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Room and pillar stability analysis using linear elastic modelling and probability of failure — a case study

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Abstract

A stability analysis was carried out for an underground narrow vein room and pillar operation using linear elastic numerical modelling combined with a probability of failure (PoF) approach. The as-mined layout was input into the model to identify areas of increased pillar stress and obtain an average pillar stress (APS) for each pillar in an identified area. Pillar dimensions were measured from the given layout to calculate an empirical pillar strength (PS) using the Hedley and Grant (1972) approach. A Factor of Safety (FS) was evaluated for each pillar from the APS and PS, and the risk of failure for selected areas was estimated based on the proportional number of pillars with a sub-critical FS. As a result, three critical areas on the operation were identified for inspection and implementation of additional monitoring and/or remedial action.

A statistical simulation was further carried out using the Monte Carlo method to evaluate the impact of measured and observed variability in pillar size on the design. The final design proposed for the operation was based on a revised pillar strength factor (K), taking into account the effect of mining practices on the empirical design.

(Note: confidentiality limitations apply such that the operation is undisclosed and target areas are arbitrarily renamed).

1 Introduction

The pillar design for a narrow vein (2.0 m mining height) room and pillar operation, situated 100 to 200 m below surface (shallow mining) employs the empirical approach of Hedley and Grant (1972) to determine the required pillar sizes at varying depth. As part of a due diligence exercise, stability of the underground pillars was assessed taking into account geotechnical conditions, pillar geometry and APS. The approach adopted a combined assessment of K, FS and an evaluation of the PoF for selected areas, highlighted by an increased proportion of pillars having critical or sub-critical FS. APS was evaluated using the linear elastic boundary element package, Map3D (Wiles 2015).

Findings include revision of the pillar strength factor, identification of areas for increased monitoring and/or rehabilitation as necessary and a revision of the pillar layout based on the PoF approach.

2 Methodology

2.1 Site characterisation

The orebody is hosted within a platinum-bearing igneous intrusion. The mineralised zone comprises pyroxenite approximately 2.0 m thick, overlain and underlain by similar pyroxenitic material, dipping between 9 and 0° (practically sub-horizontal). The mine is classified as 'dry' with no significant water ingress. The ratio of horizontal to vertical stress (k) was assumed to be unity in all directions (hydrostatic) based on visual assessment of underground conditions and in the absence of a measured stress field.

2.1.1 Mining layout

The orebody is extracted through a series of strike-orientated drives (bords) separated by rectangular pillars approximately 1.5 to 1.8 times wider than the mining height (dip dimension) with lengths approximately 3 times longer than the pillar width (strike dimension). Bord widths are reduced by more than half to 6 m in poorer ground.

2.1.2 Major structures

The orebody is intersected by several sub-vertical faults and aplite dykes of variable throw and thickness, resulting in isolated geological loss zones (pillars); however, major disruptions to the mining layout have not occurred.

The orebody is influenced by three major joint sets, two sub-vertical orthogonal sets (J1 and J2) and one sub-horizontal (J3) set. The J3 set consists of undulating shear planes creating curved, splayed or lensoid intersections with the sub-vertical joints. The shear planes may be situated in the hanging wall, ore zone, or footwall, and influences pillar stability depending on its proximity.

Infill is frequently characterised by slickensided, serpentinised or talcose material, which deteriorates on exposure to water and varies in thickness from rock wall contact (<1 mm) up to 0.05 m. Consequently, the nature of structural discontinuities in the ore zone and immediate host rock has a significant influence on pillar stability.

2.1.3 Pillar condition

The condition of a selection of pillars in current mining areas was visually assessed to observe any signs of failure, while actual dimensions were measured to evaluate the measure of compliance and/or deviation from the design.

Although the pillars appeared to be intact with no visible signs of failure, it was found that the pillar widths expressed both as minimum pillar width, w_{min}, and effective pillar width, w_e, were frequently below the minimum design requirement, with the ratio of w_{min} to height (h), w_{min}: h ≈ 1:1, whereas the design w_{min}: h = 1.5. In contrast, w_e: h ≥ 2.0 which is in close agreement with the design as a result of the pillars being approximately three times longer than their width; however, this can be misleading for actual pillar stability due to structural effects on the pillar strength. The effective pillar width, w_e is defined as the ratio of four times the pillar area (A) to pillar circumference (C) (Equation 1):

$$w_e = \frac{4A}{C} \tag{1}$$

It was also noted that the pillars selected for observation were situated close to the advancing mining front such that tributary area load was not yet applicable. Pillars in the back area were not observed, but based on outcomes from the numerical analysis, it was recommended that certain areas be selected for monitoring (Section 3).

2.2 Pillar strength factor

Whereas the study focused on a probabilistic assessment of pillar stability using APS and an empirical pillar strength formula, a basic re-evaluation of the empirical pillar strength factor, K, was undertaken using the design rock mass strength (DRMS) (1990). The DRMS is an adjustment of the rock mass strength (RMS), taking into account effects of weathering, joint orientation and blasting. Underground structural mapping was carried out on a selection of pillars, for which the DRMS was found to be 10-20 MPa lower than the applied value of K = 68 MPa for the operation. Given that the value of K = 68 MPa was based on approximately 1/3 of the UCS (~220 MPa) for an unjointed rock mass, the pillar design was re-evaluated for an average DRMS value of 52 MPa in jointed conditions (‘mining layout #2’ (ML2)).

In better ground ('mining layout #1' (ML1)), K was re-evaluated from on-mine rock mass classification data, resulting in a slightly reduced value of K = 63 MPa. ML1 contains larger inter-pillar spans than ML2, hence a higher extraction ratio, e (%).

