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Open channel transportation of thickened tailings

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Abstract

The mining industry is moving towards thickened tailings with higher density than conventional operations. As a result, tailings transportation is more challenging due to the material's tendency to exhibit non-Newtonian behaviour at high solid concentration. One of the practical ways to transport tailings, where the terrain allows, is to gravitate the tailings through open channels from the thickening plant to the tailings disposal area. But in comparison to pipe flow, the flow of non-Newtonian thickened tailings through open channels has not received sufficient attention in the literature. This paper presents a set of models developed to predict the flow behaviour in the laminar transitional and turbulent regimes through open channels. The flow behaviour predictions of these semi-theoretical models are discussed and validated against previously published experimental data.

1 Introduction

The mining industry is considering modern tailings dewatering technologies such as high-density thickeners, paste thickeners, and filtration to higher density. Hyper-concentrated thickened tailings, in addition to water recovery, helps reduce embankment construction costs and reduce environmental risks. Dewatered tailings normally travels some distance to the disposal site and is normally transported through pipe or open-top flumes or launders. But with increase in density, the tailings exhibits significant non-Newtonian behaviour and flow prediction of such material becomes challenging, when compared to Newtonian fluids (Gawu & Fourie 2004; Slatter et al. 2011).

If the terrain permits, an open channel is an economical alternative to a pipeline. However, unlike pipe flow, open channel flow of non-Newtonian fluids such as thickened tailings has not received much attention and only limited investigations have been conducted to study open channel flow (Kozicki & Tiu 1967; Wilson 1991; Coussot 1994; Haldenwang & Slatter 2006; Fitton 2007; Pirouz et al. 2013; Burger 2014). However, in many studies, the pipe flow paradigm is adopted to hydraulically design open channel.

The objective of this paper is to present a set of models as a basis to design open channel flow of thickened tailings material in the laminar and turbulent regimes. The design approach is validated by using previously published flume data (Haldenwang & Slatter 2006). Moreover, a criterion to discriminate the onset of transitional flow in open channels is developed and compared with the experimental data.

2 Laminar flow

For the laminar flow of non-Newtonian material through a channel, Kozicki and Tiu (1967) developed an analytical method. The method involves the use of two geometric parameters, a and b, to describe the channel cross section. the prediction obtained from this model showed poor agreement with the experimental results of yield pseudo-plastic non-Newtonian material (Haldenwang 2003; Javadi et al. 2014). Most of the current models associated with open channel flow design were derived from the pipe flow paradigm (Haldenwang et al. 2010; Burger et al. 2010; Haldenwang 2003).

In this study, for the laminar flow regime, a new model is presented in order to evaluate the appropriate estimate of the wall shear stress. To achieve this, the model incorporates the hydraulic and rheological properties of flow. The model adopts the flow concepts of two geometries – pipe flow and infinitely wide channel flow - to define a new formulation for the Reynolds number.

By substituting the hydraulic radius and channel slope S_o in the Darcy–Weisbach equation, the following is obtained for open channel flow of arbitrary geometry (Abulnaga 2002):

$$S_o = \frac{16}{Re}\frac{1}{R_h}\frac{V^2}{g} \tag{1}$$

Where Re is Reynolds number, R_h is hydraulic radius, V is average velocity and g is gravitational acceleration.

From Equation 1, it is quite clear that one key component to accurately predict the flow is a correct assessment of Reynolds number – which represents the ratio of inertial forces to viscous force caused by shear stress at the flow boundary. For yield pseudo-plastic fluid such as thickened tailings slurries flowing through an arbitrary geometry, this shear stress is a function of shear rate and the magnitude of yield stress.

For an infinitely wide channel, Slatter et al. (2010) defined a set of equations to evaluate the shear rate at the wall. Considering isothermal and time-independent non-Newtonian fluid flow, differentiation of velocity with respect to flow depth in the vicinity of the boundary can be related to bulk shear rate giving:

$$\dot{\gamma_w} = \frac{3V}{H}\left(\frac{2n^*+1}{3n^*}\right) \tag{2}$$

$$n^* = \frac{dln(\tau_w)}{dln(\frac{3V}{H})} \tag{3}$$

Where $\dot{\gamma_w}$ is wall shear rate, H is flow depth in the channel, n^* is fluid index and τ_w is wall shear stress.

