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Hoek–Brown m_i estimation — a comparison of multistage triaxial with single stage triaxial testing

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Abstract

This paper presents an argument that the current accepted practice of obtaining the intact Hoek–Brown parameters σ_{ci} and m_i from single stage triaxial tests suffers from several drawbacks, for instance, many specimens are needed to obtain a single estimate of σ_{ci} and mi, and specimen variability can result in difficult to interpret regression analyses. A solution is proposed in that multistage triaxial testing can be used, provided the peak compressive strengths needed are based on servo controlled termination of each testing stage based on real-time measurement of Poisson's ratio. The paper proposes a procedure for determining peak strength for each stage and tests the proposed method against single stage triaxial tests. The paper finds that the approach is valid for practical purposes but also recommends further work.

1 Introduction

The Hoek–Brown failure criterion is widely used to estimate rock mass strength in rock engineering design and requires four parameters as inputs. The first parameter is the Geological Strength Index, or GSI, which can be estimated using guidance provided by Hoek et al. (2002, 2013), and many others, such as Cai et al. (2004, 2007). The second parameter is the damage factor, or D, which can be estimated using guidance provided by Hoek et al. (2002), or back analysis of existing slopes. The third parameter is the intact uniaxial strength, or σ_{ci}, which is obtained through laboratory uniaxial and triaxial testing. And the fourth parameter is the m_i value, which is obtained through laboratory triaxial testing (Hoek et al. 2002) or through uniaxial and Brazilian tensile testing (Cai 2009, 2010) and others. This paper focuses on the third and fourth parameters, σ_{ci} and mi, which require laboratory testing to estimate.

Current practice, as proposed by Hoek et al. (2002), suggests that σ_{ci} and m_i can be estimated by plotting the results of uniaxial and triaxial rock tests in principal stress space (minor principal stress on the x-axis and major principal stress on the y-axis) and carrying out a regression analysis to determine the best fit σ_{ci} and m_i to characterise the intact Hoek–Brown curve. Each point on the graph in principal stress space represents a single stage triaxial test or uniaxial test and requires an individual test specimen. As a result, variability between points in principal stress space is significant, as each test specimen has different strength properties, and a large number of test specimens are required to obtain reliable estimates of σ_{ci} and mi. For instance, Hoek et al. (2002) suggested that at least five data points should be used. Further work carried out by Gill et al. (2005), and Ruffolo and Shakoor (2009) showed that the number of tests required varies with the coefficient of variation of the test results. Their research (i.e. Gill et al. (2005), and Ruffolo and Shakoor (2009)) showed that more than 30 tests may be needed to estimate the uniaxial compressive strength of a single material with an allowable error of 10% on the estimate of the mean. If the m_i value is needed, the number of samples required can expand rapidly.

To decrease the number of specimens required an alternative test is available to carry out multiple triaxial tests on the same sample i.e. multistage testing (ISRM 1983) and others. For each test stage the confinement pressure (minor principal stress) is set by the test operator and the test deviator stress (major principal stress) increased to just before peak stress before proceeding to the next stage. The practical weakness of this method is that the point just before peak stress is picked manually by the operator in triaxial testing machines prior to the advent of servo control. This introduces operator selection as another random variable, which results in significant variation in the results. In addition, attempting to select a point just before achieving peak stress causes a significant amount of damage to accumulate in the sample, which may distort the results of later stages.

This paper introduces an alternative type of triaxial test where each stage is terminated automatically by a testing machine capable of automatically selecting a consistent point before peak stress based on Poisson's ratio. Such a machine is capable of the required precision, as it is computer controlled, and can apply changes in testing routine based on strain measurements through servo control in response to strain measurements. Machines of this type are now coming onto the market and some are already available for commercial test work.

This paper is part of a larger testing program, the results of which are, still being compiled. The aim of the paper is to investigate whether multistage testing, using an automated Poisson's ratio cut off, can be used instead of single stage testing, to obtain intact rock parameters for design. Further publications focussing more on the testing detail, and also additional data collected, are planned. The value, should the alternative test method prove useful, is that more information can be obtained from less samples. Furthermore, a single Hoek–Brown intact rock strength curve can be obtained from a single specimen. Resulting in the reduction of inefficiencies caused by random error, as experienced when using single stage testing on multiple specimens. An added advantage is that, if multiple specimens are available, a probabilistic analysis of Hoek–Brown parameters can now be carried out.

The inefficiencies of single stage testing are demonstrated in Section 2, followed by the experiment design, results and discussion in further sections.

2 Inefficiencies of single stage testing

The inefficiencies of single stage triaxial testing is investigated by presenting the current commonly used procedure and applying that to the single stage tests carried out for this paper. The inefficiencies are then highlighted as they occur in the process.

Estimating design intact rock parameters is often carried out using guidance provided by Hoek et al. (2002). The basic procedure is to select a number of samples belonging to the same geotechnical domain and carrying out a combination of uniaxial and triaxial tests to obtain peak strengths. Peak strengths are selected as the point on the stress strain curve where the sample fails or if post peak behaviour is being tested in a stiff testing machine, the point where the sample yields if no peak is obvious (Figure 1).

