1	48	2.644	0.034	0.916	45.2
									6.984	0.077	0.863	49.2
									8.704	0.083	0.883	50.1
									11.930	0.100	0.880	51.1
									15.090	0.115	0.881	52.1
Copper 1	24	19	5	2.8	165	33	3.1	40	4.500	0.976	0.428	55.2
									9.537	1.160	0.422	58.2
									14.010	1.901	0.410	61.1
									22.040	3.173	0.396	63.9
									28.550	9.938	0.327	67.0
Copper 2	27	16	11	2.8	250	50	1.9	37	11.365	3.537	0.332	54.1
									15.405	4.999	0.314	56.2
									20.345	6.218	0.324	58.1

Tailings Sample Description	Atterberg Limits Data			SG	Particle Size Data				Rheology (Herschel–Bulkley Fit Parameters)			C_w
	LL	PL	PI		d_{90}	d_{50}	d_{10}	% < 20	τ_y	K	n	w/w
	%	%	%		μm	μm	μm	%	Pa	$Pa.s^n$		(%)
									31.850	7.761	0.333	60.1
									48.000	7.902	0.372	61.9
Copper 3	27	17	10	2.8	190	37	1.9	40	9.154	1.499	0.412	54.2
									12.460	2.502	0.373	56.2
									19.440	2.531	0.412	58.2
									30.780	2.077	0.485	60.1
									50.910	2.069	0.540	62.1
Copper 4	20	17	3	2.8	400	120	3.1	27	4.787	6.379	0.192	57.3
									16.305	2.317	0.419	63.1
									17.490	1.056	0.570	65.0
									22.800	1.336	0.591	68.1
Copper 5	21	16	5	2.8	360	85	2.4	30	2.185	0.556	0.434	57.1
									6.774	0.744	0.516	62.8
									9.672	1.114	0.527	64.9
									16.305	1.555	0.539	67.8
Copper 6	21	16	5	2.8	320	70	2	32	1.467	0.775	0.376	57.1
									6.689	0.714	0.539	62.9
									9.131	1.103	0.528	65.0
									17.400	1.864	0.523	67.9
Copper 7	17	15	2	2.8	400	130	4.3	25	2.921	1.156	0.289	62.5
									6.225	1.283	0.381	65.4
									10.695	0.930	0.526	67.7
									17.360	0.980	0.633	71.0
Copper 8	22	14	8	2.8	350	80	1.4	32	2.130	0.294	0.518	56.9
									7.175	0.382	0.653	62.9
									11.350	0.569	0.646	64.8
									20.935	0.868	0.685	68.1
Copper 9	18	0	0	2.8	300	90	2.3	30	5.967	1.226	0.281	57.1
									16.935	0.132	0.812	63.1
									21.500	0.034	1.035	65.2
									42.520	0.008	1.347	68.2

Tailings Sample Description	Atterberg Limits Data			SG	Particle Size Data				Rheology (Herschel–Bulkley Fit Parameters)			C_w
	LL	PL	PI		d_{90}	d_{50}	d_{10}	% < 20	τ_y	K	n	w/w
	%	%	%		µm	µm	µm	%	Pa	$Pa.s^n$		(%)
Copper 10	19	0	0	2.8	440	140	6	23	18.760	2.051	0.277	63.5
									23.025	0.351	0.657	64.9
									29.150	0.379	0.740	68.1
									30.415	0.382	0.774	70.6
Copper 11	20	16	4	2.8	320	70	1.3	35	1.040	0.329	0.454	57.0
									4.280	0.410	0.596	63.0
									6.936	0.415	0.654	65.1
									13.225	0.906	0.628	68.1
Copper 12	22	16	6	2.8	290	56	1.8	35	0.641	1.255	0.253	52.1
									1.069	2.225	0.236	55.3
									2.837	2.098	0.313	58.4
									8.000	1.430	0.427	61.4
									13.000	2.357	0.415	64.2
Copper 13	28	18	10	2.75	160	34	1.5	40	19.100	36.440	1.000	55.2
									35.500	60.670	1.000	58.4
									61.100	86.000	1.000	61.5
									93.500	130.00	1.000	64.3
Gold 1	22	18	4	2.85	200	29	3.1	42	0.600	0.108	0.744	62.1
									1.500	0.221	0.705	64.2
Lithium 1	22	0	0	2.65	460	180	2	25	1.048	0.254	0.963	60.2
									2.975	0.222	1.007	62.9
									6.571	0.268	1.020	65.3
									11.165	1.515	0.766	68.1
Nickel 1	49	31	18	3.37	9.5	1.8	0.4	91	1.564	0.154	0.486	29.6
									4.363	0.174	0.602	34.8
									11.990	0.511	0.535	40.1
									37.110	2.001	0.450	45.2
Uranium 1	31	26	5	2.7	370	92	2.5	30	2.769	0.065	0.755	48.0
									3.784	0.097	0.742	50.6
									5.997	0.077	0.847	52.9
									9.411	0.081	0.913	55.1