On the basis that Reynolds number is defined as the ratio of inertial forces to viscous forces and by adopting the true wall shear stress of sheet flow and hydraulic radius (R_h), one can develop a generalised formulation for Reynolds number as follows:

$$Re_{New} = \frac{8\rho V^2}{\tau_y + K(\frac{3V}{R_h}(\frac{2n^*+1}{3n^*}))^n} \tag{4}$$

$$n^* = \frac{dln(\tau_w)}{dln(\frac{3V}{R_h})} \tag{5}$$

Where τ_y, K, n are the Herschel-Bulkley rheological model parameters. This model, in comparison with Haldenwang and Slatter (2006), considers true shear rate rather than bulk shear rate in order to accurately estimate the friction caused by viscous resistance at the flow boundary.

3 Transitional flow

Because of a suspension's tendency to deposit in laminar flow, it is often desired to design a system to transport thickened tailings material in the transitional range from laminar to turbulent flow. For non-Newtonian fluid containing suspensions, the arbitrary velocity fluctuations caused by transitional flow can stir up the settling particles. Hence, it is of critical importance to accurately determine the onset of transition for such flows. However, due to the complex mechanism of transitional flow, there is insufficient knowledge to establish a general criterion for determination of the transition point.

For non-Newtonian flow in a pipe, many attempts have been made to distinguish the transition point from laminar to turbulent. By analogy to Newtonian fluid transition, some works supposed the transition occurs about Re=2,100, and manipulated the transitional criterion (Hanks 1963; Govier & Aziz 1972; Metzner & Reed 1955; Slatter 2011; Griffiths 2012). Slatter and Wasp (2000) compared several models and found that all approaches are not able to define the critical velocity over all ranges of Hedstrom number, therefore they developed three empirical models for three different ranges of Hedstrom number (Slatter & Wasp 2000). Their approaches were further analytically investigated by Wilson and Thomas (Wilson & Thomas

2006). In channel flows, limited progress has been made in comparison with pipe flow, and most of the approaches are developed either from the pipe flow criterion or experimental results obtained in flume tests (Haldenwang & Slatter 2006; Haldenwang et al. 2010; Slatter et al. 2011).

In this study an analytical criterion for the laminar/turbulent transition is presented by introducing a stability number. To define the stability number, it is assumed that at the limit of the transitional zone, the laminar shear velocity (U_L^*) approaches the turbulent regime shear velocity (U_T^*) and the stability number is defined as the ratio of these two parameters. Therefore, at regime change, the stability number (*k*) approaches unity. Shear velocity is obtained from Equation 6 for either of the laminar or the turbulent regime, depending on whether the friction factor (*f*) is calculated from the laminar or the turbulent models.

$$U_{L/T}^* = V\sqrt{\frac{f_{L/T}}{8}} \tag{6}$$

Substituting the shear velocity from the laminar model into the turbulent model, after some manipulations, the stability parameter is defined as follows:

$$k = \sqrt{S_o} - 0.14\sqrt{Re} - \ln\left(\frac{4\rho R_h^2 g^{0.5}}{3.13\ \mu_{eff}}\right) \tag{7}$$

Where, μ_{eff} is the apparent viscosity, defined here as the slope of the tangent line to the rheogram curve at a shear rate of 100 1/s.

The magnitude of *k* indicates the flow regimes and can vary from infinitely negative for extremely turbulent to infinitely positive for extreme laminar flows.

For $k < 1$ Transitional/turbulent flow.

$k > 1$ Laminar flow.

$k = 1$ Critical flow.

4 Turbulent flow

Because of high hydraulic gradient required and excessive erosion in pipes and channels, fully turbulent flow in suspension transportation can be avoided. The system is designed to operate in partially turbulent flow which is in the range of transitional flow. Therefore, it is important to employ a boundary friction evaluation for non-Newtonian thickened slurries through open channels in this range of flows. In channel flow, due to boundary shape, analytical evaluation of flow behaviour is fraught with more complexity in comparison with pipe flow (Wilson & Thomas 2006; Fitton 2007). This has led to semi-empirical models most of which are derived from the pipe flow paradigm with empirical constants obtained from experimental data (Wilson 1991; Haldenwang 2003; Haldenwang et al. 2010).

