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Comparison of predictions of beach slopes using alternative models

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Abstract

Beach slope prediction for thickened tailings is one of the key aspects to be determined by specialists before the development of the engineering phases for large scale copper sulphide ore exploitation in northern Chile. Although the main reasons for implementing thickened tailings technology in Chile are the lack of water and the high cost of bringing it from the sea, beach slope has a significant impact on the volume and cost of tailings embankments in steep terrain such as that in the Andes.

Some prediction methods assume that the beach slope equates to the slope where erosion and sedimentation reaches equilibrium. Conversely, McPhail relates the beach slope profile to 'stream power' whilst others have established models based on limit equilibrium conditions and energy conservation. Since there are no projects of over 100 ktpd in steady operation anywhere in the world at this time, these methods have not yet been proven suitable for predicting slopes for large-scale operations.

This paper compares beach slope predictions using the most common methods with results obtained from extensive pilot Plant trials involving deposition cells and flumes for a mining company in northern Chile. The paper discusses differences between the findings of each method and concludes with a commentary on the factors that may ultimately influence the actual beach slopes that will be obtained in practice. Finally, we provide recommendations for the design of tailings facilities given the absence of a proven prediction method.

1 Introduction

Chile has been the largest copper producer worldwide for decades. Many of the major copper ore mining operations in Chile are located in remote areas with extreme aridity and at high elevation. The scarcity of water and the steep terrain combine to make tailings disposal a significant proportion of mining costs; when coupled with the increasing cost of energy, these costs have made paste and thickened tailings (P and TT) a solution with significant potential benefits even when this technology have not been successfully proven for large scale operations (100 ktpd or more).

However, most of the research and practical experience associated with P and TT has been derived from relatively small operations, making the application to the production rates in Chile problematic.

The parameters required for the design of large operations are derived from pilot-scale tests that are assumed to reproduce the full scale conditions or at least form the basis for extrapolation to full scale.

During the development of a complete pilot plant campaign, tests involving thickening of the tailings are carried out along with transport and deposition tests. This paper focuses on the tests aimed at deriving the beaching-related parameters required for design purposes. Although a variety of methods exists for scaling up pilot-scale beach profile measurements, there is still no field-scale verified methodology.

This paper compares the results derived from two of the most often cited methodologies that use fundamentally different approaches to predicting beach slope. McPhail (1995, 2008) uses the principle tailings stream power and kinetic energy dissipation as the particles move down the slope of the beach to predict the settled slope, whilst Fitton et al. (2008) use a method based on the equilibrium state between the erosion and deposition of the slurry that is assumed to flow in a channel.

A third approach that has attracted interest uses lubrication theory to predict beach slope. Continuity and momentum equations have been used to predict slopes in small-scale flumes (Simms, 2007); however, the application to full scale has been discounted by some authors (Fourie and Gawu, 2010; Simms et al., 2011). The predictions using this method have produced good correlations with small scale tests and with the early stages of the deposition due to the fact that changing layer thickness and boundary conditions associated with run out of the leading edge of the deposition front of the tailings have a significant influence on equilibrium at the leading edge. The approach has promise in predicting the equilibrium conditions where the stream energy has been dissipated and the tailings stall on the beach.

This paper describes the two most commonly cited methodologies for estimating beach slopes and applies the results obtained during a pilot plant test campaign for a major operation in northern Chile. The discussion compares the methods and their application to raw data and concludes with recommendations on the use of these methods for estimating beach slope for large-scale deposition rates.

2 McPhail beach slope prediction methodology

This approach to the beach profile prediction incorporates entropy and stream power. This methodology is based on the following observations:

- Many natural phenomena can be described using entropy maximisation; hence the methodology can be applied with a high degree of representativeness to the natural deposition of slurries.
- The stream power represents the energy per unit of time consumed by a particle moving through the beach; since the discharge rate remains essentially constant over time, the stream power at any point along the beach should be consistent over time.

