

# The induced mechanism of pillar rockbursts in deep hard rock mines

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## Abstract

*Rockbursts remain one of the most serious and least understood problems in deep hard rock mines. This study is motivated by the pervasiveness of rockbursts all around the world. It is found that rockbursts are often induced by dynamic fluctuations, which are generated by mining excavations or other sources of vibrations, such as blasting in adjacent stopes. The objective of this paper is to investigate the induced mechanism of pillar rockbursts in deep hard rock mines. Rockbursts in underground mine pillars are considered as a dynamic instability problem of pillar structures. The quantitative relation between blasting and pillar rockbursts is established and parametric resonance mechanism of pillar rockbursts is proposed. This study shows that the damping in pillar rocks could reduce the rockburst occurrence and the minimum value of the excitation amplitude for rockbursts depends on the damping and natural frequency. Contrary to conventional methods where only rock materials or rock specimen were studied for rockbursts, this investigation attaches more focus on the structural effect on rockbursts, which has appeal for practicing engineers.*

## 1 Introduction

Hard rock mines are currently being excavated at great depths: more than 3,000 m in Canada and even 4,000 m in South Africa. Rockbursts are violent fracture of rocks that may occur in deep mines, which could cause an accident, or even the death of mining workers, and destroy the excavation. Although the problem of rockbursts has existed for more than two hundred years, its incidence is steadily increasing due to the increasing mining depths. For this reason, intensive study of this problem is necessary to develop a better understanding of rockburst mechanism so that improvements can be made in rockburst prediction and control strategies (Vacek et al. 2008).

From the first symposium in 1982, Rockbursts and Seismicity in Mines (RaSiM) symposia series have attempted to provide a forum for the exchange of information on rockbursts and seismicity-related problems in mining. The most recent symposium was held in September 2013 in Russia. The increased use of microseismic monitoring, improvements in the associated instrumentation and widespread applications of seismological data analysis techniques in the mining districts all over the world have led to significant advances in our understanding of rockbursts and mine seismicity in the last three decades. However, the mechanism of rockbursts is still not well understood. Wagner (1982) commented that “rockbursts are the most serious and least understood problem facing deep mining operations all around the world”. Brady (1990) made a comment that “the pervasiveness of rockbursts suggests that they remain the major unresolved ground control problem in underground mining”. Ortlepp (2005) reviewed the contribution to the understanding and control of mine rockbursts and concluded that “it is probably true that rockbursts have been the mining hazard that is the least understood and the most feared”.

To probe into the rockburst mechanism, researchers have proposed an analogy between the violent failure of a rock sample on a testing machine and the dynamic rock fracture during rockbursts in engineering practice (Cook 1965; Gill et al. 1993). Cook (1967) discussed the post-peak behaviour of a body of rock specimen in compression in relation to mine stability. In these earlier investigations, the rock specimen acts as the real fractured rock and the loading system acts as the surrounding rock mass in engineering. If the post-peak stiffness of the rock specimen is less than that of the loading system, the state of equilibrium becomes unstable and the failure of the rock specimen is violent. Otherwise, the state of equilibrium

remains stable and failure occurs gradually. There are many other theoretical and experimental results on the mechanisms of rockbursts and their prediction, e.g. the micro-gravity method, the rebound method, the drilling-yield method, the energy release method, and the microseismic method (Brady & Brown 2005; Wang et al. 2006). The primary problem of using rock specimens to study rockbursts is that they do not consider the influence of geo-structures on rockbursts, which directly leads to the drawback that it does not differentiate among pillar, roof, or tunnel rockbursts. This is due to the fact that dynamic failure characteristics of materials and structures are different (Shukla 2010). It is found that rockbursts may occur in a pillar, but may not occur in a stope roof, even the same type of rock and same state of stress are considered in a mine, and vice versa. The second problem is that the dynamically-induced effects of rockbursts are not properly accounted for. This has far-reaching implications for the study of rockbursts around the world, such that many recent researchers still use rock specimens to study rockbursts, although various theories (e.g. catastrophic theory in Wang et al. 2006) were adopted or a true-triaxial experiment was conducted (e.g. He et al. 2010), and acoustic emissions were observed. However, some useful results have been observed during experiments of rock specimens, for example, stress fluctuation is able to enhance the growth of cracks and brittle fracture (Dyskin 1999).