It is noted that Laubscher's (1990) DRMS approach is somewhat simplistic given the more recent work of Esterhuizen (2006), which takes into account the combined effects of joint orientation and joint condition to estimate a joint factor for the pillar strength. As such, there is room for further improvement to the pillar strength factor going forward.

2.3 Numerical analysis of APS and FS

In an attempt to obtain an indication of the overall stability of pillars on the mine as a whole, the mining footprint was built into the linear elastic boundary element (BEM) modelling package, Map3D (Wiles 2015). The APS was determined for each pillar for selected target areas, and related to the pillar strength (PS) to obtain a FS. PS was determined by measuring the actual pillar dimensions as provided on plan and using the Hedley and Grant (1972) formula for hard rock pillars (Equation 2).

$$PS = K \frac{w_e^{0.5}}{h^{0.75}} \tag{2}$$

where:

K = pillar strength factor (MPa).

w_e = effective pillar width (m).

h = pillar height (m).

The mining height was assumed to conform to the design, i.e. 2.0 m.

A selection of pillars from the model is illustrated in Figure 1, showing the normal principal stress distribution. The principal stress (σ_1) is assumed to be normal to the pillars (i.e. vertical) due to the shallow dip of the orebody (sub-horizontal). The area of interest encloses part of the main decline and the pillars are mined according to ML1 for better ground. A change in mining layout from ML1 in better ground to ML2 in poorer ground is visible in the image.

There are several limitations to this approach. The linear elastic approach inherently implies that the overburden behaves as a continuum, thereby discounting the potential effects of differential loading on the pillar system. However, the mining layout had not undergone major disruptions as a result of the regional geology and an elastically distributed overburden load was therefore accepted as applicable.

In addition to the behaviour of the overburden, the model also implied that the pillars behaved elastically. That means that very small pillars may be subject to spuriously high loads without undergoing failure. In reality, these small pillars will usually have failed and the load would have redistributed to the adjacent and surrounding pillars. In the model, it was assumed that the small pillars had not failed, with the result that the actual APS on surrounding pillars was likely to be higher than observed in the model, i.e. results underestimate the actual APS on intact pillars.

However, one of the primary advantages of the linear elastic approach was the capability to provide a reasonably quick, holistic estimate of the overall stress distribution for the operation, enabling the identification of vulnerable areas based on areas showing high stress.

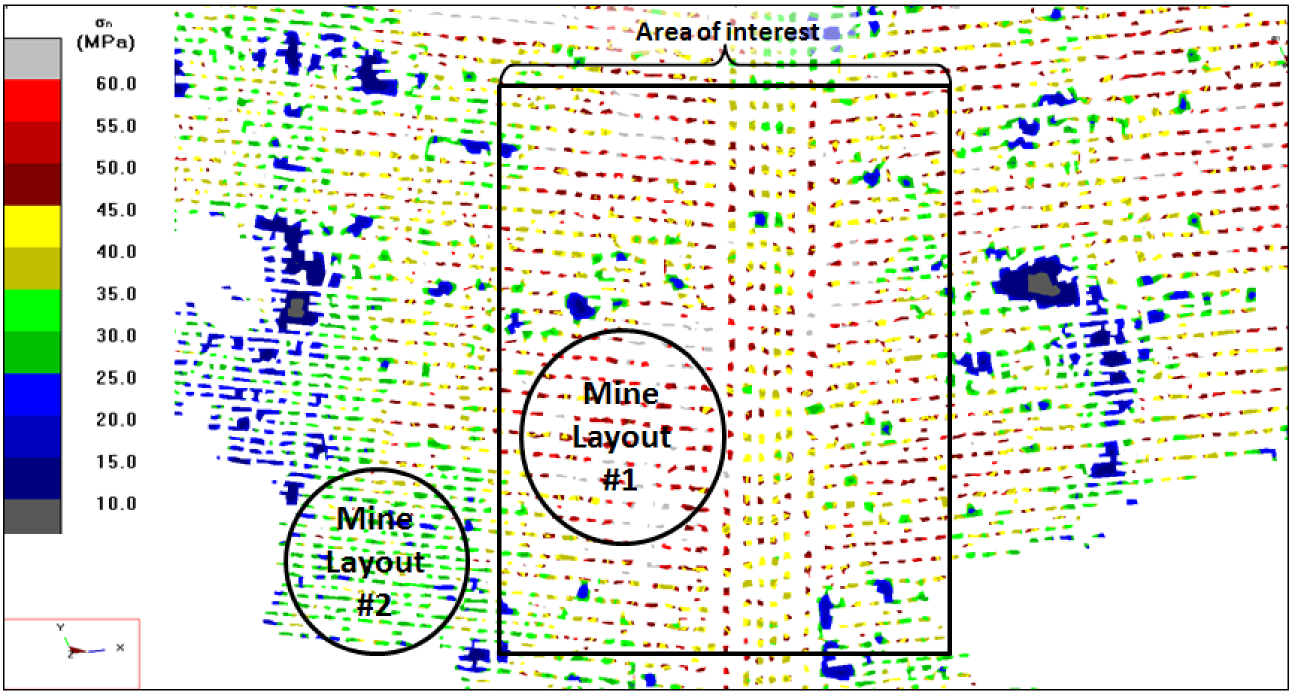

Figure 1 **Overview of normal pillar stress distribution for selected target area**

2.4 Probabilistic evaluation of pillar stability

A statistical analysis of the PoF for pillars from the selected target areas was evaluated using a Monte Carlo simulation, based on variability limits for pillar geometry as measured on plan and underground. The PoF was defined as the % number of pillars with FS < 1.0, where FS = 1.0 is considered critical for failure.