The application of the Equations of equilibrium requires the following information:

- The elevation of the beach at the discharge or start of the beach.
- The length of the beach.
- The slurry density.
- The discharge flow rate.
- An estimate of the stream power at the beginning of the beach.
- An estimate of the initial beach slope.

The first four requirements are easy to satisfy, whilst the last two must be estimated from small-scale tests.

McPhail (1995) notes that initial stream power is not equivalent to the power of the fluid exiting the pipe since conditions at the discharge (such as hydraulic jumps) dissipate part of the energy remaining in the pipeline. These conditions can change the flow regime from subcritical to supercritical and vice versa in the channel that usually forms at the point of discharge from the plunge pool. The prediction of these conditions from first principles is difficult; hence small-scale beach slope simulations can be used to estimate the flow curve for segregating slurries. This information is used to predict the initial slope of the beach at an industrial scale.

The application of this method of prediction for industrial scale beaches can be accomplished by following the steps that follow:

1. Do simulations of beach deposition at different densities, covering the range of densities expected for the project. Adjust resulting slopes using Equation of equilibrium.
2. Calculate the velocity for the known stream power and flow rate.

3. Plot shear stress-shear rate for each density down the beach.
4. Using data from density, flow rate, pipe diameter and residual pressure at industrial scale, along with the equations of the normal hydraulic for the plunge pool for initial stream power, estimate for the density and flow rate of operation.
5. Calculate the flow rate at the outlet of the plunge pool.
6. Calculate the initial slope.
7. For a given length L of the beach the beach slope can be estimated. Shear stress and shear rate can be calculated using the energy equation for open canal flows. If the generated rheogram does not fit the simple actual rheogram, then a different length must be chosen. This way calculations must be iterated for L until convergence with the rheogram is achieved. Alternatively, L can be fixed and the height of the slope varied until convergence is achieved.

3 Fitton beach slope prediction methodology

The second model for calculating the slope was developed by Fitton (2007). Fitton's model integrates sediment transport in non-Newtonian fluid flow, assuming that the maximum slope that can be achieved is equal to the slope at which the tendency to erode and settle by sedimentation reaches an equilibrium.

The equilibrium state is characterised by the minimum transport velocity of the tailings.

Based on the assumed canal geometry, the slope can be determined using the following steps:

1. Assume channel depth, p.
2. Calculate velocity as V = Q/A, where the parabolic shape width is assumed to be 5.5 p.
3. Calculate the hydraulic radius and then velocity Vc using the equation for segregating or non-segregating slurries. Repeat steps 1 and 2 until V = Vc.
4. Determine Reynolds number using the definition for a Herschel-Bulkley fluid.
5. Then, Reynolds number is used for calculating the friction factor from the Colebrook-White equation.
6. Lastly, the slope for uniform flow channel is calculated.

Fitton warns that this method does not consider the concavity in P and TT deposits that has been observed in practice, noting that the non-linearity of the slopes recorded is due to variations in the parameters characterising the tailings that are being deposited.

Thus, Fitton integrated concavity into his method by separating the length of the beach into thirds, in which the discharge rate parameters and solids concentration vary according to an assumed normal distribution. The slope of deposition is proportional to the concentration of the tailings while being inversely proportional to flow rate. Therefore, the steepest slopes are obtained for the lowest flow rates and the highest concentrations of tailings.

4 Tailings characterisation

During the development of pilot plant testing for a mining company in northern Chile, tests were carried out in order to derive relevant parameters for the design of the thickening, transportation and disposal of thickened tailings at a rate exceeding 200,000 dry tons per day.

Thickening tests involved the use of three pilot-scale high density thickeners, which were operated to produce a range of concentrations for the tailings sample.

Transport testing was used to establish parameters for gravitational and pressurised flows and for a range of concentrations and flow conditions. Tests were conducted in an open channel as well as a pressurised pipe loop. The deposition trials were performed in a flume as well as a paddock designed to suit the requirements of the McPhail methodology.

The table below shows the parameters derived from tailings characterisation tests.