As Hoek (1966) pointed out, structural rock mechanics are concerned with the stability of engineering structures in which the material is predominantly rock. Hence, if rockbursts are considered as a stability problem (Cook 1965), or instability phenomenon (Vardoulakis 1984), then the best way to study rockbursts appears to be from a structural point of view. This idea was advocated by Muller (1974), the first president of the International Society of Rock Mechanics, and was confirmed by Gu (1979) and Wang (2009). A previous study has shown that rock mass structures play a controlling role in the occurrence and intensity of rockbursts (Tan 1991). Rock mass structures are divided into natural structures and artificial structures. A mine pillar is an artificially-maintained structure between two or more underground openings.

On the other hand, it is acknowledged that a rockburst is a dynamic phenomena in highly-stressed rock massifs (Zubelewicz & Mroz 1983). Most rockbursts are induced by dynamic loads or stress fluctuations, which are caused by mining excavations or other sources of vibrations, such as blasting in adjacent stopes (Xu & Huang 2003).

This paper is aimed at studying the mechanism of pillar rockbursts induced by blasting in deep hard rock mines in a rigorous mathematical framework. This study is motivated by the events of pillar bursts occurring in many deep metal mines such as Dongguashan Copper Mine in Anhui Province, China, and the Coeur D'Alene District of Northern Idaho, USA (Whyatt et al. 2002). Contrary to conventional methods where only rock materials or rock specimens were studied, this investigation attaches more attention on the structural effects on rockbursts, which has appeal for practising engineers.

## 2 Dynamic loads on pillars

A mine pillar is a critical structure in supported mining methods in underground mines, because it is usually used to control displacements in the mine near-field and then to maintain the local stability of rocks around individual excavations (Deng et al. 2003). In deep metal mines, pillars are usually subject to both superincumbent and dynamic forces. The superincumbent forces, such as the weight of the overburden and any tectonic forces acting at depth, remain constant with time. Pillars are also subjected to dynamic loads, which vary in time. Dynamic loads generally occur in the form of stress waves or stress fluctuations. Typical dynamic loads originate from blasting nearby. The diagram of a pillar and its mechanical model are shown in Figure 1.

When an explosive charge within a blasthole is initiated, reactions take place resulting in the production of a large amount of gases at very high temperature and pressure over a very short time. The gas pressure subjects the surrounding rock to large stresses and strains. The strain pulse (stress waves) travels away from the shot point in all directions, transmitting energy and attenuating in amplitude as it propagates outward. The surrounding rock can then be divided into three zones depending on the effect of damage of the strain pulse: a crushed zone, a fractured zone, and a seismic zone. The crushed zone does not exceed

two to four blasthole radii, while the fracture zone averages 20 borehole radii away and can extend as far as 50 radii (Bhandari 1997). The seismic zone is beyond 50 radii, as shown in Figure 2. Correspondingly, stress waves around blastholes can be divided into shock waves, strong elastic waves and seismic waves. The shock wave is a high amplitude pressure pulse that travels at supersonic speeds. As it propagates from the blast hole, the shock wave reduces to a strong elastic wave and then a seismic wave with characteristics of reduction in amplitude and change in frequency.