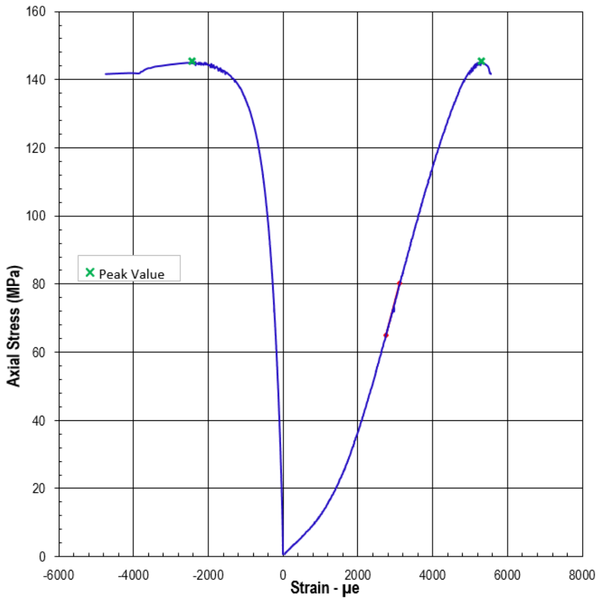


Figure 1 Typical uniaxial and triaxial test stress-strain curves

The peak strengths for all the available tests are plotted in principal stress space with minor principal stress (σ_3) on the x-axis and major principal stress (σ_1) on the y-axis (Figure 2). Figure 2(a) shows principal stress space as it is normally viewed with the x-axis scale distorted to improve visual interpretation of the trend. If both scales are kept the same, the result is Figure 2(b), which often does not display the trend very well.

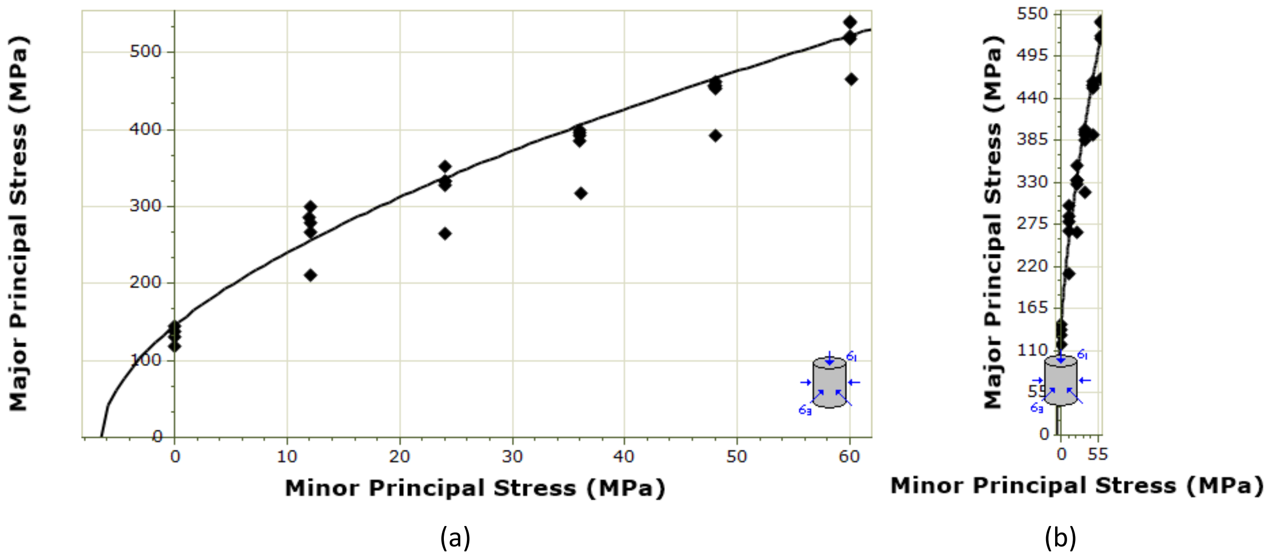
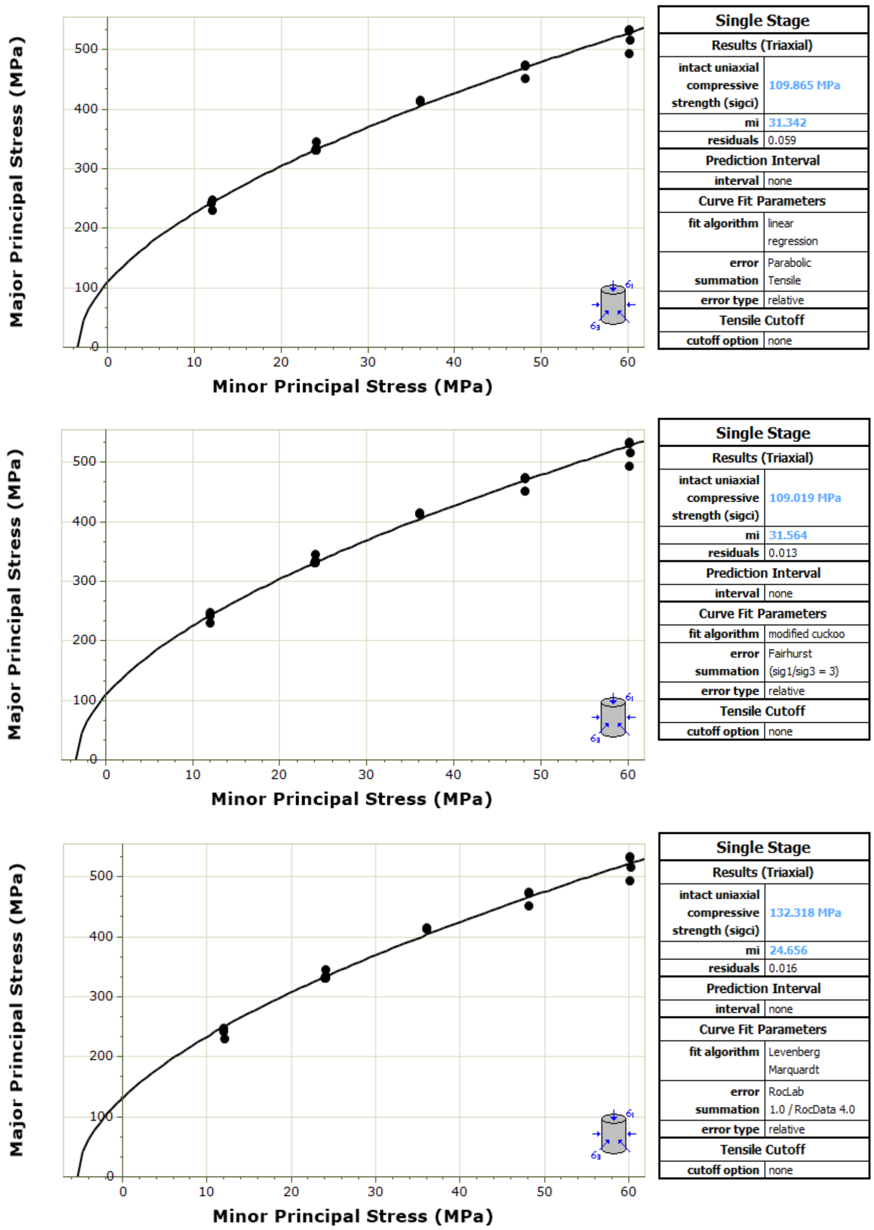