Table 1 Tailings characterisation

Parameter	Units	Value
Specific gravity		2.63
D_{90}	µm	326
D_{80}	µm	221
D_{50}	µm	86
-38 µm	%	31.5

Very high variability of the rheological data obtained in the pilot plant testing can be seen in the figures below. The data recorded during the study is presented, these rheological measurements were derived from a rheometer using concentric cylinder device (Bob and Cup) and not by measuring yield stress with the method developed by Sofra et al. (2007). Particle size distribution was made using wet sieving according to ASTM standards.

The graphs in Figures 1 and 2 show the curves obtained for both the plastic viscosity and yield stress.

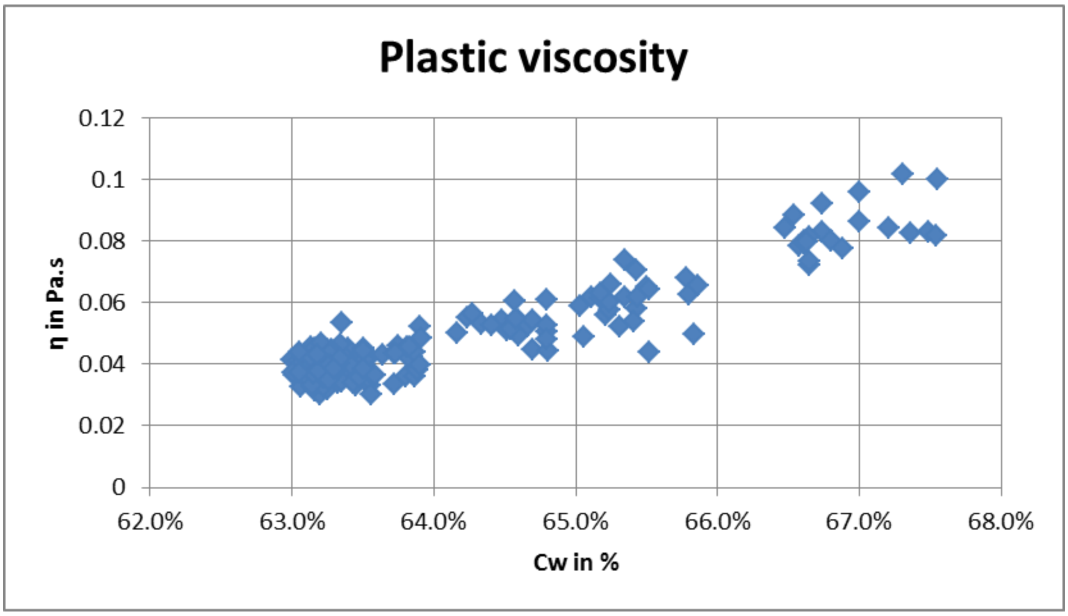