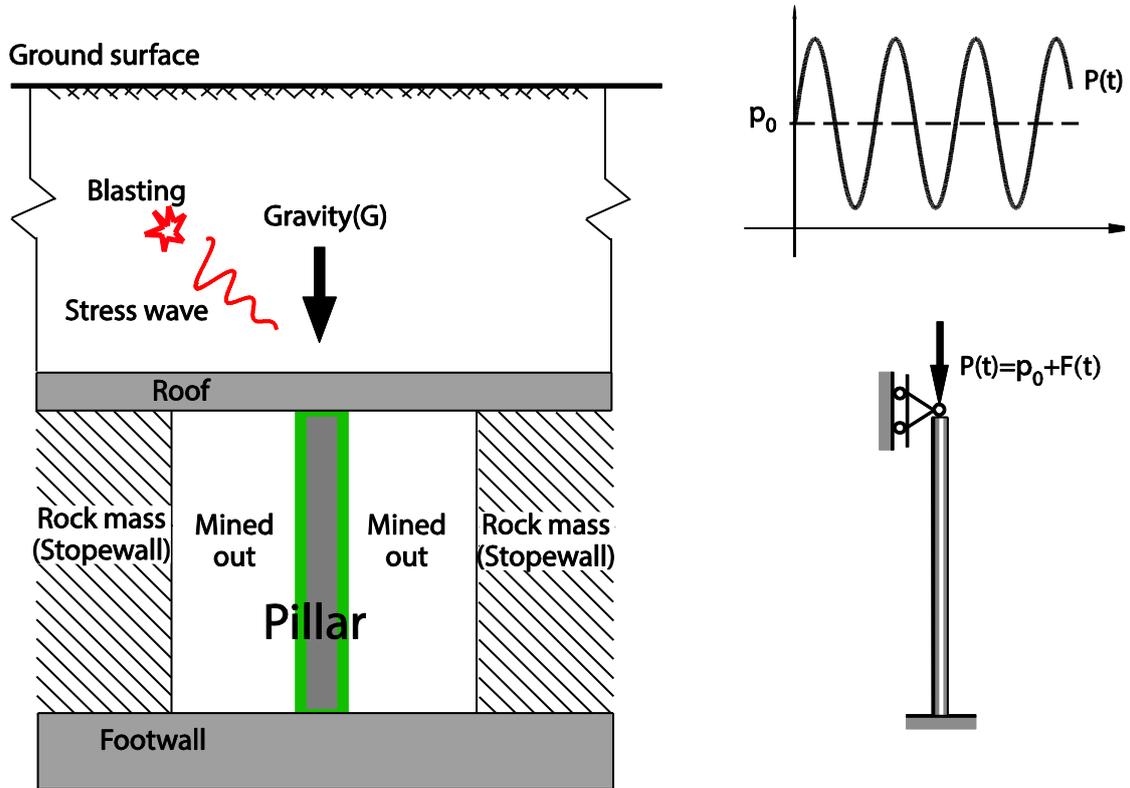


Figure 1 A mine pillar and its model

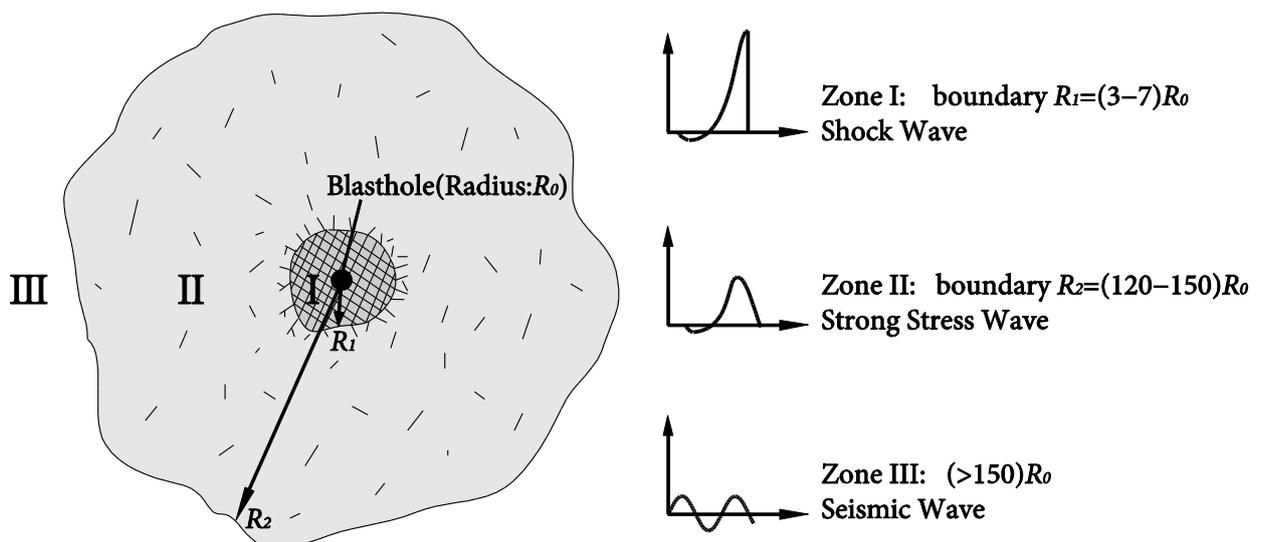


Figure 2 Stress waves around borehole rock blasting

It is very difficult to accurately predict the seismic wave from rock blasting, as the wave propagation in rocks depends on many parameters from explosives and rocks, and these parameters cannot be determined exactly. However, the amplitude of a strain pulse at a given distance from an explosion can be empirically calculated using the properties of the explosive product, the amplitude of the strain pulse

generated at the source, the distance travelled by the strain pulse, and the propagation characteristics of the rock (Duvall & Petkof 1959). In the vicinity of the source (less than  $\sim 30$  m or 100 ft), experiments have shown that the peak strain produced in rocks by the detonation of a concentrated explosive charge is given by the following equation.

$$\varepsilon = \frac{K}{l} e^{-cl}, l = \frac{R}{W^{1/3}} \quad (1)$$

Where:

- $\varepsilon$  = the peak strain in a stress wave.
- $R$  = the direct distance between the source and the point of interest.
- $l$  = the scaled distance.
- $c$  = the attenuation constant.
- $W$  = the charge weight.
- $K$  = the intercept constant,  $K = \kappa D$ .
- $D$  = the detonation pressure of the explosive charge.
- $\kappa$  = the proportionality constant.