Tailings Sample Description	Atterberg Limits Data			SG	Particle Size Data				Rheology (Herschel–Bulkley Fit Parameters)			C_w
	LL	PL	PI		d_{90}	d_{50}	d_{10}	% < 20	τ_y	K	n	w/w
	%	%	%		µm	µm	µm	%	Pa	Pa.s^n		(%)
Zinc 1	21	21	0	3.4	130	50	4.1	31	9.361	1.161	0.558	64.0
									15.885	1.253	0.577	66.0
									25.115	2.292	0.510	68.0
									35.000	14.720	0.322	70.0
Zinc 2	15	11	4	3.55	130	31	1.8	41	8.855	1.198	0.447	69.2
									17.525	1.815	0.429	72.2
									39.110	1.646	0.507	75.2
									100.26	3.858	0.479	78.1
Zinc 3	18	18	0	2.9	135	37	3	38	0.000	0.869	0.309	60.0
									1.169	0.659	0.441	62.5
									2.676	0.719	0.498	65.1
									8.006	0.890	0.577	67.6
									13.965	1.783	0.541	70.1
Zinc 4	47	22	25	2.83	38	4.8	0.3	77	0.652	0.428	0.259	25.1
									1.110	0.449	0.306	27.5
									2.267	0.300	0.435	30.2

3 Analysis

3.1 Rheological data

A plot of the yield stress against the concentration for each sample is presented as Figure 1. The Bingham yield stress (τ_{yBP}) was selected as the representative yield stress value for each slurry, rather than the Herschel–Bulkley value. This was done to reduce the number of fit parameters from three to two. The tangent point chosen for the Bingham fit line is at the shear rate value of 100 s^{-1}. It is acknowledged that this will result in a yield stress value that will be higher than the actual value, but the shear rate range of interest for most tailings handling applications is between 50 and 400 s^{-1}.

Bingham model parameters can readily be determined from Herschel–Bulkley parameters with the following two equations:

$$K_{BP} = nK_{HB}\dot{\gamma}^{n-1} \tag{1}$$

$$\tau_{yBP} = \tau_{yHB} + K_{HB}\dot{\gamma}^{n} - \dot{\gamma}(nK_{HB}\dot{\gamma}^{n-1}) \tag{2}$$

Where K_{BP} is the Bingham plastic viscosity, n is the Herschel–Bulkley flow behaviour index, K_{HB} is the Herschel–Bulkley consistency factor, τ_{yBP} is the Bingham yield stress, τ_{yHB} is the Herschel–Bulkley yield stress and $\dot{\gamma}$ is the shear rate at which the Bingham tangent contacts the Herschel–Bulkley curve. In the work presented here, a $\dot{\gamma}$ value of 100 s^{-1} has been adopted.