Figure 1 Plastic viscosity versus solids concentration by weight

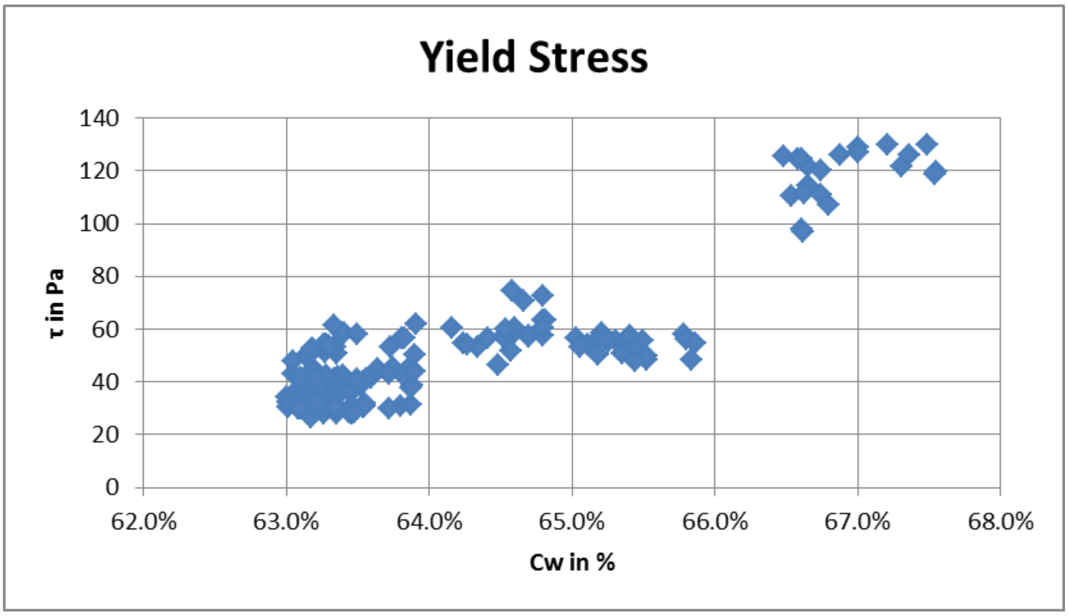


Figure 2 Yield stress versus solids concentration by weight

5 Results

The results of the predictions of the tailings beach profile are presented based on the following assumptions and conditions:

- Constant flow of dry solids 113.4 kg/s.
- Derived topography of the deposit.
- Discharge streams do not merge.
- Coefficient of variation of the flow rate in the discharge equal to 10%.
- Coefficient of variation of the concentration of solids in the discharge equal to 5%.

Figures 3 and 4 show the results of the application of the McPhail and Fitton methods for two different solids concentrations. The averages are shown in Table 2.