However, as far as can be ascertained, one of the turbulent models for non-Newtonian slurry which is being widely used for engineering purpose is the Wilson and Thomas (1985) smooth pipe model. Their model includes corrections to account for the thickening of the viscous sub-layer caused by the rheology of the material and has shown good agreement with experimental results. Their work was advanced and extended to rough boundaries (Thomas & Wilson 2007). Assuming that the same pipe sub-layer thickening occurs at the boundary of channel for thickened tailings, in our work we adopted this model in order to define a design approach in the turbulent regime as follows:

$$\frac{V}{U^*} = 2.5\ln\left(\frac{4\rho U^* R_h}{\mu_{eff}}\right) - 2.5\ln(\alpha - 1) + 11.6\,(\alpha - 1) - \Omega \tag{8}$$

Where:

$$\Omega = -2.5\ln\left(1 - \frac{\tau_y}{\tau_w}\right) - 2.5\frac{\tau_y}{\tau_w}\left(1 - \frac{\tau_y}{2\tau_w}\right) \tag{9}$$

and α is the ratio of area of a non-Newtonian fluid rheogram to that of an equivalent Newtonian fluid at the same wall shear stress. One can express α for a Herschel–Bulkley material as:

$$\alpha = \frac{2(n+1)\tau_y + 2\,(\tau_w - \tau_y)}{(n+1)\tau_w} \tag{10}$$

5 Experiment results and discussion

To validate the models, a set data of test work which was carried out at the Flow Process Research Centre at the Cape Peninsula University of Technology is used (Haldenwang 2003; Haldenwang & Slatter 2006). The test work involved the flow of kaolin and bentonite suspensions, and carboxymethyl cellulose (CMC) solutions at varying concentrations in a rectangular shaped channel. In this study, the data of the kaolin suspensions which exhibit Hershel–Bulkley rheological behaviour have been employed. Table 1 summarises the rheological properties of these kaolin suspension.

Table 1 Summary of material tested

Volumetric concentration (%)	τ_y (Pa)	k $(Pa\,S^n)$	n
10	21.311	0.428	0.468
7.1	9.431	0.625	0.388
6	6.840	0.148	0.517

In Figure 1, the predicted average velocity for the new model (Equations 4 and 5) in the laminar regime is illustrated. To evaluate the new model, the predictions were compared with the measured velocity. It is quite evident that the agreement is very good for kaolin suspension at varying concentration in the laminar regime. The absolute error of our predictions, with few exceptions, does not exceed 10%.