The parameters  $\kappa$  and  $c$  are constants for a given rock type. Hence, for a particular rock and a particular explosive product, the peak strain in rock can be calculated from Equation (1) at a given distance from the explosion source. If the hard rock is further assumed to be elastic, then the peak stress can be obtained from the linear constitutive relation of the rock. It is assumed that the load on a pillar is a sinusoidal function of the form:

$$P(t) = p_0 + F(t) = p_0 + p_1 \cos(\nu t), p_1 = E\varepsilon \quad (2)$$

Where:

- $P(t)$  = the load on a pillar.
- $p_0$  = the superincumbent constant load.
- $F(t)$  = the dynamic fluctuation on a pillar.
- $p_1$  = the amplitude of the dynamic fluctuation.
- $E$  = the elastic modulus of the rock mass.
- $\nu$  = the frequency of dynamic load (excitation frequency).

The next section will discuss the pillar rockburst under the excitation of this dynamic load, by considering the rockburst as a dynamic instability problem.

### 3 Analysis

#### 3.1 Formulation

In deep hard rock mines, a pillar is often the in situ rock between two or more underground openings and the in situ rock is often of good quality without significant joints. To render the analysis tractable, the pillar is assumed to be simply supported and of uniform cross section along its length. We will make the usual assumptions that Hooke's elastic law holds for hard rocks and plane sections remain planar. Referring to Figure 3, one can set up the pertinent equations:

$$\Delta S + (\rho A \Delta x) \ddot{v}(x, t) + \beta_0 \Delta x \dot{v}(x, t) = 0, \Delta M(x, t) - S \Delta x + P(t) \Delta v(x, t) = 0 \quad (3)$$

From which one may arrive at:

$$\frac{\partial^2 M(x,t)}{\partial x^2} = -\rho A \ddot{v}(x,t) - \beta_0 \dot{v}(x,t) - P(t) \frac{\partial^2 v(x,t)}{\partial x^2} \quad (4)$$