3 Findings

3.1 Numerical analysis of APS and FS

A total of 511 pillars from 11 representative areas were selected for evaluation, an average of 46 pillars per area. Each area was evaluated independently due to the pillar design being variable with depth below surface and ground condition. Results for the analysis area ML1-171N shown in Figure 2 are detailed for reference. In the example, 39 pillars were evaluated, representative of ML1, i.e. better ground conditions where K = 63 MPa.

Variability in pillar properties for the example (Figure 2) is summarised in Table 1. Variability for each parameter follows an approximately normal distribution (Figures 3, 4 and 5) with outliers as a result of spuriously small or large pillars, respectively. It is evident from Table 1 that the average for each of the parameters does not exceed (or fall below, whichever is critically relevant) the design criterion for that parameter; pillars are for the most part mined slightly larger than required, resulting in a marginally lower extraction ratio than the design and a higher overall FS.

This implies a reasonable confidence in the state of stability of pillars on the operation. However, it will be seen that large variability in pillar dimensions due to poor constraints on mining practice implies the potential for areas in which a significant proportion of pillars may be sub-critical and susceptible to failure.

The example area of interest is situated centrally within the mining footprint such that tributary area loading can be expected to apply, i.e. outside the influence of solid abutments at the mining front. However, the APS per pillar is somewhat lower than expected when compared to the design, as a result of unmined areas due to geological losses and more closely spaced pillars in ML2, that relieve a proportion of overburden load outside of the area of interest.

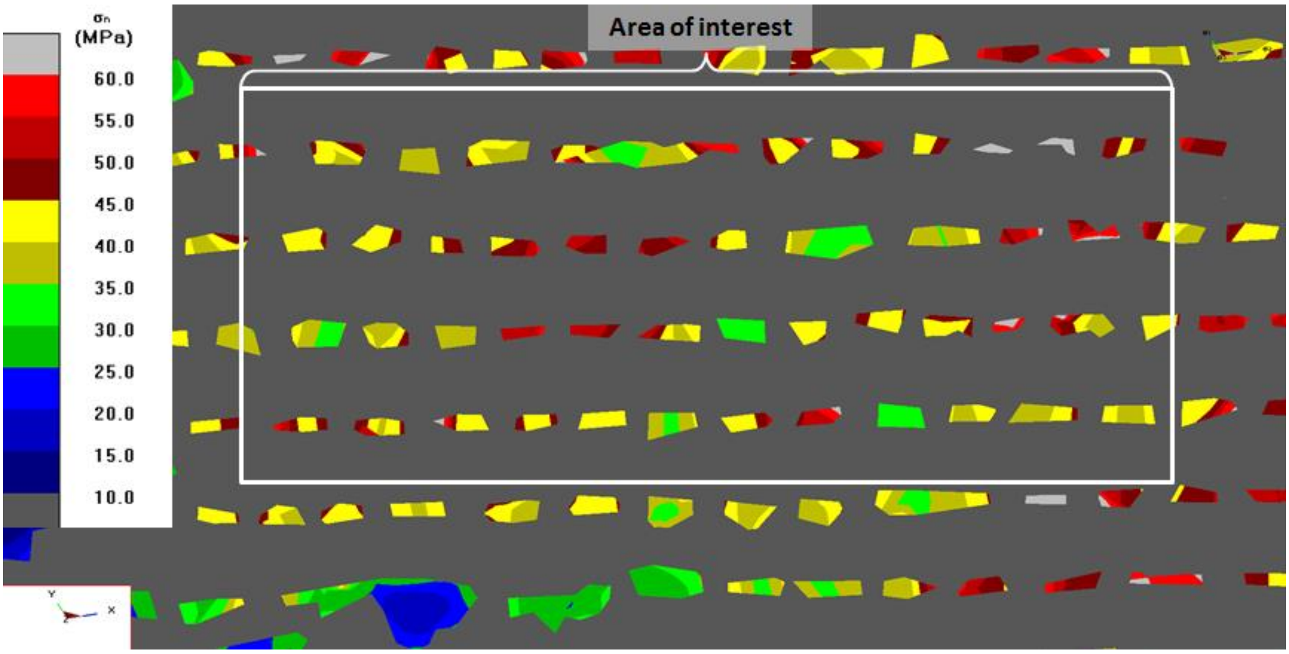

Figure 2 **Normal pillar stress (σ_1) for selected area of interest, ML1-171N**

Table 1 **Variability in pillar properties for the selected area of interest ML1-171N**

Parameter	Extraction ratio (%)	Average pillar stress (MPa)	Effective pillar width (m)	Pillar strength (MPa)	Factor of Safety
Design	90	51.5	4.6	80.5	1.6
Actual average	88	45.2	5.0	82.9	1.9
Minimum	58	33.9	2.1	54.3	0.7
Maximum	97	78.6	10.2	119.6	3.2
Standard deviation (stdev)	7	9.1	1.5	12.0	0.5
Number of cases in excess or below design (FS < 1.6)	17	6	14	16	9
Per cent in excess or below design (%)	44	15	36	41	23
Number of cases subcritical (FS < 1.0)					2
PoF (%)					5

There is significant variability in the effective pillar width (Figure 3) w_e (coefficient of variability = 30%), which implies loose attention to pillar shape and dimensions (mining practice). A particular concern with high variability in w_e is that the long length of pillars in relation to the width (length >3 × pillar width) can mask the effect of actual minimum pillar width, w_{min} without being directly evident from w_e. This was visually confirmed underground and was evident on the plan. A further interesting effect of variability in w_e is the critical influence on PoF, which can be seen in the results from the PoF analysis (Section 3.2).

