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Design of open channels for non-Newtonian fluids

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Abstract

The use of pipes for transporting non-Newtonian fluids has been widely studied and applied, and it can be said that a practical engineering level of knowledge and understanding has been reached for designing these types of systems. Useful design guides can be found for the design of pumped slurry systems. However, the scenario for open channels is different. Despite the fact that this topic has been studied by several investigators, it is difficult to find a clear explanation of the available methodology that establishes the steps necessary for designing non-Newtonian flow in open channels, delineating their range of application and presenting recommendations or comparisons as to which model would be better to use, depending on the particular application.

To help present guidance for design engineers, a range of published experimental and operational results have been compiled from existing literature to allow categorisation of the different flow behaviours of non-Newtonian flows in open channels. Due to the nature of non-Newtonian flows, particularly those with yield stresses, there are analytical and practical limits for the dimensions of open channels. Analysis of the initiation of movement for slurries that exhibit a yield stress indicates that there are restrictions on the hydraulic radius to width ratios for open channels. Furthermore, since it is often desirable to operate industrial conduits transporting non-Newtonian slurries for stability and economic reasons near the transition from laminar to turbulent flow, it is important to accurately define the design of these channels near this transition point. The combination of the dimensional restrictions, together with the operational limits observed from the data, has been used to establish guidance for the dimensioning of open channels to convey non-Newtonian fluids. Following the proposed design guidelines, a sensitivity analysis has been performed to assess the impact of different rheological properties on the resulting open channel design. This paper will present the background to the analysis undertaken, the developed guidelines for open channel design, and the results of the sensitivity analysis.

1 Introduction

The main objective of this paper is to define a basic basis for the design of open channels that conveys non-Newtonian slurries in process industries based on information from the literature. Technical recommendations will be provided for a consistent design methodology that can be utilised as a first approximation of the design.

The design recommendations will be focused on two key issues that should be always considered in the design of an open channel that conveys non-Newtonian fluids. These are the hydraulic characteristics of the flow (flow depth and flow velocity) and its flow regimes determined by the transition velocity between laminar and turbulent flow. These characteristics determine the capability of the proposed system to convey the slurry, and whether or not deposition may occur that could lead to plugging the system or unacceptable hydraulic performance (overtopping, excessive spills and splashing).

2 Study approach

Typically, for heterogeneous and settling slurries the channel designs are developed under a turbulent regime, because the laminar regime may present risks of solids segregation. In this sense, the designs have usually been based on either the Manning or Chézy equations for modelling turbulent flow of low concentrated slurries, with some success. However, this criterion would be incorrect for highly concentrated slurries where the flow in the channel is no longer turbulent.

Several authors have adapted existing models or developed new models for non-Newtonian fluids to determine flow depth, flow velocity or friction factor for a given geometry and slope. Since these parameters constitute the basis for defining the section dimensions of a channel, choosing the most appropriate open flow model within the design stage is essential to reduce the risks of exceeding the channel capacity.

On the other hand, the selection of the proper model also depends on the flow regime, which may be laminar or turbulent. For water in an open channel, the transition zone corresponds to Reynolds numbers in the range 2,000 to 2,400. However, according to the experimental data, for non-Newtonian fluids the transition zone definition is not that clear.

The most relevant theoretical aspects between models will be reviewed further on.

2.1 Literature review

Alderman and Haldenwang (2007) give a good summary of existing models and also give a critical review of the work carried out on Newtonian and non-Newtonian flow of pseudo-homogeneous non-settling slurries in open channels, for both laminar and turbulent regimes.

From the comparison of the models, it is clear that further research is needed, since the differences between the predicted and actual velocities are not fully explained.

Every flow model used estimates either the Fanning friction factor or the average velocity. Table 1 shows the applicability of each these models depending on the rheological features of the slurry.

Table 1 Flow models applicable for different rheological models

Regime	Model	Herschel & Bulkley	Bingham plastic	Power Law
Laminar	Hanks	☑	☑	☑
	Martinez	☑	☑	☑
Turbulent	Slatter	☑	☑	☑
	Haldenwang	☑	☑	☑
	Torrance	☑	☑	☑
	Darby		☑	
	Wilson and Thomas	☑	☑	☑
	Naik		☑	

Most of the existing flow models have been deduced to estimate the height losses in pipes and have been adapted for an open channel flow using the transformation:

$$D = 4 \cdot R_h \tag{1}$$

Where:

D = diameter (m).

R_h = hydraulic radius (m).

The validity of this relationship to transform a pressured pipe flow formula into an open channel formula has been studied by several authors (Haldenwang, 2003; refer Section 2.5.1.2) and is not part of this work.