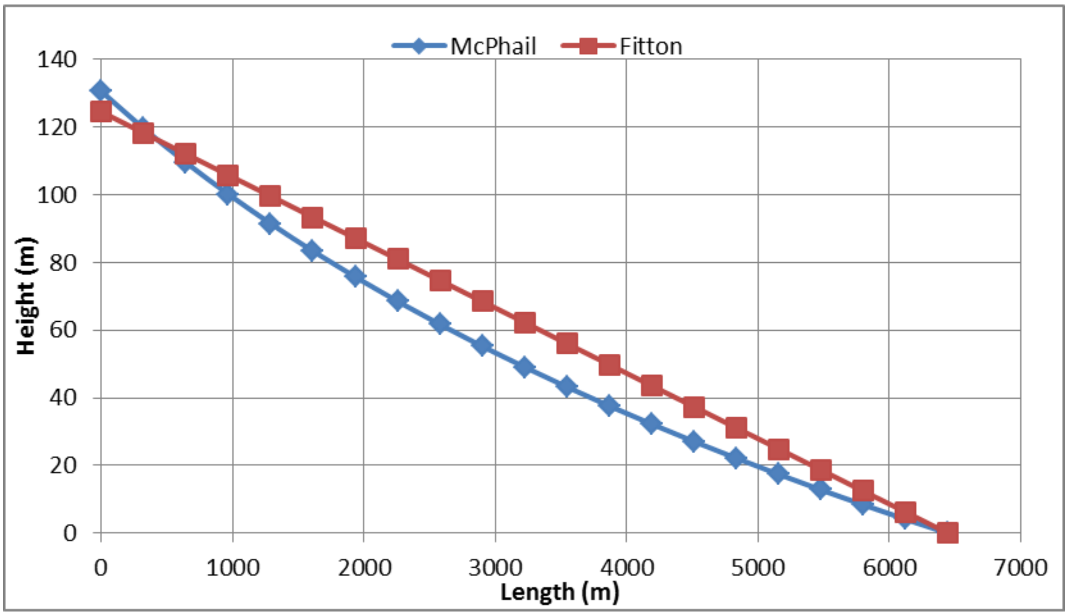


Figure 3 Comparison of prediction of beach slope at Cw = 65%

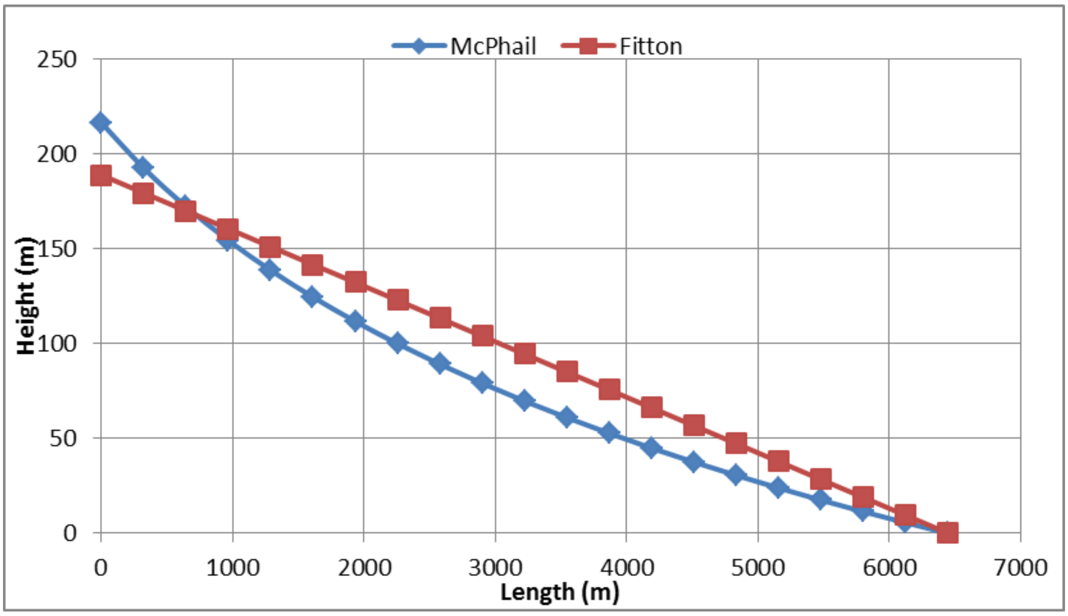


Figure 4 Comparison of prediction of beach slope at Cw = 68%

Table 2 Slope prediction results

Methodology	Cw (%)	Mean slope (%)
McPhail	65%	2.03%
Fitton	65%	1.94%
McPhail	68%	3.36%
Fitton	68%	2.94%

The results of the two methods correlate well with respect to average slope; however, there is a significant difference in the concavity predicted by the two methods.

Figures 5 and 6 show a reworking of the results to include the adjustment for concavity proposed by Fitton. Table 3 compares the averages.

Since the pilot plant tailings are produced at a fixed solid content, we have assumed a coefficient of variation of 10% and 5% for flow rate and solids concentration by weight, respectively in order to adapt the Fitton predictions. These coefficients are the same as those Fitton derived from operational data from five mines around the globe in 2011.