Where the superscript dot denotes the derivative with respect to time. Using:

$$M(x, t) = EI \frac{\partial^2 v(x, t)}{\partial x^2} \tag{5}$$

one can obtain the equation of motion:

$$EI \frac{\partial^4 v(x, t)}{\partial x^4} + \rho A \frac{\partial^2 v(x, t)}{\partial t^2} + \beta_0 \frac{\partial v(x, t)}{\partial t} + P(t) \frac{\partial^2 v(x, t)}{\partial x^2} = 0 \tag{6}$$

Where:

- $\rho$  = the mass density per unit volume of the pillar rock.
- $A$  = the cross-sectional area of the pillar.
- $v$  = the transverse displacement of the central axis.
- $\beta_0$  = the damping constant.
- $EI$  = the flexural rigidity of the pillar.
- $M(x, t)$  = the moment at cross-section  $x$ .

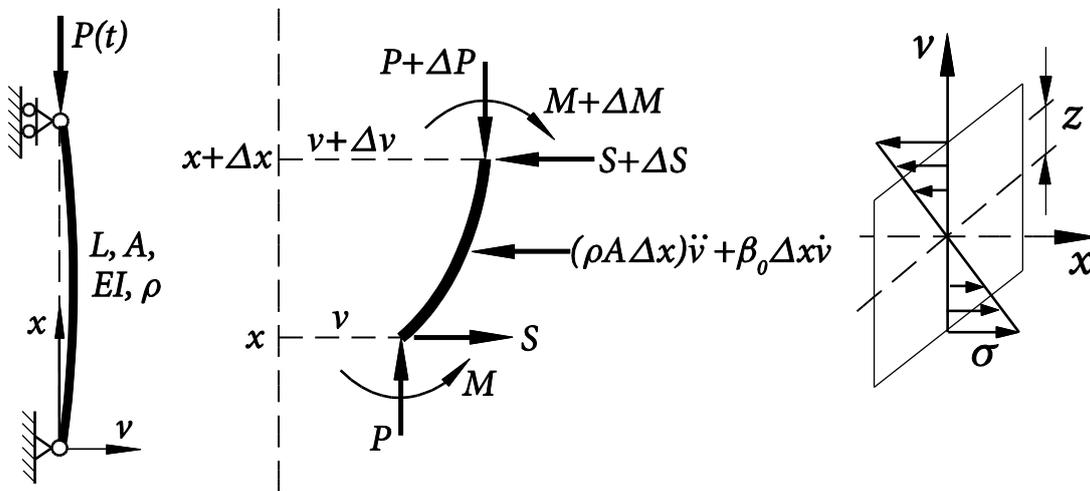


Figure 3 A pillar model under axial compressive load

The boundary conditions are: at  $x = 0, v(0, t) = \frac{\partial^2 v(0, t)}{\partial x^2} = 0$ ; and at  $x = L, v(L, t) = \frac{\partial^2 v(L, t)}{\partial x^2} = 0$

Substituting a solution in the form:

$$v(x, t) = \sum_{n=1}^{\infty} q_n(t) \sin \frac{n\pi x}{L} \tag{7}$$

into (6) leads to the equations of motion:

$$\ddot{q}_n(t) + 2\beta \dot{q}_n(t) + \omega_n^2 \left[ 1 - \frac{P(t)}{P_n} \right] q_n(t) = 0 \tag{8}$$

Where:

$$\beta = \frac{\beta_0}{2\rho A}, \omega_n^2 = \frac{EI}{\rho A} \left( \frac{n\pi}{L} \right)^4, P_n = EI \left( \frac{n\pi}{L} \right)^2 \tag{9}$$

If only the  $n^{\text{th}}$  mode is considered, and the dynamic load  $P(t)$  is assumed to be the form of Equation (2), one may obtain the equation of motion for the pillar:

$$\ddot{q}(t) + 2\beta \dot{q}(t) + \omega^2 (1 - 2\mu \cos vt) q(t) = 0 \tag{10}$$