Figure 2 Typical uniaxial and triaxial test peak strengths in principal stress space. (a) x-scale different from y-scale; (b) x and y-scales are the same

Hoek (2007) suggests that at least five samples are required and, also that, the minor principal stresses for the samples be in well-spaced intervals, up to 50% of the uniaxial compressive strength (Hoek et al. 2002). Normally, in the case of geotechnical engineering, the rule is that laboratory testing is carried out at the same stress conditions as what occurs in the field. In the case of intact rock specimens for slope stability analysis, however, following the rule will result in incorrect estimations of m_i. The reason is that the scatter in peak strengths is almost always greater than the confinement range. For example, coefficients of variation of 20 to 40% are common, which, for a uniaxial strength of 40 MPa, results in data points with a peak strength variation of 8 to 16 MPa or more. For higher uniaxial strengths, the effect is even greater when compared to the typical minor principal stress in slopes, which rarely exceeds 2 MPa. The data cloud generated is, therefore, a sub-vertical elongated shape but the Hoek–Brown intact rock curve, which is applied through regression analysis, is a sub-horizontal function. The data is therefore mismatched to the equation with the result that regression analysis simply finds a line that goes through the centre of the data cloud but the angle of the line is almost arbitrary and bears no resemblance to the intact rock properties.

The next step is to carry out a regression analysis on the data. Several methods are available and, due to the high data variability typically found in rocks, significantly different regression results are typical. Even with almost perfect data, the different regression methods produce widely varying results as shown by Figure 3.



Note: different curve fitting methods provide` different values for σ_{ci} and m_i

Figure 3 Single stage regression results

In conclusion, four inefficiencies have been found in the current practice of obtaining σ_{ci} and m_i estimates using single stage triaxial testing combined with uniaxial tests. These are:

- Inefficiency 1: regression analysis is sensitive to data point placement on the minor principal stress axis with the quality of the analysis decreasing as the natural variability in test results increases compared to the spacing of data points on the x-axis. To partially correct Inefficiency 1, the minor principal stresses used for tests have to be selected much higher than would normally be preferred, thereby making the tests unrepresentative of field conditions.
- Inefficiency 2: to obtain statistically significant results large numbers of samples are required to obtain a single Hoek–Brown intact rock curve estimate.
- Inefficiency 3: statistical analysis of Hoek–Brown parameters for probabilistic design is hampered as groups of tests are needed to obtain a single estimate for m_i and σ_{ci}. This is especially severe for

the parameter m_i as uniaxial strength data can be used to estimate variability for the Hoek–Brown parameter σ_{ci} but no alternative for m_i is available.