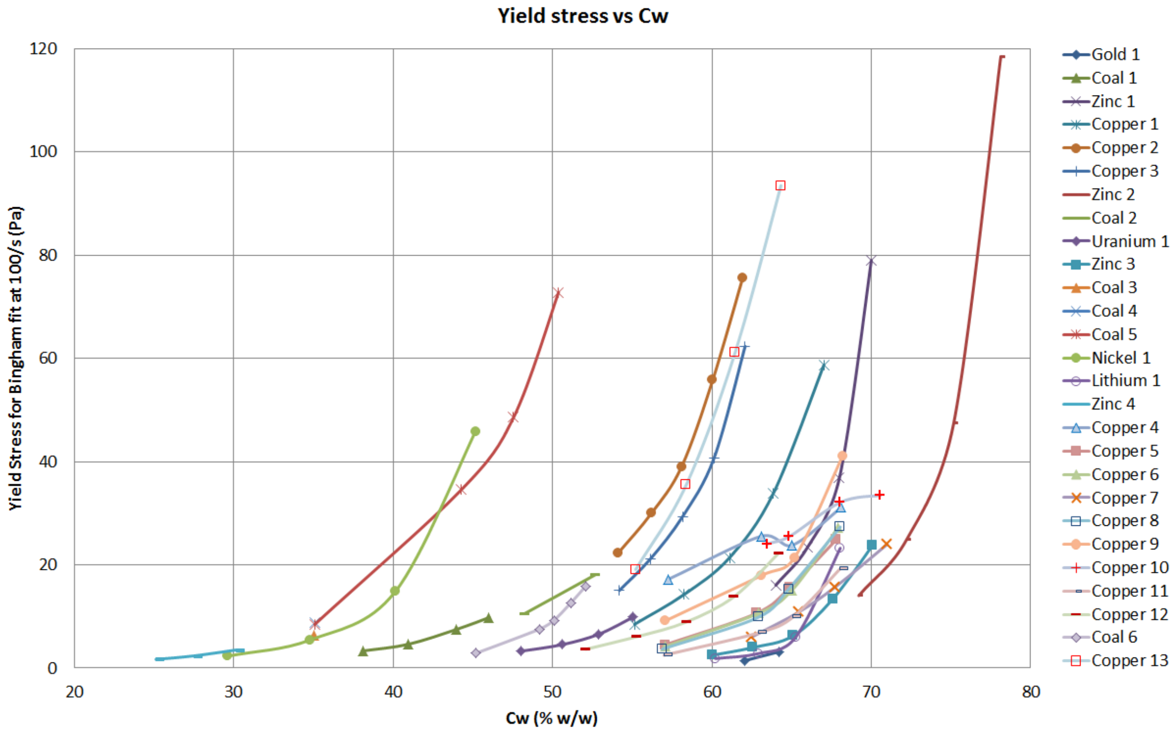

Figure 1 Plot of Bingham yield stress values as a function of concentration

It is noted that this plot is very similar in style to those presented by others, e.g. Sofrá and Boger (2002).

3.2 Yield stress/LL criterion

The assumption that the yield stress of the slurry at the LL is 1,700 Pa allows an additional point to be plotted on all yield stress plots.

Care must be taken not to confuse the definitions of soil moisture content and slurry concentration. The soil mechanic definition of (gravimetric) moisture content (w) is as follows:

$$w = \frac{W_w}{W_s} \tag{3}$$

Where W_w is the weight of water present in a soil sample, and W_s is the weight of the soil solids in the sample. Note that this is typically expressed as a percentage, rather than a ratio as expressed in Equation (3). Consequently, the conventional definition of LL is in terms of gravimetric moisture content as a percentage.

Chemists, rheologists and process engineers define C_w (the concentration by weight) as follows:

$$C_w = \frac{W_s}{W_s + W_w} \tag{4}$$

For the LL to be compatible with the concentration figures used by rheologists, it is necessary that the gravimetric moisture content be converted to its equivalent concentration. This can be achieved with the following equation:

$$C_{wLL} = \frac{1}{1 + LL/100} \tag{5}$$

Where C_{wLL} is defined as the concentration by weight at LL, as a ratio.

By calculating the equivalent concentrations of the moisture contents at LL, these points can now be added to the yield stress plot as an additional point for each slurry, with all of them having the yield stress of 1,700 Pa. See Figure 2 for this plot. This in itself does little to reduce the scatter of curves.