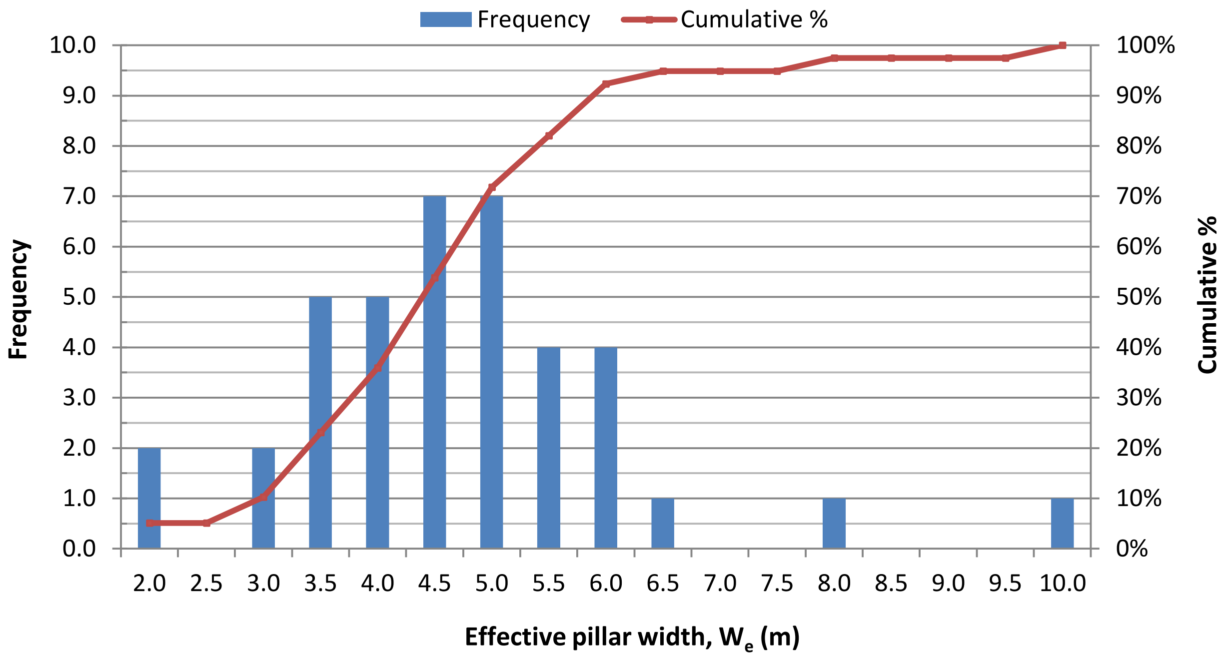

Figure 3 Effective pillar width distribution for the analysis area ML1-171N

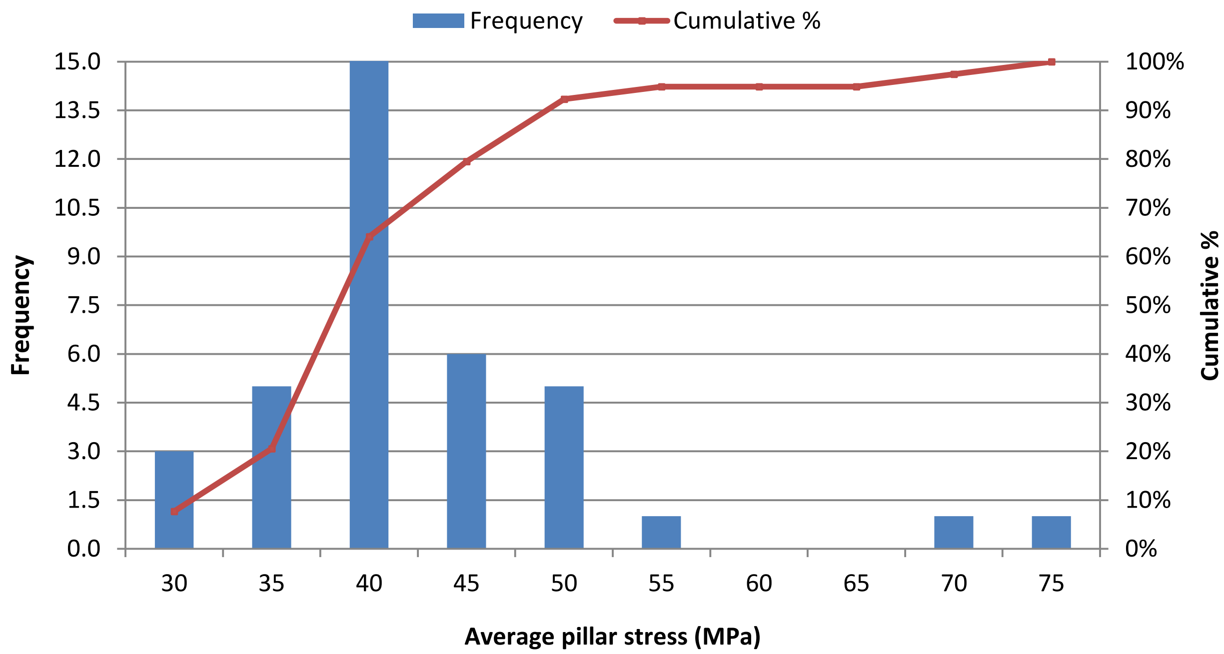

Figure 4 Average pillar stress distribution for the analysis area ML1-171N

For the FS distribution (Figure 5), 23% (nine out of 39) of the pillars have a FS less than the design; however, two pillars are sub-critical, which represents a 5% PoF. For reference, Salamon et al. (2005) recommend for pillar design that a probability of survival of 99% (one failure in 100 cases, i.e. 1% PoF) should be acceptable. The same recommendation has been applied to this analysis. In other words, in this particular area, the PoF exceeded the recommended tolerable risk threshold.