2.2 Rheological models used

Basically, the rheological model used in this work is the Herschel and Bulkley model:

$$\tau = \tau_y + K \cdot \gamma^n \tag{2}$$

Where:

τ = shear Stress (Pa).

τ_y = yield Stress (Pa).

K = consistency index (Pa·s^n).

n = flow behaviour index.

γ = shear rate in (s^{-1}).

From this generalised model, the following two particular models were studied: Power Law (τ_y = 0) and Bingham plastic (n = 1).

2.3 Experimental data

The following range of measurements from experiments published in the literature was considered:

Table 2 Experimental data range summary

Author	Fluid	Ss	Cw	τ_y	K	n	W	Q	s0	Shape
Haldenwang (2003)	CMC	1.58	1.7%	0	0.06	0.655	75	0.180–74.1	1.7–8.7%	Rect
			2.8%	0	0.087	0.765	75, 150			
					0.105	0.775	300			
			4.3%	0	0.197	0.758	75			
					0.769	0.197	150			
					0.368	0.658	300			
			5.8%	0	0.453	0.724	150			
					0.606	0.678	75,300			
	Bentonite	1.53	4.3%	1.00	0.003	1	75, 150	0.043–81.8	1.7–8.7%	Rect
			6.8%	4.40	0,006	1	75, 150, 300			
			9.0%	8.19	0.006	1	150			
				12.70	0,006	1	75, 300			
	Kaolin	2.66	7.6%	1.73	0.004	0.955	300	0.043–162.6	1.7–8.7%	Rect
				1.84	0.002	1.062	75, 150			
			11.2%	3.51	0.012	0.836	75, 150, 300			
			13.0%	4.99	0.03	0.717	150, 300			
			14.5%	4.99	0.03	0.717	300			
				6.84	0.148	0.517	75, 150			
			16.9%	10.55	0.834	0.387	300			
			18.8%	14.63	0.057	0.694	75, 150, 300			
			20.8%	20.45	0.002	0.535	150			
			22.7%	21.31	0.524	0.468	150, 300			
			16.9%	9.43	0.625	0.388	150			
Martinez (2010)	Tailings Copper	2.8	54–64%	5.9 to 25.9		1	220	167–343	1%, 2%	Trap
			54–64%	6.2 to 26.9		1	350	194–341	2%	Rect
			53%–68%	3.7 to 35.5		1	560	156–344	1.5%, 2%, 3%	Rect

Thinking of the average mining slurry draws attention the low concentrations of the slurries tested by Haldenwang.

3 Preliminary analysis

The analysis was made in the following order:

1. A calculation sheet was implemented with a selection of the most common models used for laminar and turbulent regime.
2. Martinez (2010) and Haldenwang (2003) experimental data was considered.
3. From the experimental data, the characteristics parameters of the flow were obtained.
4. A point-by-point comparison was made of the experimental dataset against a selection of the models mentioned above. The analysis was made on a Fanning friction factor versus two types of Reynolds number graphic.
5. A simple sensibility analysis was conducted over changes in parameters of rheological model.

3.1 Minimum width and height to initiate movement

To calculate the height of a flow and ensure the convergence of most of the models, the initial height should be a height capable of generating a flow. This is achieved with $\tau_w > \tau_y$, where τ_w is the wall shear stress in (Pa) and is calculated as follows:

$$\tau_w = R_h \cdot \rho \cdot g \cdot \sin(\theta) \tag{3}$$

Where:

ρ = fluid or slurry density in (kg/m^3).

g = gravitational acceleration = 9.8 (m/s^2).

θ = slope angle.

Therefore, considering a limit situation in which $\tau_w = \tau_y$:

$$R_h > \frac{\tau_y}{\rho \cdot g \cdot \sin(\theta)} \tag{4}$$

Leaving aside the fact that the slope is generally conditioned to topographic and economic restrictions, in theory the channel's slope could increase as much as is required to achieve the inequality presented above. However, this cannot be done with the hydraulic radius because of the geometric dependence; it would reach a maximum value as shown in Figure 1.