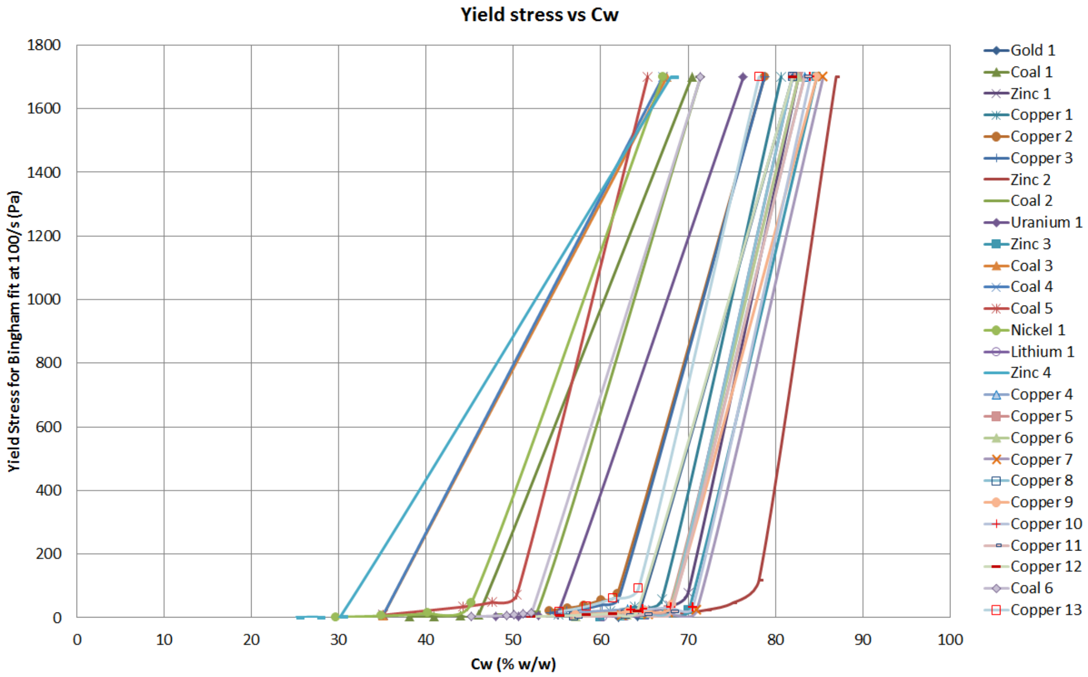

Figure 2 Plot of Bingham yield stress values as a function of concentration

3.3 Normalisation process

The concentration values for all points can be normalised by dividing each concentration value by the LL concentration (CwLL). This creates a common point on all of the curves, running through the LL.

The interesting result of this normalisation is that the yield stress data for each slurry, when plotted on a log scale against the range of normalised concentrations tested, generally falls on a straight line radiating out from the focal point (Figure 3). There is clearly less scatter in this normalised plot compared to the raw results (Figure 1).