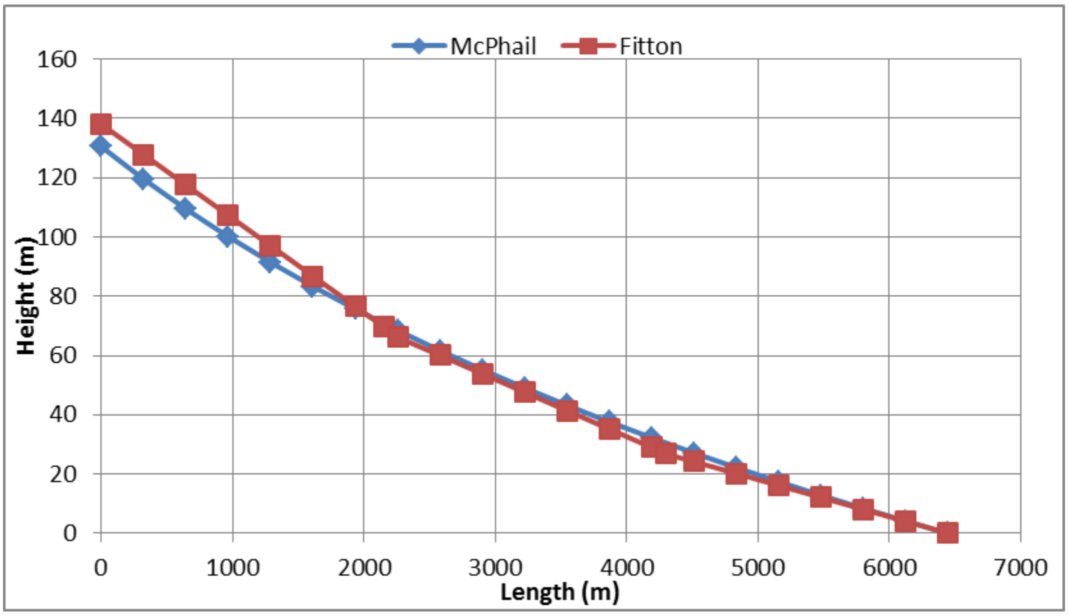


Figure 5 Comparison of beach slope prediction after adjustment for concavity at Cw = 65%

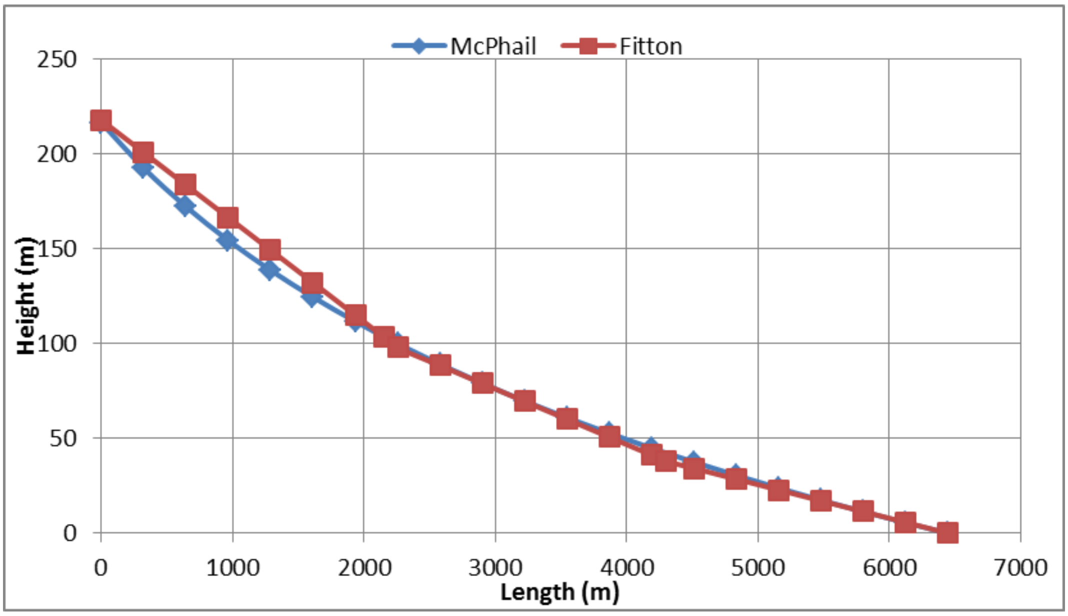


Figure 6 Comparison of beach slope prediction after adjustment for concavity at Cw = 68%

Table 3 Slope prediction results after adjustment for concavity

Methodology	Cw (%)	Mean slope (%)	Concavity (m)
McPhail	65%	2.03%	16.2
Fitton Upper	65%	1.25%	
Fitton Middle	65%	1.93%	14.6
Fitton Lower	65%	3.18%	
McPhail	68%	3.36%	38.6
Fitton Upper	68%	1.77%	
Fitton Middle	68%	2.93%	24.9
Fitton Lower	68%	5.32%	

6 Discussion and conclusions

The methodologies described in this paper are based on fundamentally different models that use different parameters. Notwithstanding these differences, both methods have been refined over time to the point where they can be used to provide an approximation of the beach slopes that will be obtained in the field at industrial scale.

There are a great number of parameters required for the calculation, and a great amount of work is required to obtain them. Given these restrictions, the calculation is only affordable for companies with sufficient resources, in contexts where the scale of the projects justifies obtaining the sample sizes required for the tests.