Where:

$$\omega^2 = \omega_n^2 \left( 1 - \frac{p_0}{P_n} \right), 2\mu = \frac{p_1}{P_n - p_0} \tag{11}$$

### 3.2 Rockburst analysis

The phenomenon of pillar bursting is taken as a problem of dynamic instability. It is found that the stability boundaries of Equation (10) correspond to periodic solutions, i.e. periodic solutions of period  $2T$ ,  $T = 2\pi/\nu$  and periodic solutions of period  $T$ .

Periodic solutions of period  $2T$  can be expressed as a Fourier series of the form:

$$q(t) = \sum_{k=1,3,5}^{\infty} (a_k \sin \frac{k\pi t}{T} + b_k \cos \frac{k\pi t}{T}) = \sum_{k=1,3,5}^{\infty} (a_k \sin \frac{k\nu t}{2} + b_k \cos \frac{k\nu t}{2}). \quad (12)$$

Substituting (12) into (10) and setting the coefficients of  $\sin \frac{k\nu t}{2}$  to zero result in equations for coefficients of  $a_k$ :

$$\begin{aligned} k = 1: & \quad (1 + \mu - r^2)a_1 - \mu a_3 - \frac{2\beta r}{\omega} b_1 = 0; \\ k = 3, 5, \dots: & \quad (1 - k^2 r^2)a_k - \mu(a_{k-2} + a_{k+2}) - \frac{2k\beta r}{\omega} b_k = 0 \end{aligned} \quad (13)$$

And equating coefficients of  $\cos \frac{k\nu t}{2}$  to zero results in equations for coefficients of  $b_k$ :

$$\begin{aligned} k = 1: & \quad (1 - \mu - r^2)b_1 - \mu b_3 + \frac{2\beta r}{\omega} a_1 = 0; \\ k = 3, 5, \dots: & \quad (1 - k^2 r^2)b_k - \mu(b_{k-2} + b_{k+2}) + \frac{2k\beta r}{\omega} a_k = 0 \end{aligned} \quad (14)$$

Where  $r = \frac{\nu}{2\omega}$ . To assure non-trivial solutions, the determinant of the coefficient matrix of vector  $\{\dots, a_3, a_1, b_1, b_3, \dots\}^T$  is set to zero:

$$\Delta_{2T} = \begin{vmatrix} \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & 1 - 9r^2 & -\mu & 0 & -6\beta r/\omega & \dots \\ \dots & -\mu & 1 + \mu - r^2 & -2\beta r/\omega & 0 & \dots \\ \dots & 0 & +2\beta r/\omega & 1 - \mu - r^2 & -\mu & \dots \\ \dots & +6\beta r/\omega & 0 & -\mu & 1 - 9r^2 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{vmatrix} = 0 \quad (15)$$

The first-order approximation uses the  $2 \times 2$  submatrix in Equation (15) to correspond to the coefficients  $\{a_1, b_1\}^T$  for  $k = 1$ :

$$\begin{vmatrix} 1 + \mu - r^2 & -2\beta r/\omega \\ 2\beta r/\omega & 1 - \mu - r^2 \end{vmatrix} = 0 \quad (16)$$

which can be written in an equation form:

$$(1 - r^2)^2 - \mu^2 + (2\beta r/\omega)^2 = 0 \quad (17)$$

Similarly, substituting periodic solutions of period  $T$  in a Fourier series form:

$$q(t) = b_0 + \sum_{k=2,4,6}^{\infty} (a_k \sin \frac{k\pi t}{T} + b_k \cos \frac{k\pi t}{T}) \quad (18)$$

Into (10) and setting the coefficients of  $\sin \frac{k\nu t}{2}$  to zero result in equations:

$$k = 2, 4, 6, \dots: \quad -\frac{2k\beta r}{\omega} b_k + (1 - k^2 r^2)a_k - \mu(a_{k+2} + a_{k-2}) = 0 \quad (19)$$