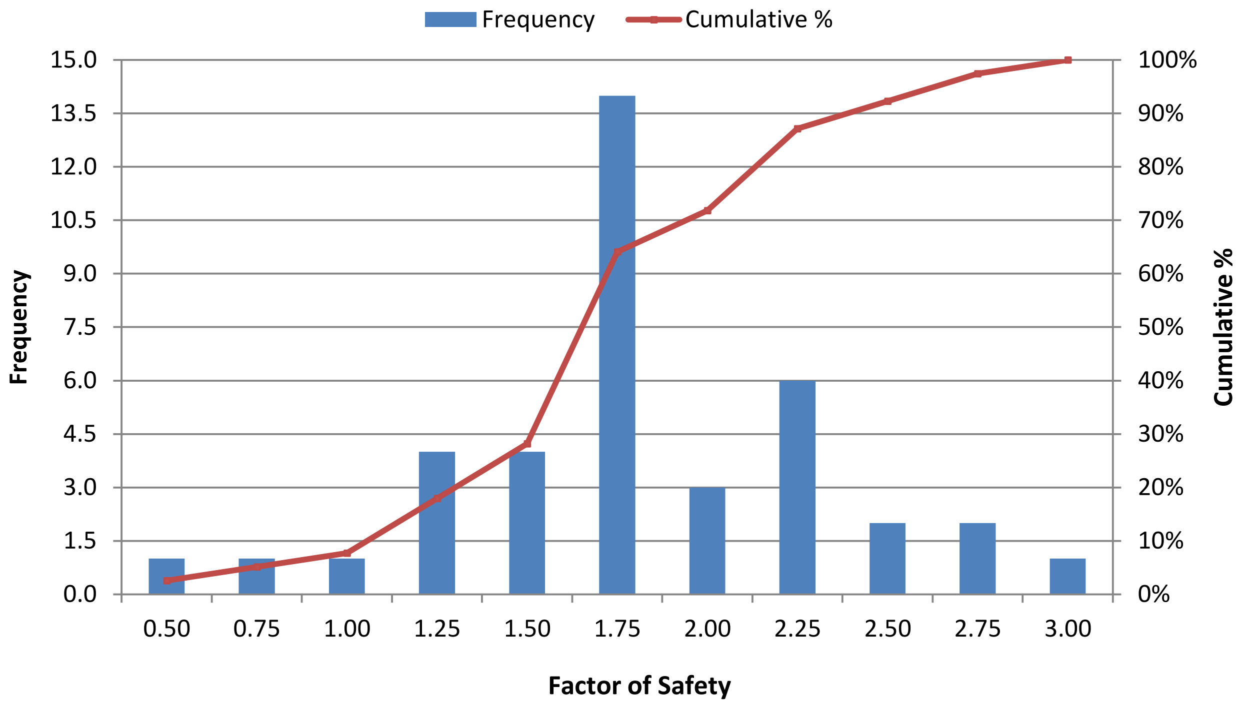

Figure 5 FS distribution for the analysis area ML1-171N

The same exercise was undertaken for each of the areas of interest, results of which are summarised in Figure 6 for FS. Out of the 11 analysis areas, four are located in ML1, five in ML2 and two in decline selections, at varying depths below surface. Three areas were identified for focused pillar monitoring and compliance auditing based on the presence of pillars with a FS < 1.0 (Figure 6), all of which were characterised by w_e less than the design. Out of a total of 511 pillars evaluated:

- 1.3% (seven pillars) had FS < 1.0 (i.e. PoF = 1.3%), which is slightly in excess of the recommended maximum of 1% PoF. Most of these occurrences are situated towards the centre of the mining footprint in ML1.
- 14.3% (73 pillars) had FS < 1.6 which suggests that ~85% of the pillars are cut to size or in excess of the design.
- In the two decline analysis areas, none of the pillars had sub-critical FS (<1.0); however, a large proportion of the pillars (>40%) are less than the design recommendation of FS ≥ 2.0. This implies that focussed attention to pillar cutting practices in the declines is required.

While the FS distribution for the operation suggests that there is currently no critical cause for concern, it is important to note that the large majority of pillars within ML2 are situated less than twice the mining depth from the advancing perimeter and hence are subject to reduced normal stress by the effects of the perimeter abutment. The PoF is therefore likely to increase as the mining perimeter advances and pillar load increases towards tributary area loading. The identified areas where PoF > 1.0% were recommended for monitoring and/or rehabilitation as necessary, together with strategic siting of barrier pillars to protect the central footprint for life of mine stability.