- Inefficiency 4: single stage testing of samples operate on the assumption that specimens are approximately similar and the test results obtained can, therefore, be included in the same regression analysis. In practice, however, specimens of the same lithology can vary significantly in terms of mineral types and distribution, mineral grain size, micro fracturing, defects and many other attributes. As a result, it is nearly impossible to obtain similar specimens in most lithologies. Even if similar specimens are obtained, a sample bias is likely to result as the specimens will no longer be representative of the population.

A possible improvement in efficiency may result, if multistage triaxial testing is used, as the previous four inefficiencies are eliminated. There are, however, several issues that need to be resolved before this can occur.

- Multistage testing is hampered by the operator of the testing machine having to pick a failure point for each stage. Being subjective, and based on visual observation of test results, a large variation is evident, which negates many advantages of multistage triaxial testing.
- During multistage testing a certain amount of damage is accumulated by the triaxial sample, in each stage, which may influence the test result of subsequent stages.

The experiment design is aimed at eliminating these issues, as demonstrated in Section 3.

3 Experiment design

The test specimens used for the testing were donated by Lonmin Pty Ltd at Marikana, in South Africa. The Norite at Marikana is a medium grained igneous rock from the Marginal zone in the Rustenburg layered suite in the Western Limb of the Bushveld Complex (Cawthorn et al. 2006). Bushveld Complex Norite were selected as these rocks are renowned for their uniformity, isotropy and homogeneity (Figure 4). The intent of selecting uniform rocks was to eliminate random error, as far as possible, by selecting perfect samples.

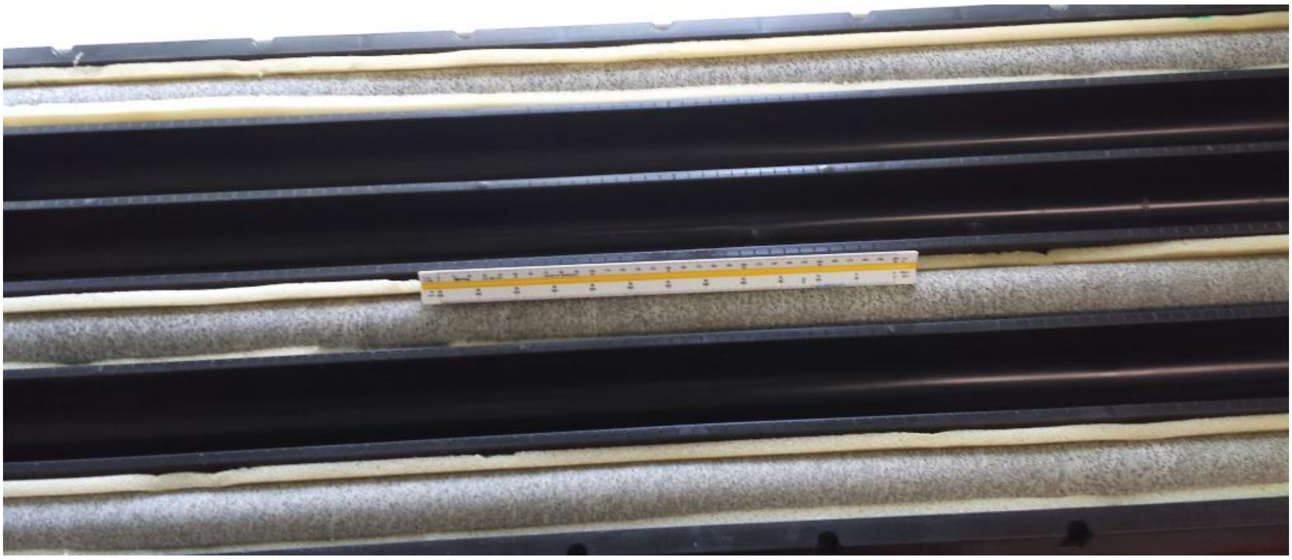


Figure 4 Photo of samples in core tray

Four experiments were carried out to evaluate the validity of the alternative multistage testing method proposed in this paper. The first experiment is a set of four uniaxial tests to determine the appropriate range of confinement for carrying out the triaxial tests. The second experiment is a set of single stage triaxial tests and the third experiment is a set of multistage triaxial tests. The final experiment consisted of a cumulative damage test to determine whether the selection of a Poisson's ratio of 0.4, as a trigger to terminate a testing stage in multistage testing, is sufficient to limit specimen damage to tolerable levels for further stages. The test programme and results are summarised as Table 1.