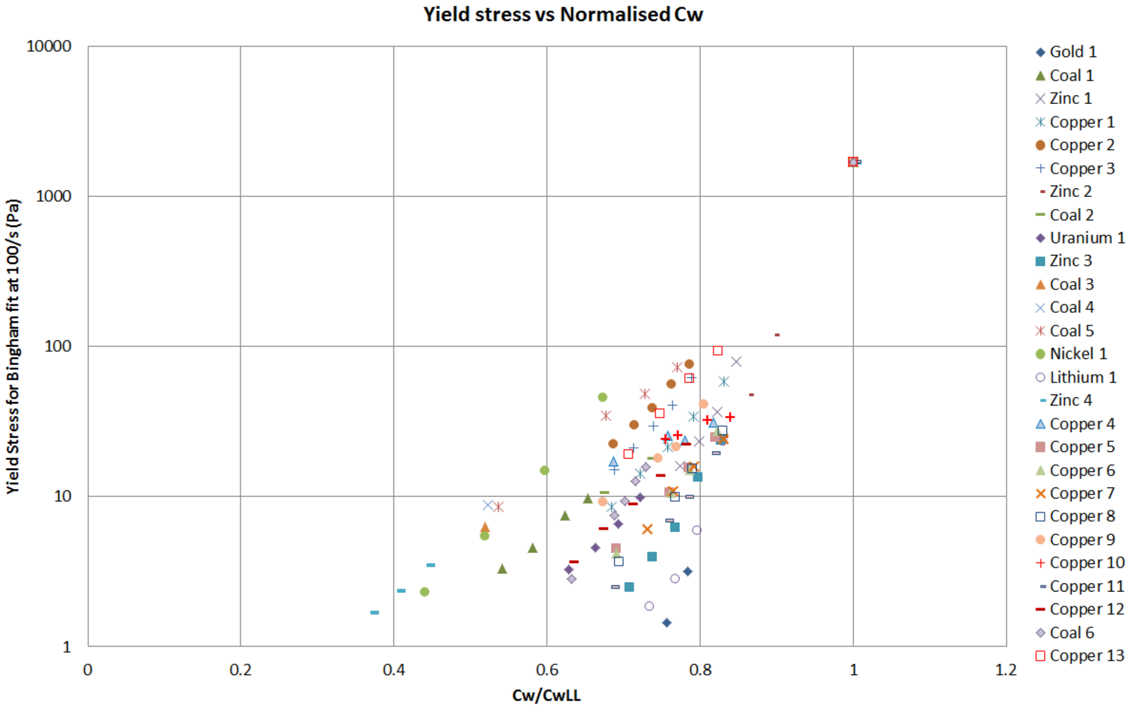

Figure 3 Log scale plot of Bingham yield stress values as a function of normalised concentration

3.4 Particle size effect

The scatter still apparent in Figure 3 indicates that the LL alone does not provide a full solution. It is postulated that finer grained materials will be likely to have a higher clay content, which would be expected to exhibit greater shear stress at lower concentrations. To this end, further analysis found that a correlation exists between the particle size and the slope of a line radiating out from the LL point on Figure 3. The diameter of the 10th percentile particle in a slurry (d_{10}) was found to show the strongest correlation amongst the four PSD statistics that are presented in the data table. This finding is consistent with the literature and previous experience, where it is generally agreed that it is the fine fraction of a slurry that has the greatest impact on the rheology.

This trend has been graphically presented in Figure 4, with each of the radial lines indicating the fit for the yield stress behaviour of slurries of various d_{10} values.