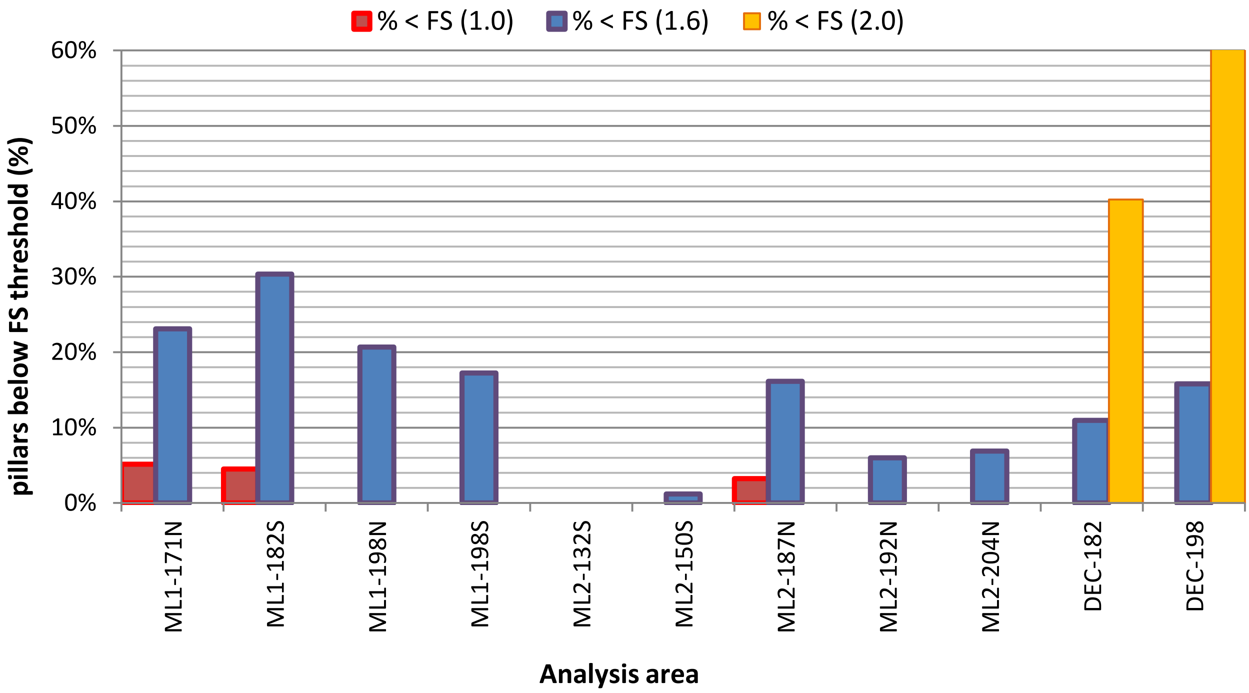

Figure 6 Per cent of pillars with FS below threshold per analysis area

3.2 Probability of failure analysis

Probability of failure of the design was tested using the Monte Carlo method of statistical analysis (Metropolis & Ulam 1949) and variability limits determined from the actual mining layout. The PoF for each mining section was evaluated assuming a maximum depth below surface for all of the pillars in each respective section, tributary area loading and pillar strength factors for ML1 and ML2 ('good' and 'jointed' ground respectively). A final pillar layout was put forward for the operation based on a PoF ≤ 1.0% for each section and recommended variability limits for pillar cutting discipline. The PoF was defined as the per cent probability for pillars to have a FS ≤ 1.0, where FS = 1.0 is considered critical for failure.

Variability limits for a section at the maximum depth of mining (204 m depth below surface) are presented in Table 2. A normal distribution for each of the contributing parameters was assumed (Section 3.1).

Table 2 Variability limits for evaluation of PoF measured underground and on plan

Parameter	Mean	Stdev	Coefficient of variation	Source
h (m)	2.0	0.2	10	Underground measurements
w_e (m)	5.0	1.6	32	Measured on plan
Extraction ratio, e (%)	76	17	22	Measured on plan (tributary area layout)

Results for ML2 are presented in Table 3 (actual practice and current design) and Table 4 (proposed design) and illustrated in Figures 7 and 8 (FS distribution truncated at FS = 5.0). Sensitivity of the current design to variability in w_e and h was tested and found to be significantly sensitive to w_e (large change in PoF for a small change in stdev) and less so to h (negligible change in PoF).

Table 3 PoF for ML2 — actual practice and current design

Parameter	As measured (actual practice)	Current design, current stdev	Current design, limited stdev (w_e)	Current design, limited stdev (w_e and h)	Current design, more limited stdev (w_e)
Mean w_e(m)	5.00	5.20	5.20	5.20	5.20
Stdev, w_e	1.60	1.60	1.00	1.00	0.75
Mean h (m)	2.00	2.00	2.00	2.00	2.00
Stdev, h	0.20	0.20	0.20	0.05	0.20
e (%)	0.76	0.77	0.77	0.77	0.77
Stdev, e	0.17	0.17	0.08	0.08	0.05
Deterministic FS	2.5	2.5	2.5	2.5	2.5
Average FS	2.5	2.5	2.5	2.5	2.5
Most frequent FS	2.5	2.2	2.4	2.4	2.5
Prob. FS < limit (%)	20.6	21.6	3.7	3.6	0.4