And equating coefficients of  $\cos \frac{k\nu t}{2}$  to zero results in equations:

$$\begin{aligned} k = 0: & \quad b_0 - \mu b_2 = 0; \\ k = 2: & \quad \frac{4\beta r}{\omega} a_2 + (1 - 4r^2)b_2 - 2\mu b_0 - \mu b_4 = 0 \\ k = 4, 6, \dots: & \quad \frac{2k\beta r}{\omega} a_k + (1 - k^2 r^2)b_k - \mu(b_{k-2} + b_{k+2}) = 0 \end{aligned} \quad (20)$$

Where  $a_0 = 0$ . To assure non-trivial solutions, the determinant of the coefficient matrix of vector  $\{\dots, a_4, a_2, b_0, b_2, b_4, \dots\}^T$  is set to zero:

$$\Delta_{2T} = \begin{vmatrix} \dots & \dots \\ \dots & 1 - 16r^2 & -\mu & 0 & 0 & -\frac{8\beta r}{\omega} & 0 & \dots \\ \dots & -\mu & 1 - 4r^2 & 0 & -\frac{4\beta r}{\omega} & 0 & 0 & \dots \\ \dots & 0 & 0 & 1 & -\mu & 0 & 0 & \dots \\ \dots & 0 & \frac{4\beta r}{\omega} & -2\mu & 1 - 4r^2 & -\mu & 0 & \dots \\ \dots & \frac{8\beta r}{\omega} & 0 & 0 & -\mu & 1 - 16r^2 & -\mu & \dots \\ \dots & \dots \end{vmatrix} = 0 \quad (21)$$

The second part of the first-order approximation uses the  $3 \times 3$  submatrix in Equation (21) to correspond to the coefficients  $\{a_1, b_1\}^T$  for  $k = 1$ :

$$\begin{vmatrix} 1 - 4r^2 & 0 & -\frac{4\beta r}{\omega} \\ 0 & 1 & -\mu \\ \frac{4\beta r}{\omega} & -2\mu & 1 - 4r^2 \end{vmatrix} = 0 \quad (22)$$

which can be written in an equation form:

$$(1 - 4r^2)^2 - 2\mu^2(1 - 4r^2) + (4\beta r/\omega)^2 = 0 \quad (23)$$

Hence, the first-order rockburst region can be obtained from Equations (17) and (23), shown in Figure 4. It has been proved that that this approximation converges as  $k \rightarrow \infty$  (Xie 2006). In the stable region, the pillar is stable with bounded responses, which means that no rockbursts occur. On the contrary, in the instable region the pillar system is unstable, and the response grows exponentially with unbounded motion, which shows that rockbursts might occur. Thus, it is reasonable to take the boundary between pillar stability and instability as the dividing line between rockbursts and non-rockbursts.

From Figure 4, one could determine the condition of the pillar rockburst occurrence. It is found that when the excitation frequency  $\nu$  is twice the pillar natural frequency  $\omega$ , rockbursts may occur even if the excitation amplitude  $\mu$  is small. The mechanism is called parametric resonance, which is quite different from the ordinary forced resonance where the frequency relationship is  $\nu = \omega$  (Chopra 2011). Figure 4 also shows that with the increase of damping, the minimum excitation amplitude increases accordingly, which suggests damping could reduce the rockburst occurrence. Equation (17) can be recast as:

$$r = \sqrt{1 \pm \sqrt{\mu^2 - \left(\frac{2\beta r}{\omega}\right)^2}} \quad (24)$$

which shows the minimum value of the excitation amplitude,  $\mu_{cr}$ , for rockbursts is given by  $\mu_{cr} = \frac{2\beta}{\omega}$ .

Similarly, one can obtain the second minimum value  $\mu_{cr} = \sqrt{\frac{2\beta}{\omega}}$ .