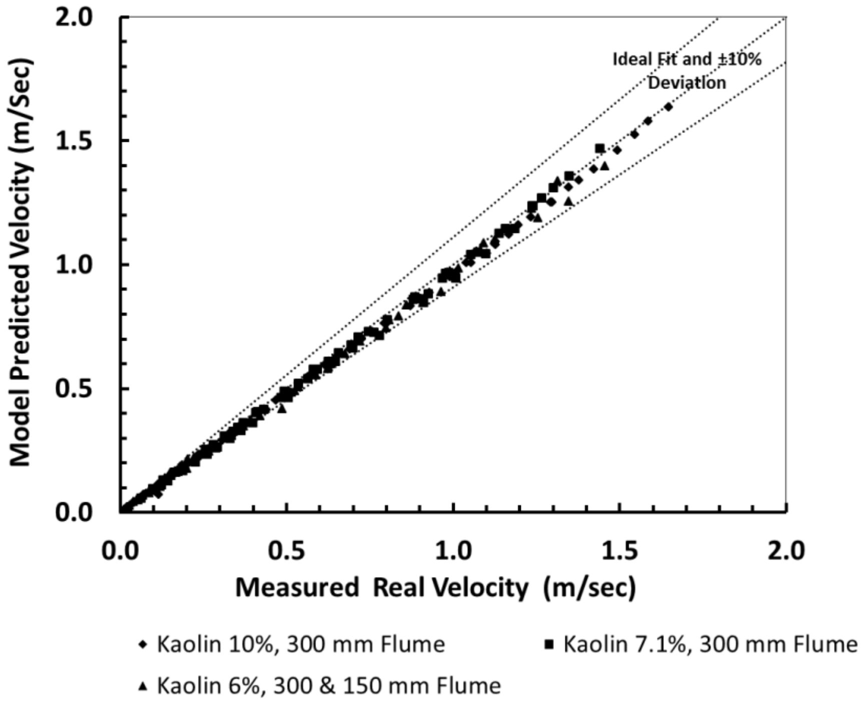

Figure 1 Predicted and experimental average velocity in the laminar regime for the tested fluid

The new model was evaluated further against two previously defined models that were developed based on the pipe flow paradigm, namely the Haldenwang–Slatter model and Chilton–Stainsby model. These comparisons are presented in Figure 2 and indicate that the most agreed predictions with the measured average velocity are obtained for the new model.

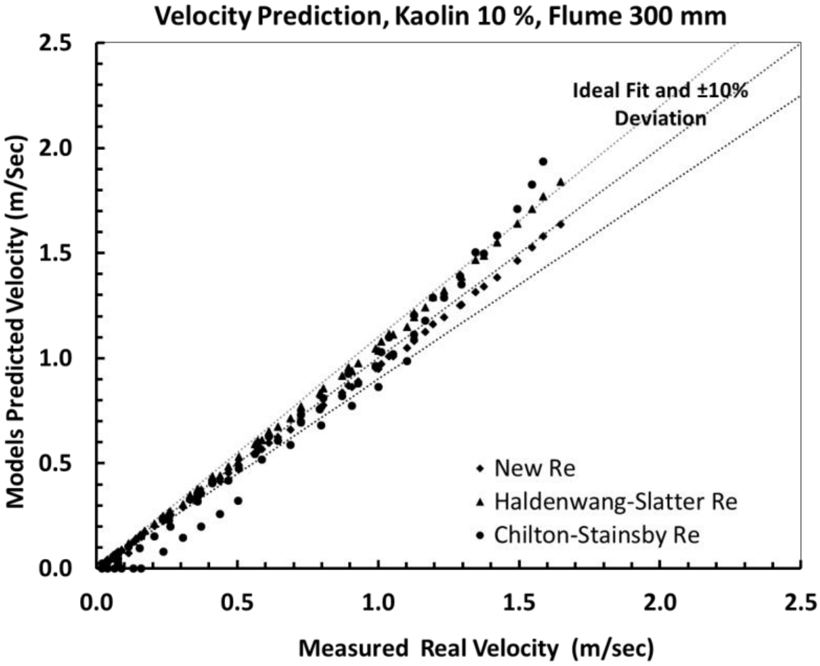

Figure 2 Predicted and experimental average velocity for Kaolin suspension in the laminar regime (10% volumetric concentration)

As shown in Figure 2, there is a clear tendency for the Haldenwang–Slatter model to over-predict the average velocity. The Haldenwang–Slatter Re over predicts the average velocity due to under-predicting the friction on the wall as velocity increases. This is because of a limitation of the Haldenwang–Slatter Re at high velocity. Haldenwang–Slatter Reynolds number employs the bulk shear rate instead of true shear rate in order to estimate the friction factor at the flow boundary. As the bulk shear rate always is lower than the true shear rate at the wall for these non-Newtonian materials, the friction factor obtained from this model is underestimated. Hence, this leads to over-prediction in the average velocity obtained from this model (Javadi et al. 2014).

Figures 3 and 4 illustrate the transition point discriminated by the stability number *k* that is presented in this paper for kaolin suspensions flowing at slopes of 4 and 5°. The onset of transitional flow occurs at the deflection point of plots of wall shear stress against bulk shear rate on the logarithmic axis (Wilson 1991; Haldenwang 2003). The *k* values for the measured data is calculated from Equation 7. For each set of data there is a horizontal line indicating the *k*=1 and the locus of the measured points compared to this line indicates the flow regime. Flow conditions of the points located above the line *k*=1 (e.g. *k*=0.1 in Figure 3 and *k*=0.799 in Figure 4) are in the transitional/turbulent range while the other points are in the laminar region. Given that the stability number *k*=1 represents the transition from laminar to turbulent, it is clear that the stability parameter quite accurately predicts the onset of transition for the data obtained from the experimental results. However, this approach needs to be validated with more data especially from real tailings flume data.