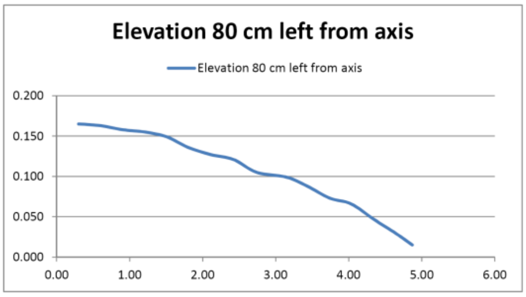
Since this paper describes a project that has no equivalent elsewhere in the world, it has not been possible to compare the predictions with field data to provide for a more grounded prediction of the outcome that could be expected at full scale. For the purposes of scaling up to full scale it is therefore necessary to recognise that the actual slopes may differ substantially from those predicted by application of theory to the pilot scale results and to provide for this contingency in the design.

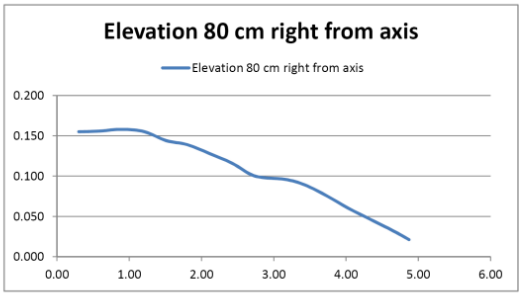
Notwithstanding the problems associated with projecting the results to full scale, it has been informative to compare the results of the two prediction methods. McPhail's method requires more inputs for calibration, including the three-dimensional topography of the trial deposit. Figure 7 shows an original topographic survey from one of the McPhail tests conducted during the campaign. McPhail also reproduces the natural concavity that has been observed in thickened tailings deposits, thus producing a prediction that best replicates the results observed in the field. Fitton's method, on the other hand, requires fewer parameters and does not depend on a pilot-scale test result for calibration. The results also correlate well with the McPhail predictions, save for the concavity predicted by McPhail.

Distance from discharge point [m]	Elevation 80 cm right from axis
0.30	0.155
0.60	0.156
0.90	0.158
1.21	0.155
1.51	0.144
1.80	0.139
2.11	0.128
2.42	0.116
2.74	0.100
3.15	0.096
3.43	0.089
3.74	0.075
4.03	0.060
4.34	0.046
4.65	0.032
4.87	0.021

Distance from discharge point [m]	Elevation from axis
0.30	0.174
0.60	0.174
0.90	0.173
1.21	0.163
1.51	0.153
1.80	0.143
2.11	0.136
2.42	0.121
2.74	0.116
3.15	0.104
3.43	0.095
3.74	0.077
4.03	0.066
4.34	0.050
4.65	0.019
4.87	0.006

Distance from discharge point [m]	Elevation 80 cm left from axis
0.30	0.165
0.60	0.163
0.90	0.158
1.21	0.155
1.51	0.149
1.80	0.136
2.11	0.127
2.42	0.121
2.74	0.105
3.15	0.099
3.43	0.088
3.74	0.073
4.03	0.066
4.34	0.047
4.65	0.029
4.87	0.015





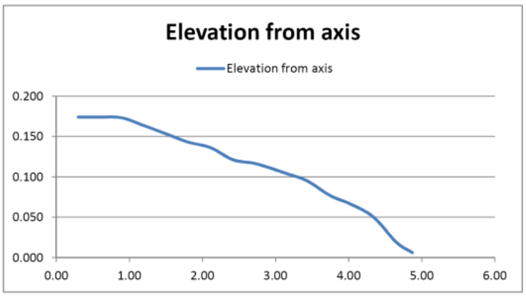


Figure 7 McPhail beaching trial topographic survey data

It should be noted that thickened tailings are normally non-segregating; therefore, one would not expect concavity. The authors' observations concur with Fitton's finding that concavity is explained best by variations in the slurry yield stress, variations that lead to the formation of flatter slopes from low yield stress slurry and steeper slopes from the higher yield stress slurry. This explanation provides a basis for predicting the slope based on a range of yield stresses and flow rates that are expected in practice. Hence one is able to differentiate between the slope that would be derived from the wide range of yield stress that would typically arise from a thickener, and the narrower range that would arise from filters.