Table 1 Test programme description

Test type (experiment number)	Number of specimens	Number of stages	Minor principal stress
Uniaxial (1)	4	1	0 MPa
Single stage triaxial (2)	25	1	12, 24, 36, 48, 60 MPa
Multistage triaxial (3)	5	5	12, 24, 36, 48, 60 MPa
Multistage triaxial (4)	1	5	12 MPa

The current standard for m_i and σ_{ci} determination, as set by Hoek et al. (2002) and Eberhardt (2012), is based on single stage triaxial testing with confinement pressures determined using uniaxial compressive testing. To facilitate the triaxial testing for Experiments 2 and 3 four uniaxial tests were carried out as Experiment 1 to determine the target confinement of triaxial testing. The baseline testing presented as Experiment 2, to be used for validating the alternative multistage test method (Experiment 3), consisted of 29 test specimens divided into five lots of five specimens. Each lot was tested at a specified minor principal stress, with the maximum minor principal stress determined using the uniaxial test results from Experiment 1. The minor principal stresses used for testing were: 12, 24, 36, 48 and 60 MPa, based on a typical uniaxial test strength of 120 MPa. In each case, the specimen was instrumented with axial and radial strain measurement instrumentation to determine Poisson's ratio.

An additional five samples were tested using multistage testing as Experiment 3. These five samples were tested in five stages, each with the minor principal stress corresponding to those of the single stage tests i.e. 12, 24, 36, 48 and 60 MPa. The instrumentation used was the same as Experiment 2.

For the multistage tests of Experiment 3, the test was stopped automatically by the test machine when the strain instrumentation indicated that the sample had deformed to a Poisson's ratio of 0.4. Given that rock typically has an elastic Poisson's ratio of less than 0.4, this value was taken to indicate the onset of dilation as a precursor to yielding. Also, a very tight threshold of 0.4, as opposed to 0.5, was selected as it was hoped that the stage will be stopped before sufficient damage accumulated to influence the result of further stages (see Experiment 4). The last stage for each test was continued until failure of the sample was reached.

As a result of stopping each stage test as soon as the specimen started dilating, the peak strengths obtained at a Poisson's ratio of 0.4 cannot be assumed to represent failure. To remedy this situation, the peak strength of each stage was estimated by multiplying by an adjustment factor called the Peak to $\text{Poisson}_{0.4}$. The Peak to $\text{Poisson}_{0.4}$ adjustment factor was obtained by dividing the peak strength for the last stage by the deviator stress at a Poisson of 0.4 for the last stage. This ratio was taken to hold as a constant for a given sample.

The validity of this approach was tested by carrying out a standalone test, as Experiment 4, consisting of a single specimen, confined to a stress of 12 MPa and then loaded axially until a Poisson's ratio of 0.4 was achieved. The sample was then de-stressed axially to a stress of 12 MPa and reloaded to a Poisson of 0.4. The procedure was repeated five times, after which, the sample was loaded axially to failure. The Peak to $\text{Poisson}_{0.4}$ adjustment factor was then determined for each stage and the results compared.

After applying the Peak to $\text{Poisson}_{0.4}$ adjustment factors, the Hoek–Brown parameters for Experiments 2 and 3 were then determined using regression analysis as implemented by the software program Rocdata (Rocscience 2015). In all cases, the Levenberg-Marquardt regression equation was used except for Specimen 2 of the multistage testing (Experiment 3) where the Simplex regression equation was used. This variation was carried out as the Levenberg-Marquardt regression equation failed to provide a regression result for Specimen 2.

The results of Experiments 2 and 3 were then compared to determine if the Hoek–Brown properties, derived from single stage testing using 35 test specimens, could be adequately obtained using multistage testing of five test specimens.

4 Results and discussion

The results of the testing, carried out for this paper, comprised four uniaxial test results presented as Experiment 1, 25 single stage triaxial test peak shear strength results (Experiment 2), 25 multistage shear strength results at a Poisson's ratio of 0.4 (Experiment 3), five multistage triaxial peak shear strength results (Experiment 3), and five multistage triaxial shear strengths at a confinement of 12 MPa and an accompanying peak shear strength (Experiment 4).

The uniaxial compressive strength test results of Experiment 1 were: 145, 137, 131 and 119 MPa, with an average of 133 MPa. Based on the guidance provided by Hoek et al. (2002), the range of minor principal stresses for triaxial testing should be well spaced between zero and 50% of the uniaxial compressive strength. The representative uniaxial compressive strength selected for the testing was 120 MPa and the highest confinement used for the tests was 60 MPa.

The triaxial compressive strength test results for Experiments 1 and 2 are presented as Figure 5. From Figure 5 it can be seen that the multistage results have significantly lower major principal stresses than the single stage tests. The reason being is that the multistage results are at a Poisson's ratio of 0.4 while the single stage results are at peak strength.