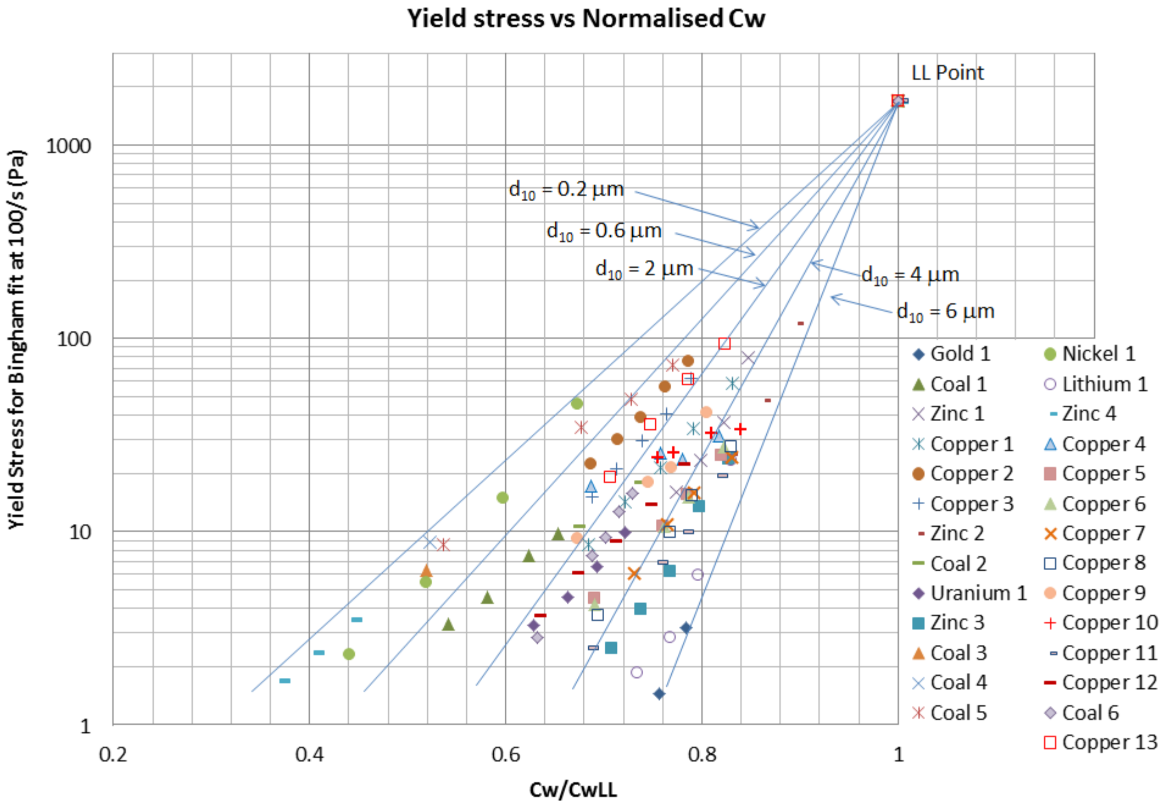

Figure 4 Influence of particle size

Figure 4 provides a powerful tool for enabling the Bingham yield stress of a slurry at any given concentration to be predicted from the LL and d_{10} particle size.

4 Applications

4.1 Prediction of Bingham plastic viscosity

From the data set presented in this work, an empirical method of predicting the Bingham plastic viscosity (K_{BP}) from the Bingham yield stress is presented in Figure 5.

The fit equation to this data can be used to predict the Bingham plastic viscosity as a function of the Bingham yield stress. These two graphs therefore enable a Bingham model linear rheogram to be predicted for any tailings slurry for which the LL and d_{10} is known.