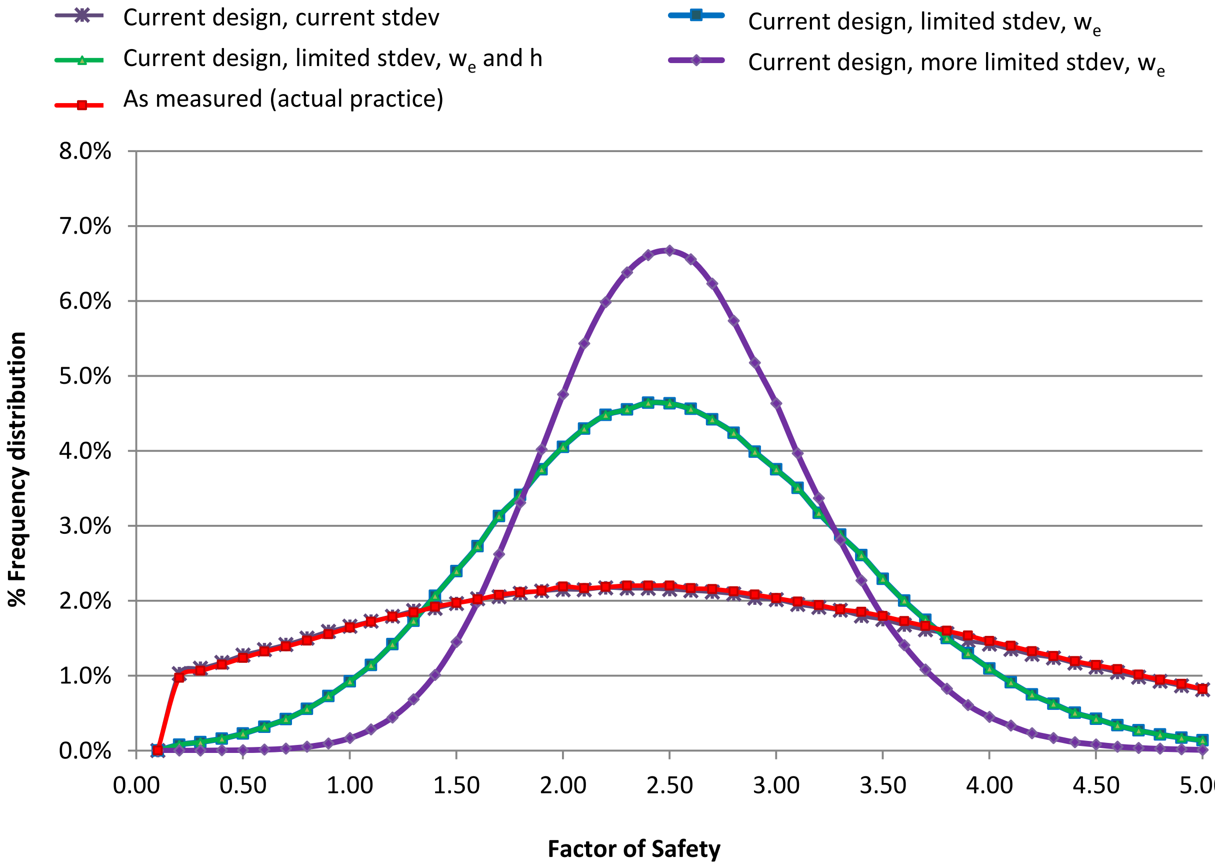

Figure 7 FS distribution for ML2 — actual practice and current design

An interesting result is PoF = 21% for current practice based on measured variability in pillar dimensions and tributary area (TAT) loading. This, compared with the overall PoF = 1.3% based on actual results for pillars from numerical analysis of the applied stress distribution, implies that the estimate of PoF using tributary area loading is conservative for the current conditions. The contribution of geological loss pillars to a reduction in applied pillar stress is inherently not accounted for in TAT simulation. Nonetheless, as the mining front advances, the average (normal) pillar stress is expected to increase as the overburden load approaches tributary area loading. The Monte Carlo method result is therefore useful to define an upper bound for PoF of the design, and drives a disciplined approach to maintain low variability in pillar cutting.

The current design, subject to variability in pillar dimensions as per current practice, has a high potential for instability, having a PoF approaching 25%. Limiting the variability in w_e has a significant effect in reducing the PoF, such that a 17% reduction in stdev in w_e from 1.60 to 0.75 m (reduction in the coefficient of variation from 32 to 15%) reduces the PoF to below critical (PoF = 0.4). Simultaneously, variability in extraction ratio is similarly reduced. In contrast, a similar reduction in variability of pillar height has a much lower effect on the PoF outcome.

However, a particular shortcoming of the current design is the narrow pillar width in one dimension which is masked by the calculation of w_e. A proposed pillar layout was put forward with dimensions approaching 'squat' (square) pillars by increasing the dip dimension and shortening the strike length such that w_{min} is not less than twice the mining height. The proposed design was tested for PoF using the current variability limits determined from actual practice and systematically limiting the variability in w_e (and similarly limiting e) until the required PoF ≤ 1.0 was achieved. Results are presented in Table 4.

Assuming that the mining practice continues to result in the current variability, the required pillar dimensions to meet PoF ≤ 1.0 were estimated; however, it soon became evident (Table 4) that a dramatic increase in pillar size would be required, with a substantial reduction in extraction ratio such that the benefit of improved pillar cutting practice is self-evident.

Table 1 PoF for ML2 — proposed design

Parameter	Required design with current stdev to meet PoF ≤ 1.0	Proposed design, current stdev	Proposed design, limited stdev	Proposed design, required stdev constraints for PoF ≤ 1.0
Mean w_e(m)	9.60	5.50	5.50	5.50
Stdev, w_e	1.60	1.60	1.00	0.75
Mean h (m)	2.00	2.00	2.00	2.00
Stdev, h	0.20	0.20	0.20	0.20
e (%)	0.62	0.77	0.77	0.77
Stdev, e	0.17	0.17	0.08	0.05
Deterministic FS	5.6	2.5	2.5	2.5
Average FS	5.6	2.5	2.5	2.5
Most frequent FS	5.0	2.2	2.4	2.5
Prob. FS < limit (%)	3.3	21.8	3.6	0.4