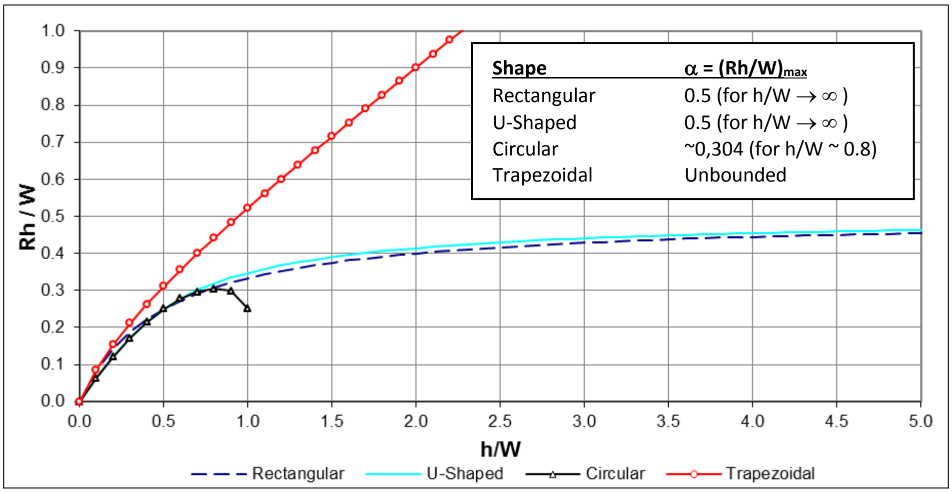

Figure 1 Hydraulic radius–width ratio versus flow depth,with ratio for rectangular, U-shaped, circular, and trapezoidal channels

For a given width W, the maximum hydraulic radius is:

$$R_{h\max} = \alpha \cdot W \tag{5a}$$

Where α is given in Figure 1. Therefore, considering a limit situation in which $R_h = R_{h\max}$:

$$W_{\min} = \frac{\tau_y}{\alpha \cdot \rho \cdot g \cdot \sin(\theta)} \tag{5b}$$

This also means that for slurries with a yield stress over zero, there is a minimum width (*W_{min})* that allows the initiation of movement. Note that this does not depend on the flow.

Once the width restriction is overcome ($W > W_{\min}$), it is adopted as a criterion that the height that will start the iterative process will be the one that generates a wall stress 5% over the yield stress.

It was found that adopting this criterion improves the convergence and decreases the risk of incorrectly calculating a flow height. In general, every model has more than one solution, and not every solution is physically possible.

3.2 Flow characterisation associated to the experimental data

To characterise the experimental data, Figure 2 shows the Fanning friction factors (*f*) versus Re deduced from the experimental data as follows:

$$f = \frac{2 \cdot R_h \cdot g \cdot \sin(\theta)}{v^2} \tag{6}$$

$$\mathrm{Re} = 8 \cdot \left(\frac{n}{1+3n} \right)^n \cdot \frac{\rho \cdot v^2}{K \cdot (2v/D)^n} \quad \text{(Hanks, 1978)} \tag{7}$$

Where:

v = mean velocity (m/s).

Equation (7) can be written in terms of hydraulic radius with Equation (1). Velocity and hydraulic radius are calculated considering both experimental flow depth data and the channel geometry.