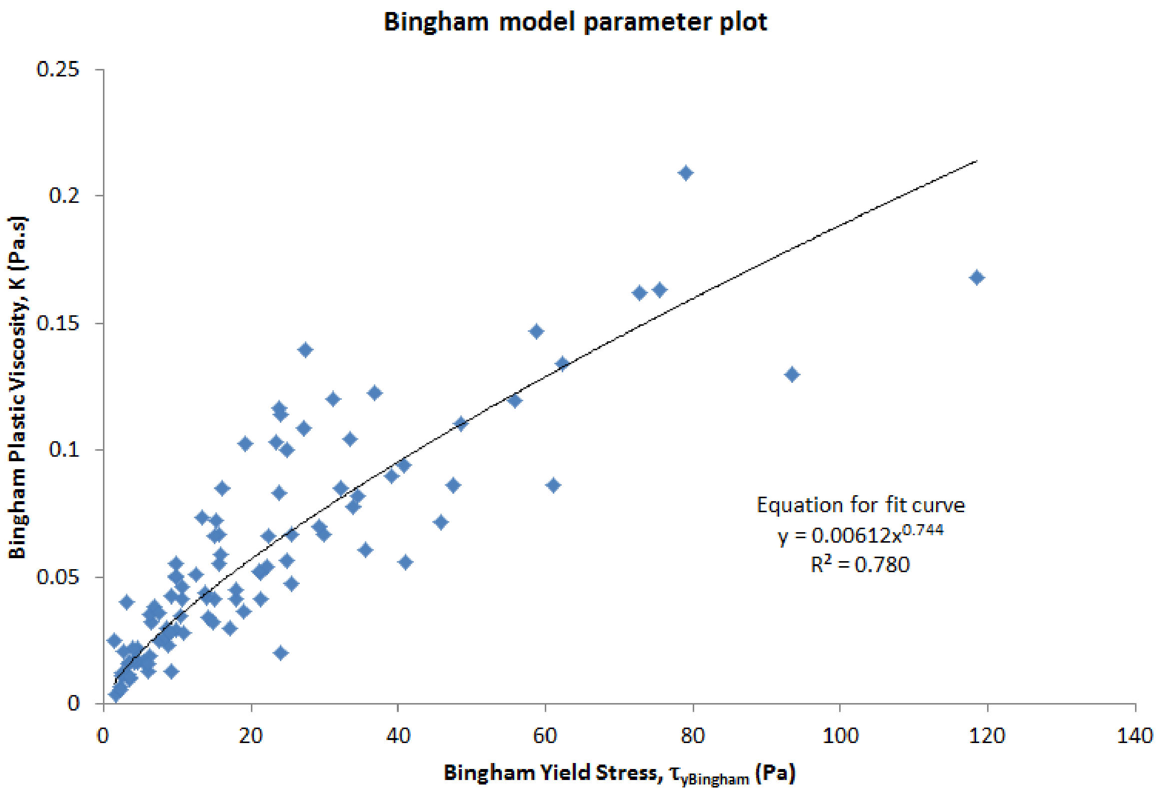

Figure 5 Correlation of Bingham yield stress points against corresponding Bingham plastic viscosity data, with empirical fit inscribed

4.2 Rheology prediction example

In the early stages of project studies, it is not uncommon to have only limited data available for tailings properties. In preliminary studies where only PSD and plasticity data are available, a preliminary estimate of rheology may be estimated as shown in this example.

An example of this empirical prediction method is presented for the prediction of Bingham model rheology data for a hypothetical tailings slurry:

Tailings parameters: $d_{10} = 0.6\ \mu m$; LL = 45%. We want to predict the Bingham model parameters for slurry concentrations of 0.4, 0.5 and 0.6 (w/w).

First, the C_{wLL} must be calculated. This is done using Equation (6).

$$C_{wLL} = 1 / (1 + 45/100) = 0.69 \text{ (or 69\%)} \tag{6}$$

Now the proposed slurry concentrations of 40, 50 and 60% must be normalised to produce C_w/C_{wLL} figures. This involves dividing each concentration by C_{wLL}:

$$40/69 = 0.58 \quad 50/69 = 0.72 \quad 60/69 = 0.87 \tag{7}$$

The Bingham yield stress corresponding to each of these values can now be read off Figure 4. Since the tailings has a d_{10} value of 0.6 μm, the 0.6 μm line is used. The three yield stress values obtained are 7.5 Pa, 43 Pa and 320 Pa.

Now the empirical fit presented in Figure 5 can be used to predict the Bingham plastic viscosity for each point. The fit equation obtains K_{BP} values of 0.027, 0.100 and 0.447 Pa.s respectively.

4.3 Interpolation and extrapolation of rheology data

Another common problem is the case where rheology tests are available, but do not extend to the slurry concentrations of interest. The pronounced non-linearity of rheological parameters with increasing concentration (as illustrated on Figure 1) has always presented a problem with respect to extrapolation of results from low concentrations to high values. The normalisation process (Figure 3) provides a rational approach to this problem.

If the rheology of a slurry had been measured for one or more concentrations, a normalised plot (as per Figure 3 or Figure 4) could be prepared, upon which the rheology for any other concentration could be predicted.

If no rheology and no PSD data were available, the plot still serves in indicating a likely range of concentrations for any desired yield stress value (for example, if a yield stress value of 100 Pa was desired, the plot indicates that a C_w/C_{wLL} range of 0.75–0.90 would be expected).

Whist this scenario is less likely, Figure 4 could (in principle) also be used for predicting the Atterberg limits for a tailings slurry when only the rheology data exists, so long as yield stress measurements exist for two or more concentrations of the slurry. A trial and error approach is used to guess various values for the LL. The predicted LL value is reached when the normalised yield stress points line up with the focal point.

The predicted LL value can then be used in turn to predict a PI value, based on an empirical fit presented in Figure 6.