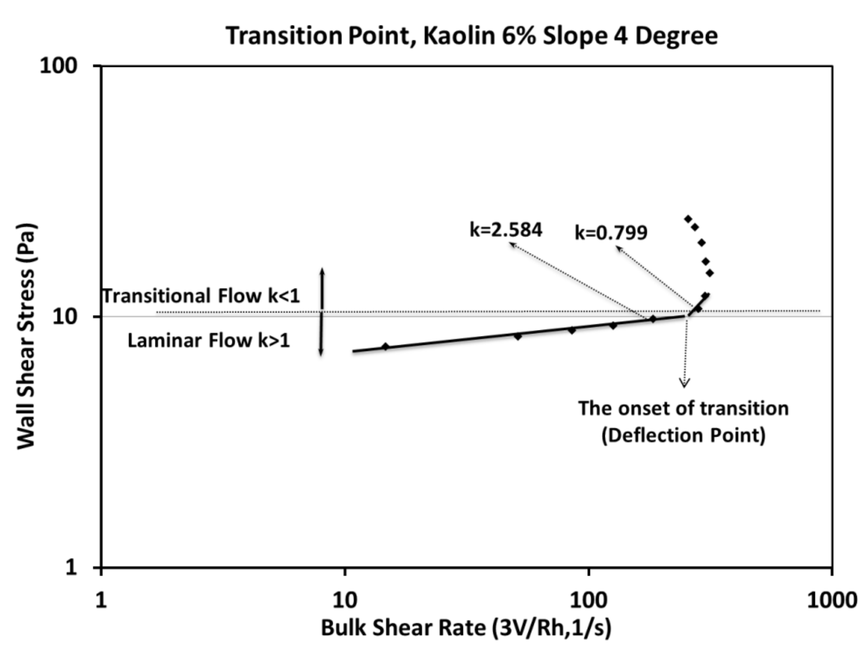

Figure 3 Locus of transition point (Kaolin 6% slope 4°)

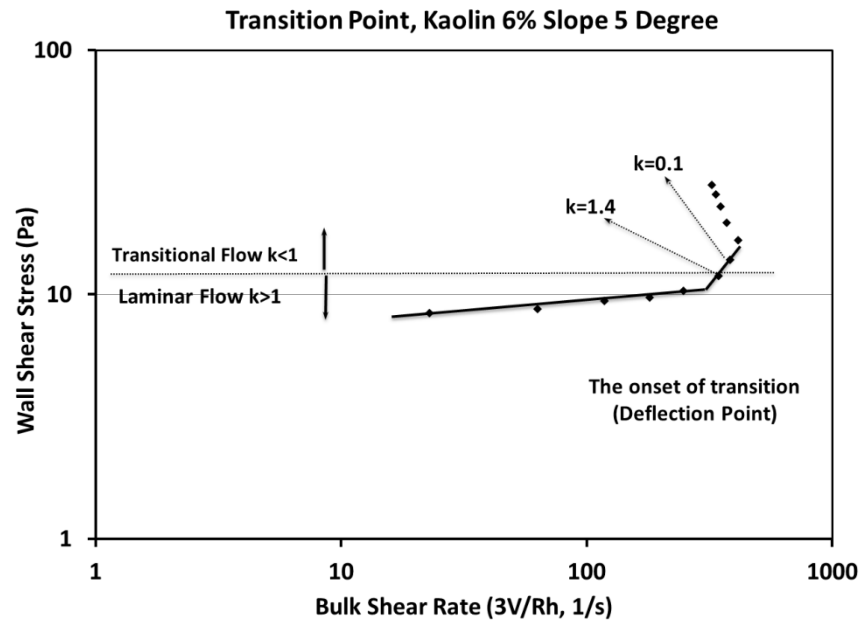

Figure 4 Locus of transition point (Kaolin 6% slope 5°)