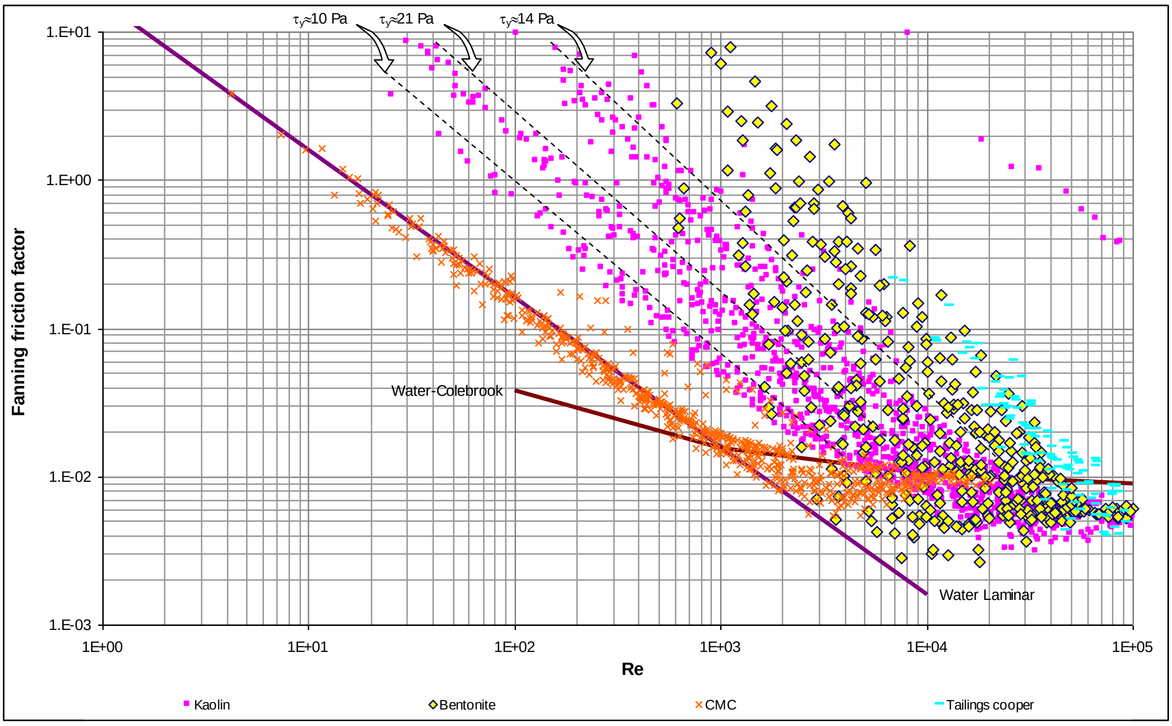

Figure 2 Fanning friction factor versus Reynolds number associated to the experimental data

In Figure 2 it is not easy to distinguish an order for the curves. While an alignment tendency can be distinguished around specific yield stress values, curves are not necessarily sorted from lowest to highest τ_y. This is because Equation (7) does not take this parameter directly into account.

4 Discussion

4.1 Reynolds 2 number

Several authors (Haldenwang, 2003; Martinez, 2010; Fernández et al., 2010) propose the existence of a unique relationship between the Fanning friction factor and an alternative Reynolds number Re_2 defined by Slatter and Lazarus (1993) as:

$$\mathrm{Re}_2 = \frac{8 \cdot \rho \cdot v^2}{\tau_y + K\left(\frac{2 \cdot v}{R_h}\right)^n} \tag{8}$$

Reviewing Figure 3, the first obvious advantage that one finds with this kind of relationship is that, unlike what happens in Figure 2, almost independently of the rheology of the fluid, all curves collapse in a single band around the theoretical Fanning friction factor in laminar regime and the Fanning number for a smooth conduit (Blasius). The reason for that might be found in that the Re_2, unlike the Re, directly measures the yield stress and not through an equivalent apparent viscosity.

From Figure 3, the following can be concluded:

- Experimental data fits quite well to a band between half and twice the Fanning friction factor and Blasius, both calculated using Re_2:

$$f_L = \frac{k}{\mathrm{Re}_2} \tag{9}$$

(Fanning, k = 16, valid for laminar zone)

$$f_T = \frac{0{,}079}{\mathrm{Re}_2^{\,0{,}25}} \tag{10}$$

(Blasius, valid for smooth wall in turbulent zone)

- It is confirmed that few experimental data exist outside the transition zone. Therefore, for the moment it is possible that not enough data exists to support one or the other of the flow models that have been deduced for turbulent regime.
- A transition zone for Re_2 is observed within the interception between f_L and f_T, and between interception $2f_L$ and $f_T/2$. That is to say that the transition zone could be between $Re_{2\,crit}$ = 1,200–8,000.