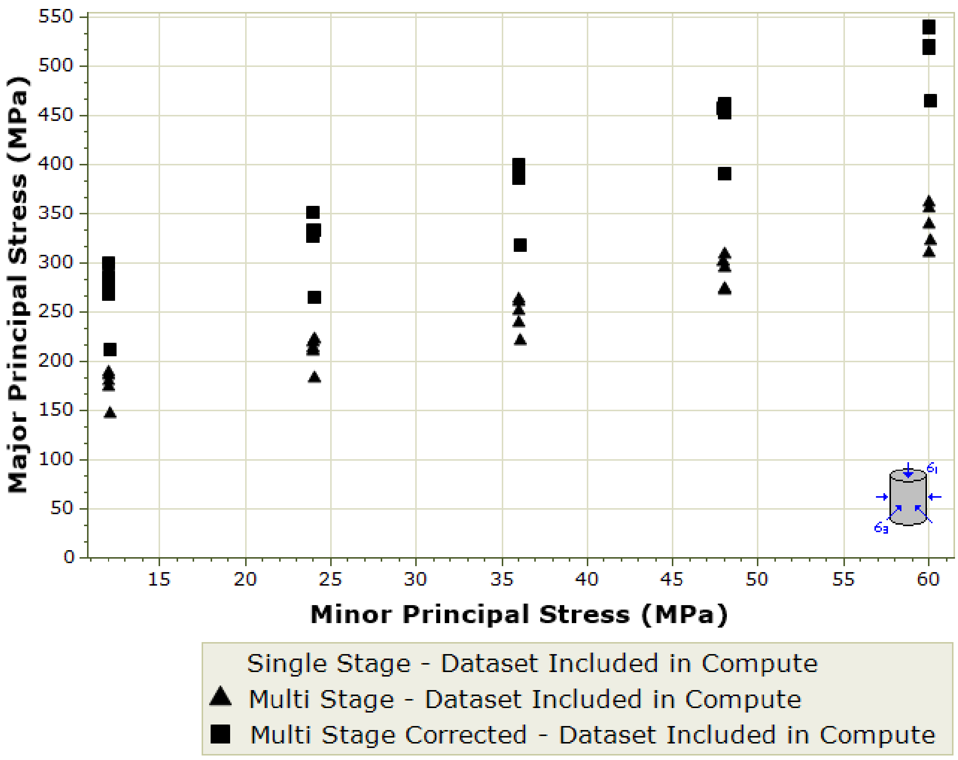


Figure 5 Principal stress space

To correct this effect, the multistage triaxial test results were multiplied by the Peak Poisson$_{0.4}$ adjustment factor. Based on the comparison between the peak deviator stress and the deviator stress at a Poisson of 0.4, the Peak Poisson$_{0.4}$ adjustment factors were: 1.67, 1.44, 1.49, 1.54 and 1.51 for each of the test specimen respectively. The adjusted multistage and the original multistage results from Experiment 3, as well as the single stage triaxial test results from Experiment 2, are presented as Figure 6. It can be seen visually that the adjusted multistage results are very close to the single stage test results except for that of multistage Specimen 2, which lies slightly below.

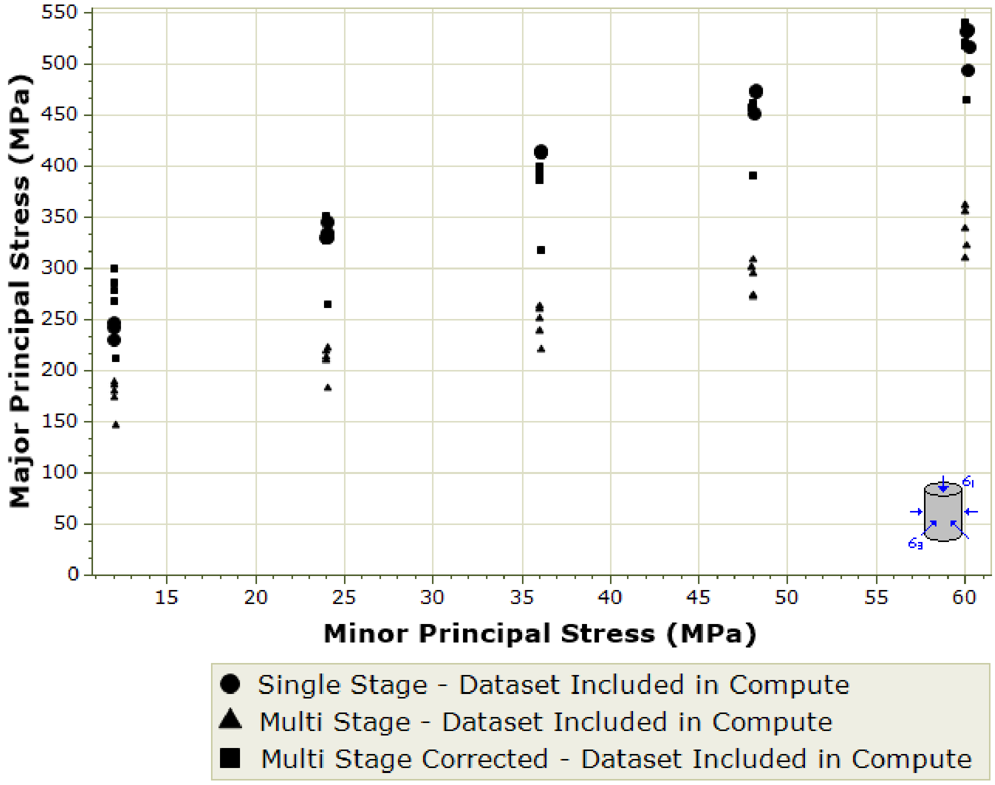


Figure 6 Adjusted multistage test results

The next step, in processing the data, was to carry out a regression analysis on the single stage test results of Experiment 1, to determine the Hoek–Brown σ_{ci} and m_i parameters according to Hoek et al. (2002) and Eberhardt (2012). The results of this regression are presented as Figure 7 and the resulting σ_{ci} is 154 MPa and m_i is 19.