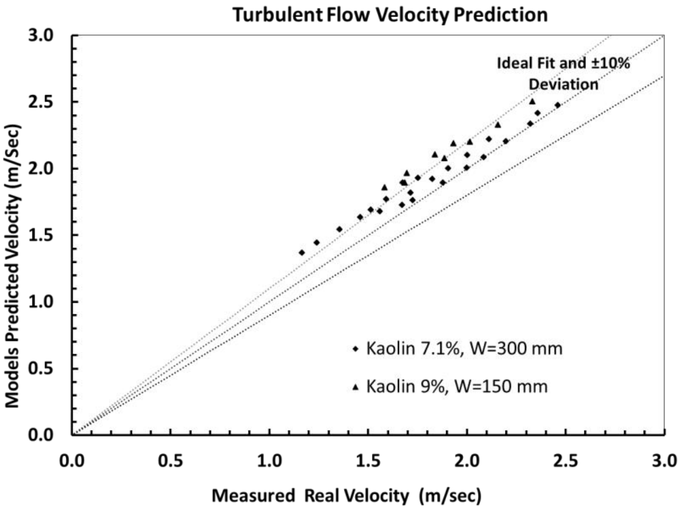

Figure 5 Turbulent flow velocity prediction (Kaolin 7.1 and 9%)

Figure 5 presents the average velocity predicted by the modified Wilson–Thomas model. For the range of velocity at two volumetric concentrations of 7.1 and 9%, the average velocity obtained with this model has a deviation of 1% to slightly over 10% to that from the experiment. The deviation becomes lower as the average velocity decreases. In other words, the friction factor is under-predicted by the modified Wilson–Thomas model as the velocity decreases.

This is believed to be attributable to either the shortcoming of the original model or unestablished turbulent flow at lower velocity. Figure 6 compares the friction factor predicted from the model with the experiment data in the pipe from original paper of Wilson and Thomas (1985) and it shows that the prediction is lower than the measured data. The lower the friction factor is given by the models, the higher the prediction of average velocity.

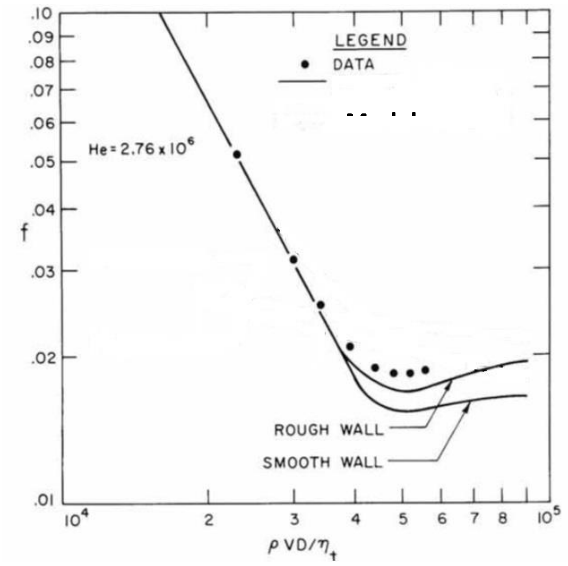

Figure 6 Wilson–Thomas model friction factor prediction for limestone slurry (from Wilson & Thomas 1985)

6 Conclusion

A set of semi-theoretical models in laminar, transitional and turbulent flow of non-Newtonian material have been presented in this paper. These models were found to agree closely with the experimental flume data that employed kaolin suspensions at several concentrations. In the laminar regime, the velocity prediction from the new Reynolds number agrees with the measured flume data with agreement improving as the velocity increases.

A new analysis to establish the onset of transitional flow has been presented and validated using the experimental data. This approach accurately predicts the laminar/turbulent transition.

For the transitional and fully turbulent flow regimes, a modified model based on the Wilson-Thomas model is presented. This model accounts for channel shape and the rheology of Herschel Bulkley material. Comparison of the model and measured flume data indicated a maximum absolute error of 15% in average velocity prediction.

However, further confirmation of the validity of the models presented is required using real tailings rather than model fluids such as kaolin and bentonite suspensions.

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