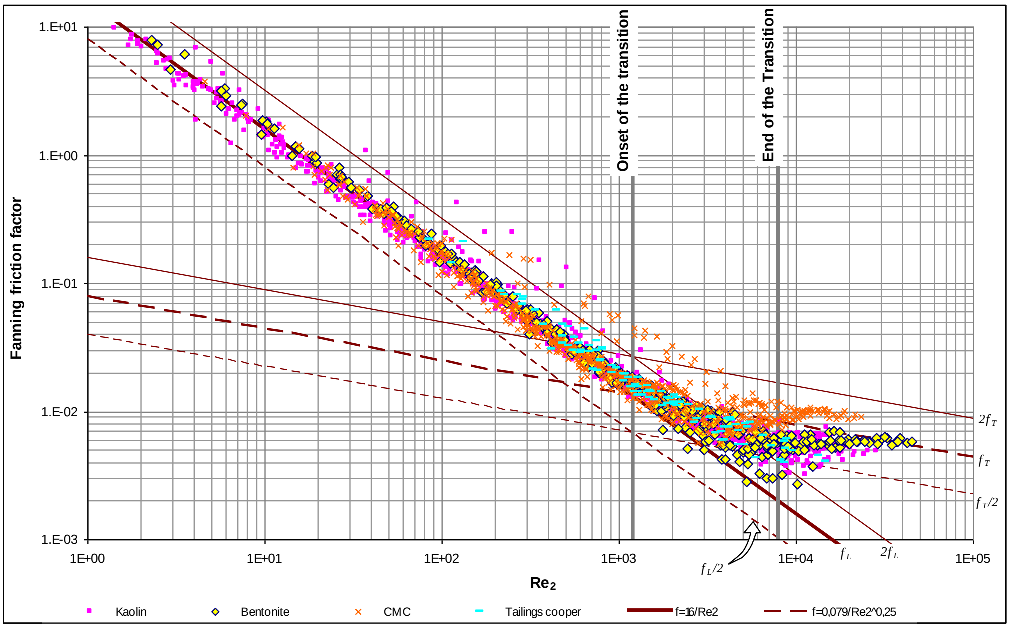

Figure 3 Fanning friction factor versus Reynolds 2 number associated to the experimental data

4.2 Laminar-turbulent transition

In order to know what the transition zone means in terms of velocity and flow height, Equation (8) can be re-written as follows:

$$\mathrm{Re}_2 = \frac{8 \cdot \rho \cdot v^2}{\tau_y + K\left(\frac{2 \cdot v}{R_h}\right)^n} > \mathrm{Re}_{2crit} \tag{10}$$

$$\Rightarrow R_h > \frac{2 \cdot v}{\left[\frac{1}{K} \cdot \left(\frac{8 \cdot \rho \cdot v^2}{\mathrm{Re}_{2crit}} - \tau_y\right)\right]^{1/n}} \tag{11}$$

The following relationship must be met so that the denominator in Equation (11) is defined:

$$v = \sqrt{\frac{\mathrm{Re}_{2\,crit}}{\underbrace{8}_{K1}}} \cdot \sqrt{\frac{\tau_y}{\rho}} \tag{12}$$

This last equation has the same form to the velocity of transition for pipes proposed by Thomas (1963), where K1 values are presented in Table 3.

Table 3 K1 values for the transition velocities

	K1
Slatter and Lazarus, 1993	26
Thomas, 1963	19–22
This work (for $\mathrm{Re}_{2\ crit}$ 1200–8000)	12–32

On the other hand, this critical velocity is an asymptote for the hydraulic radius, as shown in the example in Figure 4.