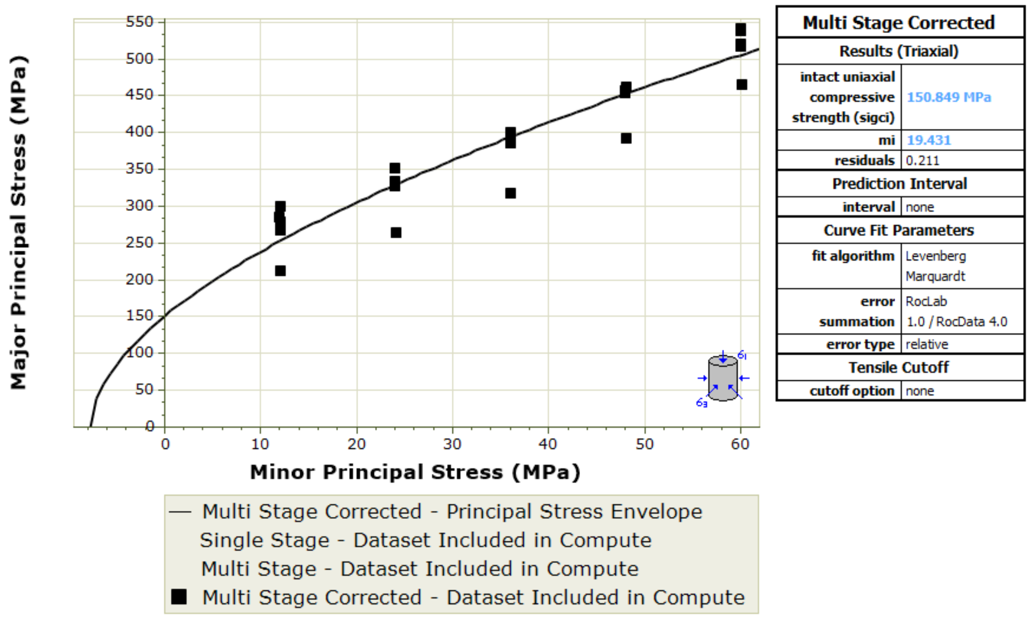


Figure 7 Single stage regression analysis result

The results of the multistage regression analysis from Experiment 3 are presented as Figures 8 to 12. For the multistage tests, the triaxial results from each multistage test was analysed using regression analysis. This is one of the advantages of using this method of multistage testing.