Prediction of beach slopes remains more of an art than a science. There have been no large-scale operations against which to calibrate the models that are currently in use. The results of predictions based on these models should therefore be used for guidance only. This recommendation is supported by the review of current practice published by Jewell (2012). Some practical observations based on the author's experience are set out below.

The beach forms at the point that the tailings slurry stalls at the end of its run. This phenomenon, observed in practice in all instances, is poorly described by both the models considered here. Researchers and modellers are urged to study the behaviour of the tailings slurry at the point where the beach is formed (not at the point of discharge or along the beach where the slurry is in motion) to better understand equilibrium behaviour and thereby provide the basis for a predictive tool that uses fundamental geotechnical and rheological slurry properties to predict slopes.

The tailings slurry generally flows in channels to the point where it begins to fan out and slow down. The models reviewed in this paper describe this process but do not describe the equilibrium conditions that occur at the end of the run out when the tailings stalls.

If beach slopes are to be produced and maintained to meet a design target, then the distribution system must be designed to split the flow into separate discharges that are sufficiently spaced to minimise coalescence over the larger part of the beach. The selection of discharge rate and spacing is also more of an art than a science. Designs should therefore be flexible in order to enable the operational personnel to find the best way to manage discharge so as to achieve the target objectives. Observations at Bulyanhulu in Tanzania suggest that discharge rates be limited to no more than 3,000 dry tons per day per discharge point to obtain the steepest beach slopes possible.

Layer thickness plays an important role in optimising beach slopes. The steepest slopes will be obtained with the thinnest layers. In practice, layer thickness cannot be controlled everywhere on a deposit, and depends on the footprint shape and topography. Nevertheless, consideration should be given to this important variable in order to maximise beach slope and to achieve the required degree of consolidation and, if necessary, desiccation of the tailings.

In this case study the variability of results obtained for the rheological properties was particularly concerning and raises a question about the repeatability of the results. Although it is likely that the sample had been well mixed for the tests and that ore variations can be eliminated as an explanation for the variability of the properties. Little is known about the effects of reusing the same sample a number of times, each time with the addition of flocculants. Uncertainty also surrounds the effectiveness of shearing prior to each of the trials. These two aspects could explain the variability encountered in the trials.

McPhail and Fitton’s methods give similar results for the average beach slope and for concavity after adjustment of the Fitton methodology. Since concavity is an important determinant of the volume stored in the basin for a given discharge arrangement, it is important to be able to predict the concavity that has been observed in practice.

McPhail’s method requires many parameters and sufficient samples to carry out tests from which to obtain the required parameters, making it less feasible for small projects and for projects in the early stages of project development.

Fitton’s method, on the other hand, requires little information, making it a good approach to guide design in the early stages of project development. Fitton’s adjustment for predicting the concavity in the slope is based on a coefficient of variation derived from information from five operations around the world. This provides a starting point, but requires validation from large-scale operations to improve reliability.

Implementation of pilot trials for the generation of reliable data for design is not practical for most Greenfields projects and is not affordable for small projects. However, these tests are beneficial for calibrating the models used for beach prediction, and are recommended where the scale of the investment justifies the additional insight that the trials provide. Trials are particularly useful where the properties of the tailings are atypical and where the cost of the deposition facility will be sensitive to variations in the beach slopes.

The authors have found through field observation that beaches form when the tailings fans out and stalls. Layer thickness also plays an important part. Neither of the two methods evaluated model the observed mechanisms well. It is therefore recommended that further research be done to better understand the equilibrium conditions pertaining during beach formation with a view to finding a method to facilitate slope prediction from first principles using the tailings properties.

The representativeness of the ore from which the tailings for the trials are produced should always be examined. The impact of sample reuse and completeness of shearing can be significant. It is therefore recommended that these two aspects be carefully considered and managed for future trials.

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