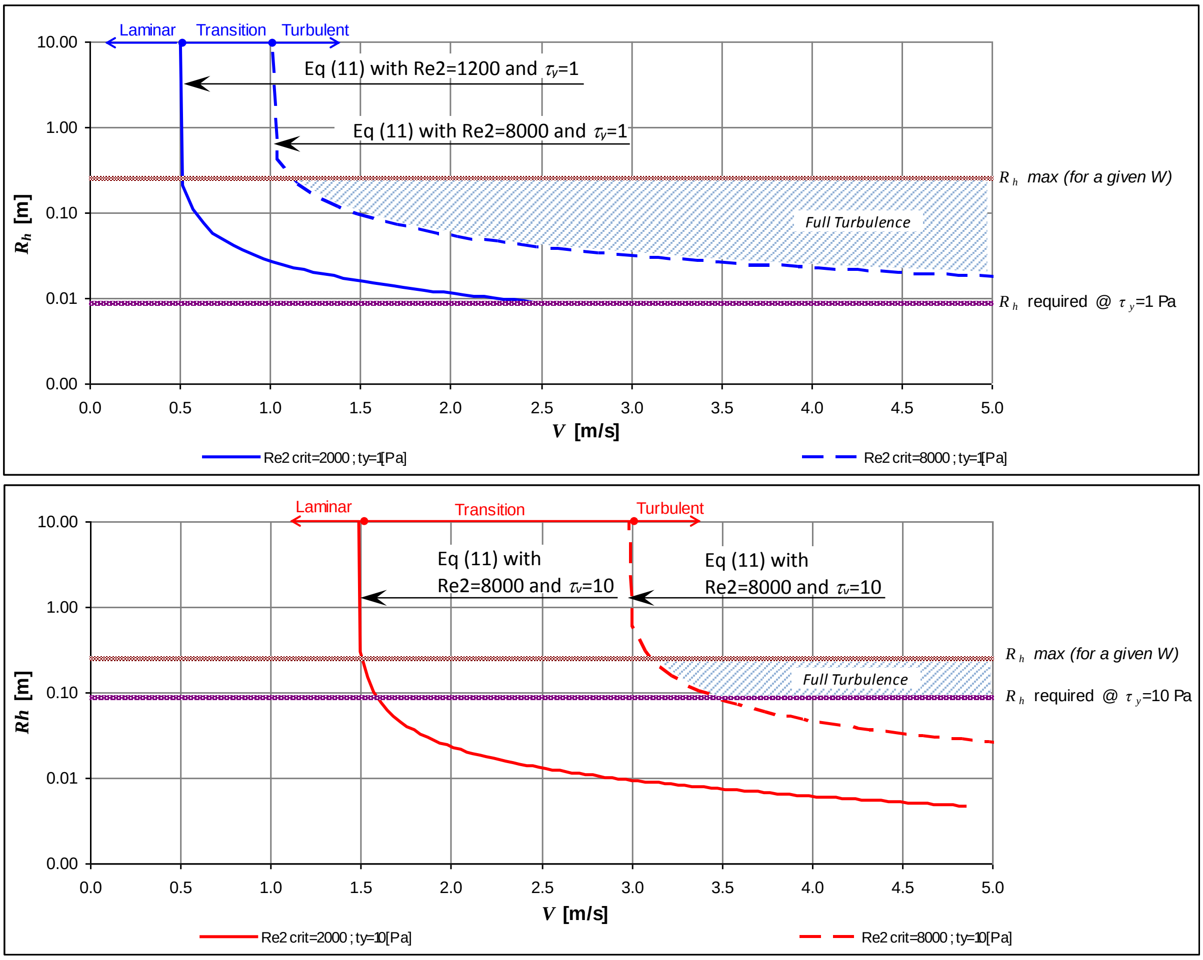

Figure 4 Hydraulic radius versus mean velocity for a rectangular channel, slope 1%

Of these results the following can be concluded:

- In order for a non-Newtonian fluid on an open conduit to be conducted on a turbulent regime, the hydraulic radius must be greater than the most restrictive between Equation (11) and Equation (4).

- As both limits increase with the yield stress, the zone where turbulence can be found gets reduced.
- Therefore, the domain where turbulence can exist on these conditions is quite restricted. This is consistent with the experimental data.

4.3 Sensitivity of the models to the type of rheology

The objective of this exercise is to discover how sensitive the laminar models might be in the hypothetical case that one rheogram could have more than one valid adjustment. Laminar model defined by Equation (9) is compared with the theoretical equations for laminar flow for a Herschel-Bulkley fluid defined as follows (Hanks, 1978):

$$f = \frac{16}{F \cdot \mathrm{Re}} \tag{13a}$$

With:

$$F = (1+3n)^n \cdot (1-\xi)^{n+1} \cdot \left[\frac{(1-\xi)^2}{1+3n} + 2\xi \cdot \frac{1-\xi}{1+2n} + \frac{\xi^2}{1+n} \right]^n \tag{13b}$$

Where:

$\xi = \tau_y / \tau_w$.

For this exercise a rectangular channel of W = 1,000 mm and a slope of s_0 = 3% was considered. Three different fluids were used (hypothetically).

Table 4 Rheological parameters of sensitivity analysis

Type of fluid	τ_y	K	n
Bingham plastic	9.0	0.525	1.000
Herschel & Bulkley	4.3	2.70	0.570
Power Law	0.0	5.60	0.408

Figure 1 summarises all three fluids considered. It can be appreciated that in all three cases the shear stresses are practically the same after γ = 8 [s^{-1}], where the shear stress is approximately 13 Pa.

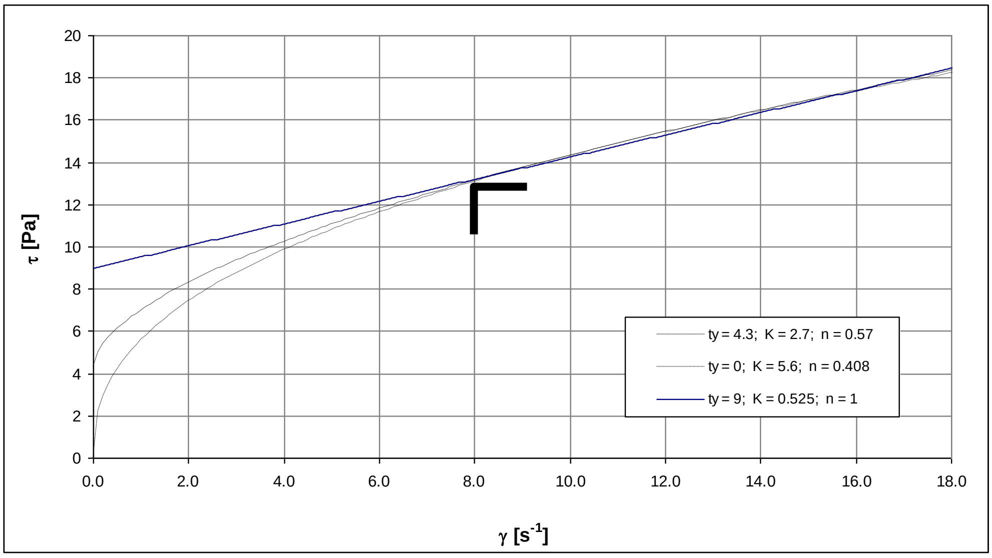

Figure 5 Example rheogram, sensitivity over the rheological models

The results of this exercise are summarised in Figures 6 and 7. From these results, the following can be concluded:

- To the contrary of what was expected for one flow, different regime flow depths were obtained depending on the type of rheological model selected.
- In this particular example, the Bingham rheological model proved to be the most conservative.

For a same flow condition, Hanks model gives friction factors greater than $16/Re_2$ model, whatever the rheological model used. However, the situation is different for $f = 24/Re_2$. Therefore, from a conservative point of view it seems recommendable to use $f = k/Re_2$ laminar model, with k = 24 (Martinez, 2010) or k = 32 (Figure 3).

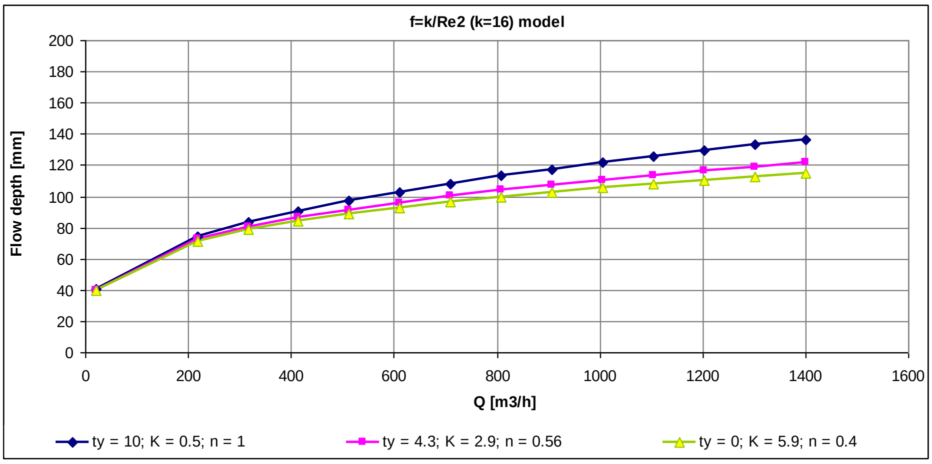

Figure 6a Sensitivity analysis – H versus Q for Bingham plastic, W = 300 mm, S0 = 3%, $f = 16/Re_2$ laminar model

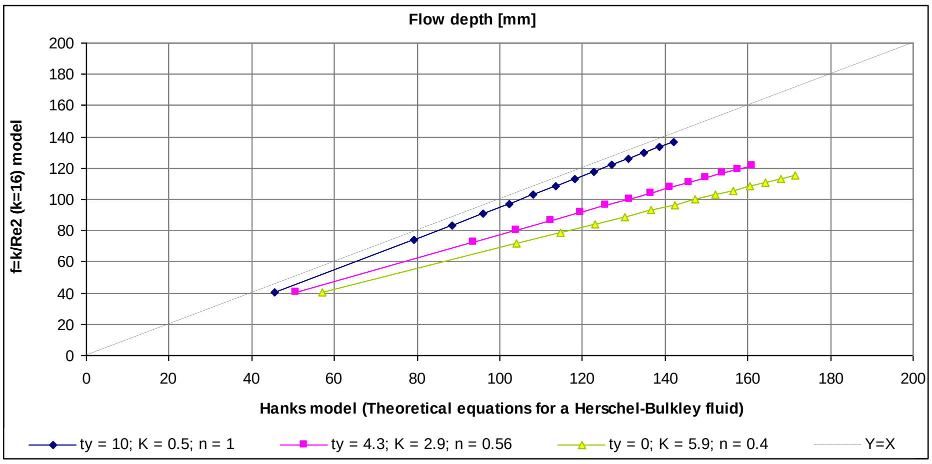

Figure 6b **Sensitivity analysis – flow depth using f = 16/Re$_2$ model versus Hanks model for Bingham plastic, W = 300 mm, S0 = 3%**