Parameters  $\mu$  and  $\nu$  in Equation (10) are related to the parameters  $(p_0, p_1)$  of dynamic load  $P(t)$  on the pillar. In addition, Equation (2) shows that the load  $P(t)$  is determined by the explosives administered in engineering. Hence, the quantitative relation between pillar rockburst and the explosives in blasting engineering is established. Using this important relation, one can predict the pillar rockburst and control it by adjusting blasting parameters to rock properties.

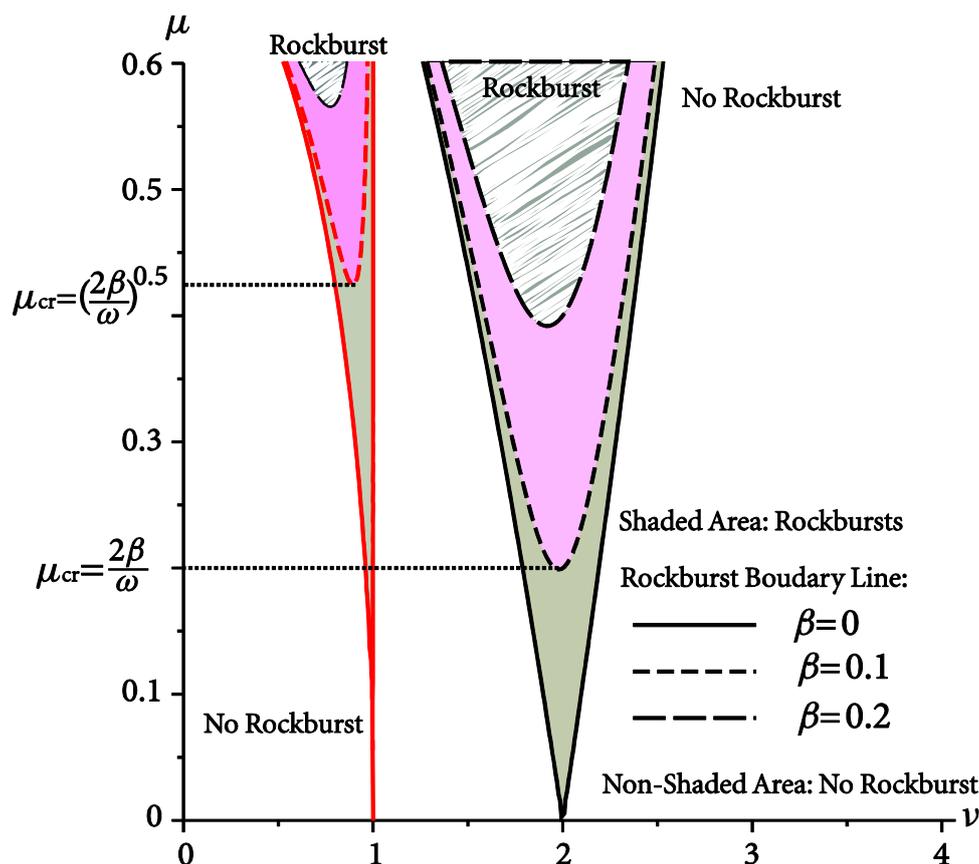


Figure 4 Pillar rockburst boundary

## 4 Conclusions

This paper considers pillar rockbursts in underground mine pillars as a dynamic instability problem related to the pillar structure itself. Dynamic load on pillars from nearby blasting is introduced. This dynamic load or stress fluctuation may induce pillar rockbursts. Under the excitation of the stress fluctuation, the dynamic equation of motion is developed, from which the boundaries of pillar rockburst occurrence are obtained by deriving the conditions for dynamic instability. Parametric resonance mechanism of pillar rockbursts is addressed and the quantitative relation between blasting and pillar rockbursts is established. The damping could reduce the rockburst occurrence and the minimum value of the stress amplitude for rockbursts is given. Contrary to conventional methods where only rock materials or rock specimens are considered for rockburst potential investigations, this approach pays more attention to the structural effect on rockbursts, which tends to appeal to practicing engineers. The second-order rockburst region can be obtained in a similar way.

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