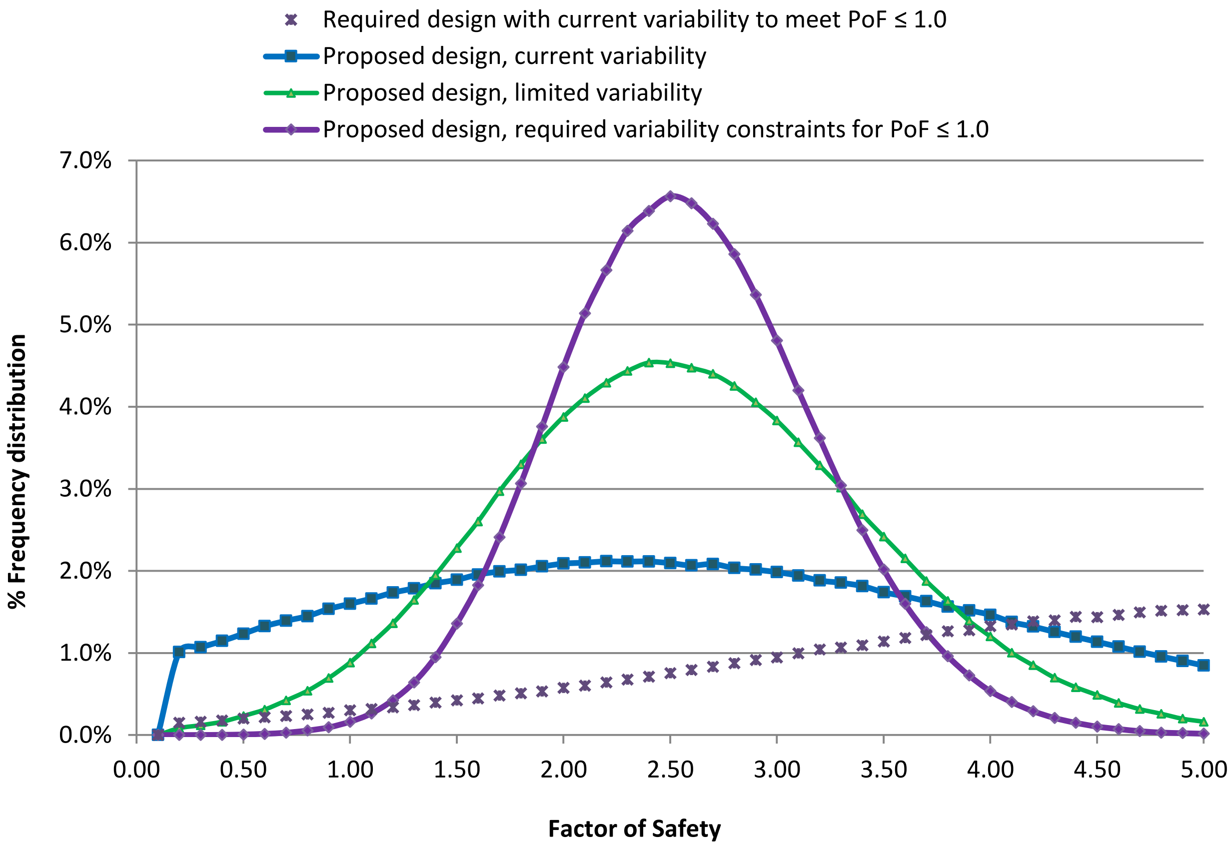

Figure 8 FS distribution for ML2 — proposed design

Similar to the current design, the proposed design requires that variability in pillar width be reduced by 17% from 1.60 to 0.75 m (with a similar reduction in variability for e) to maintain an overall PoF ≤ 1.0.

Intuitively, variability in w_e is difficult to conceptualise in practice; hence, to guide mining practice going forward, the effect of variability in actual pillar width and length should be quantified by obtaining the variability limits from observed data and substituting the results into the Monte Carlo analysis. Nonetheless, it is evident that improved pillar cutting discipline is required to limit the PoF for the operation to an acceptable threshold.

4 Conclusion

4.1 Numerical stress distribution analysis

Numerical analysis of average normal pillar stress was carried out using Map3D (Wiles 2015) to estimate the FS distribution for selections of pillars from various representative areas on the operation. A PoF for each section and for all the pillars collectively was obtained where 'failure' is defined as FS < 1.0. It was found that three areas contained 3% or more pillars with FS < 1.0, i.e. PoF ≥ 3%. Based on a limiting tolerable PoF = 1% for pillars, these sections were identified for monitoring and/or rehabilitation. The overall PoF for all of the pillars evaluated was found to be 1.3%, which is slightly in excess of the recommended maximum limit.

Whereas results from the numerical analysis suggest that there is currently no critical cause for concern, the large majority of pillars that were analysed are situated at a distance less than twice the mining depth from the advancing perimeter and hence subject to reduced normal stress by the effects of the perimeter abutment. The PoF is therefore likely to increase as the mining perimeter advances and pillar load increases towards tributary area loading. As a result, pillar monitoring and rehabilitation, as necessary, has been recommended in the identified areas, combined with the inclusion of barrier pillars to protect the central mining footprint for life of mine stability.

4.2 Probability of failure analysis

Probability of failure of the design was estimated using the Monte Carlo statistical method, assuming tributary area loading and variability in pillar dimensions determined from observed practice. It was evident that current practice results in a PoF ≈ 21% and that a substantial reduction in variability of pillar width (w_e) from stdev = 32% of w_e to stdev = 15% of w_e is required to constrain PoF < 1.0%.

However, the current design is characterised by long, narrow pillars such that w_e masks the effect of the minimum pillar width, $w_{min,}$ in one dimension. In the context of a jointed orebody, the minimum pillar width is easily reduced to unacceptably narrow limits due to wedge failure. A proposed pillar layout was therefore put forward such that w_{min} is not less than twice the mining height and the pillar shape approaches a 'squat' (square) form rather than an elongate form. Similar to the current design, variability in w_e must be constrained such that the stdev is not more than 15% of w_e to ensure PoF ≤ 1.0% for the proposed design.

4.3 Going forward

Several improvements to the analysis and the design have yet to be incorporated going forward. Amongst these are (i) a revision of the very high extraction ratio (>85%) for ML1; (ii) improved estimate of variability in pillar strength factor, K, the variability of which should be similarly incorporated into the statistical estimate of PoF; (iii) an estimate of the variance between plotted (plan) pillar geometry and actual underground geometry; and (iv) estimating the actual constraints on pillar width and length required to maintain a minimum PoF limit to assist with implementation of the design (mining practice).

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