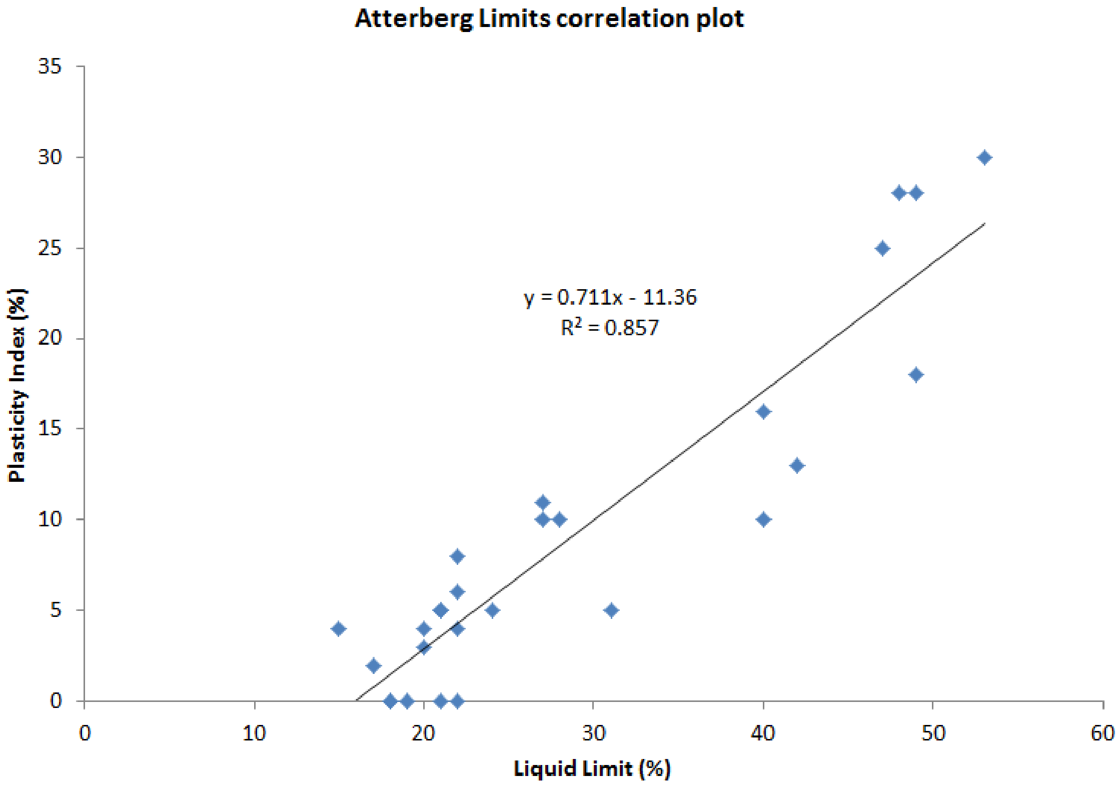

Figure 6 Correlation of LL data against corresponding PL data, with empirical fit inscribed

5 Conclusion

This paper presents a useful data set containing SG, partial Atterberg limits, PSD and rheological fit parameters for a range of 27 different tailings slurries. From this data, an empirical method has been presented that enables rheology data to be predicted for a slurry when only the Atterberg limits and PSD data is known. A reciprocal method of predicting the Atterberg limits based on the rheology has also been presented.

Like the slump test, the LL test provides another method of evaluating the yield stress of a material. The slump test was commonly used for many decades in the concreting industry before rheologists realised its application to yield stress measurement. The LL test has been widely used by geotechnical engineers for many decades, but the crossover benefits to rheologists has not been realised. Unlike the slump test, which only measures the yield stress of a sample of a material, the LL test is a more onerous test that seeks to determine the moisture content of a soil at which it starts to exhibit liquid behaviour. This result has a powerful ramification for rheological applications, which is exploited in this paper.

Acknowledgements

The many years of work by Peter Lam and John Walker in the ATC Williams laboratory are gratefully acknowledged, particularly in light of the requirement for consistency of the testing methods and technique in the lab results used in this work. Numerous engineers at ATC Williams also deserve recognition for their significant (and essential) part in the grander scheme of acquiring and managing the clients that have supplied the tailings slurry. In particular, Paul Williams, Steve Murphy and Trevor Osborne are acknowledged for their long standing service to ATC Williams, and to industry at large, together amassing more than 120 years of experience. Recognition also goes to Winthrop Professor Andy Fourie of The University of Western Australia for first suggesting this channel of inquiry.

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