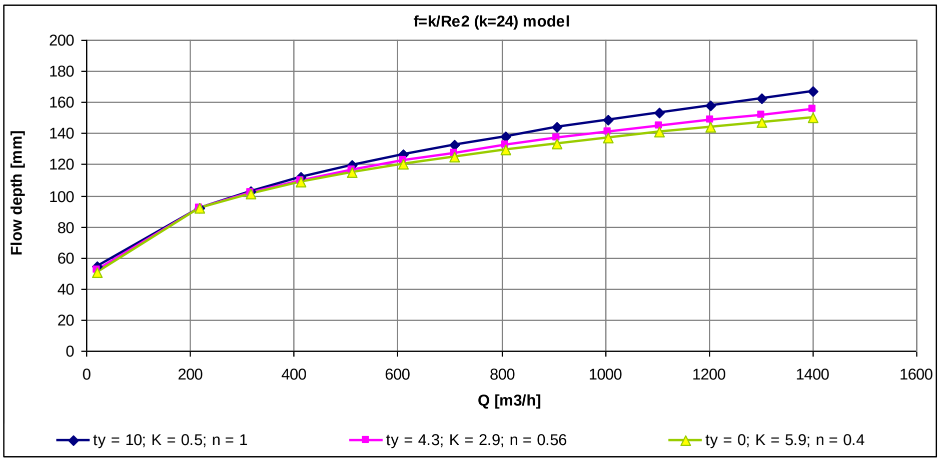

Figure 7a **Sensitivity analysis – H versus Q for Bingham plastic, W = 300 mm, S0 = 3%, f = 24/Re$_2$ laminar model**

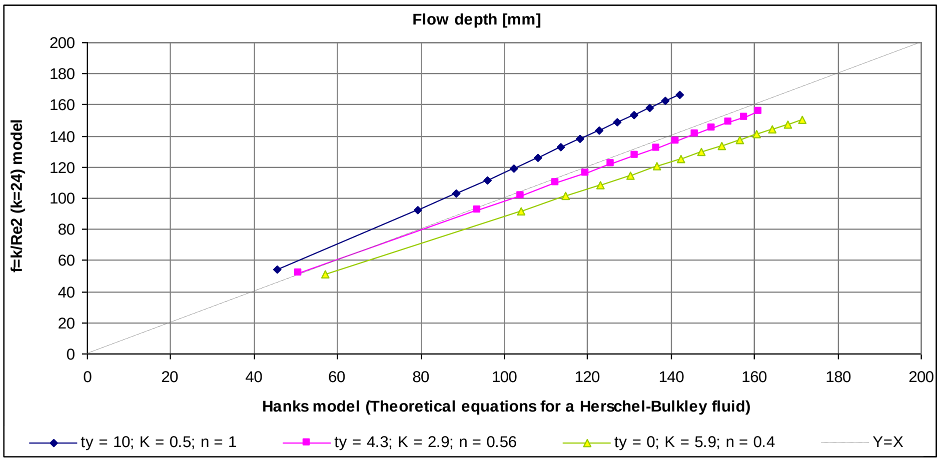

Figure 7b **Sensitivity analysis – flow depth using f = 24/Re$_2$ model versus Hanks model for Bingham plastic, W = 300 mm, S0 = 3%**

5 Conclusions

For fluids with a yield stress other than zero that are supposed to be transported in rectangular pipes or U-shaped channels, the minimum width given by Equation (5b) should be guaranteed. This minimum width is needed to generate the movement.

It is estimated that about 20% of the experimental data are found to be over the lower limit of the laminar-turbulent transition. Therefore the experimental data might be found on a turbulent zone too nascent to conclude anything about the validity of any other more sophisticated model discussed in this paper.

For an open channel the use of Re_2 is more useful, probably because it measures the yield stress directly and not through an apparent viscosity or just through of consistency index K.

However, this number does not meet the dimensionless analysis, as is indicated on Appendix E of Haldenwang, 2003. It is suggested therefore that a study formalises the validity of the number.

It is possible to distinguish a transition zone between $Re_{2\ crit}$ = 1,200–8,000. It exists beside a critical velocity and has the same structure as the proposed relationships for Thomas, 1963In a Re_2 versus v graphic, the feasible zone for a non-Newtonian fluid on a contour is bounded. Therefore, if the design criteria are defined in such a way that some slurry must be transported on a turbulent regime, it must be taken that the geometric design of the channel meets the restrictions given by the equations Equation. (4), Equation (5a) and Equation (11).

Finally, the Bingham rheological model and laminar model $f = 24/Re_2$ seems to be the most conservative from a design point of view. This was obtained through a theoretical exercise focussed on knowing how sensitive the used flow models are in the hypothetical case that one rheogram could have more than one valid adjustment.

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