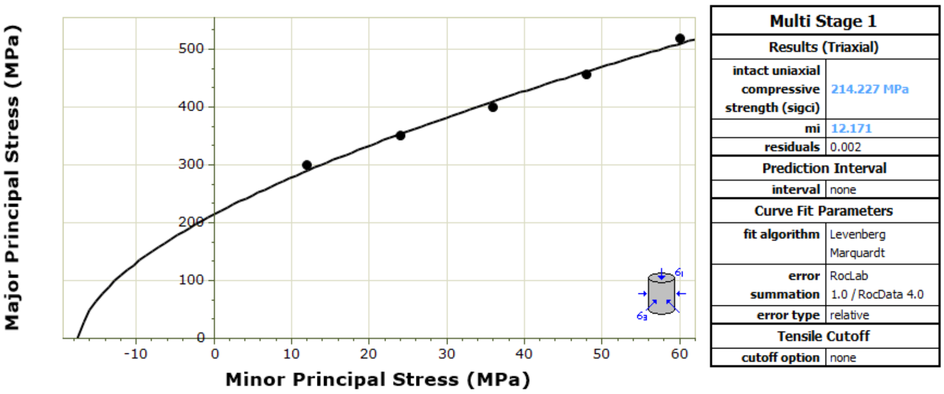


Figure 8 Multistage sample 1 regression analysis

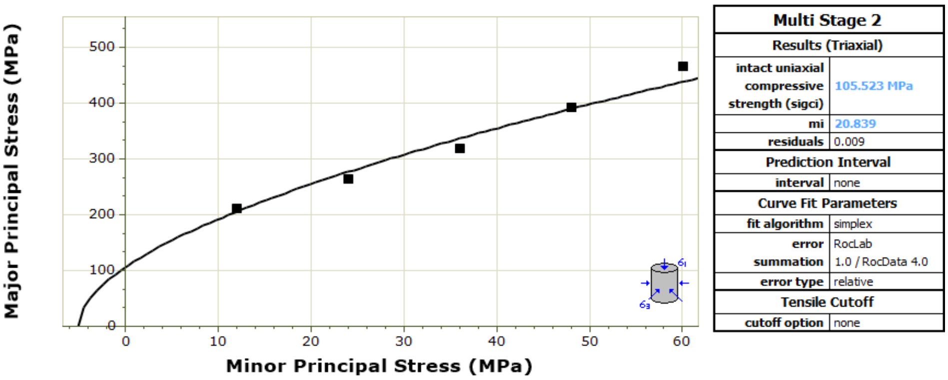


Figure 9 Multistage sample 2 regression analysis

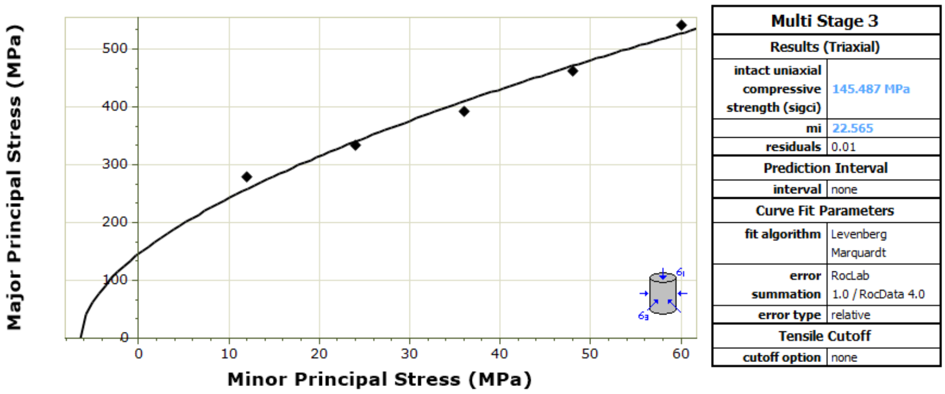


Figure 10 Multistage sample 3 regression analysis

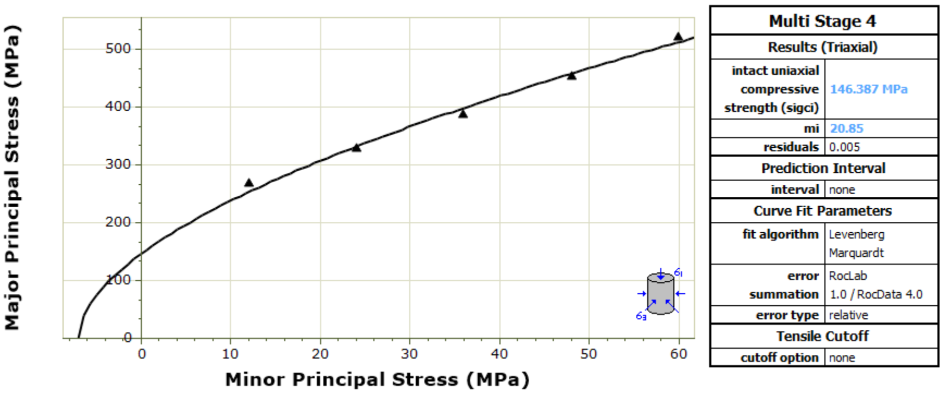


Figure 11 Multistage sample 4 regression analysis

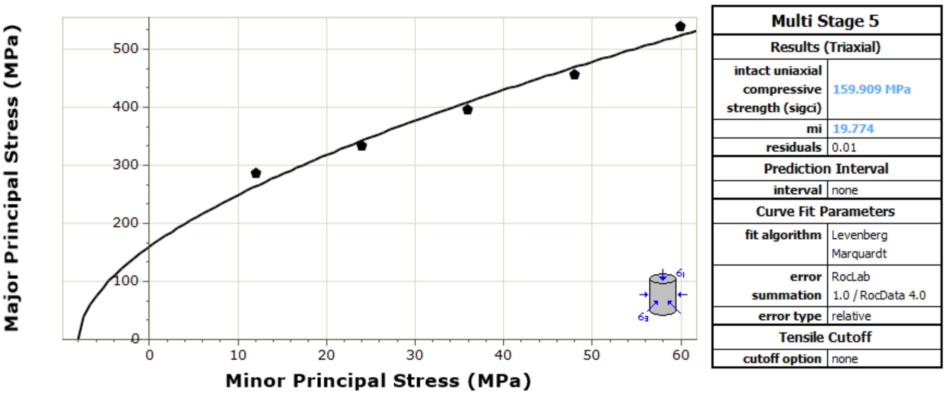


Figure 12 Multistage sample 5 regression analysis

The results of Experiments 2 and 3 are also presented and summarised as Table 2. Table 2 shows that for the specimens tested, the σ_{ci} and m_i values derived from the single stage tests of Experiment 2, corresponds almost perfectly to the average σ_{ci} and m_i values derived using the single specimen multistage tests of Experiment 3. Another interesting observation is that, while a visual inspection of the data points in principal stress space shows that all the values are very similar, the derived σ_{ci} and m_i parameters vary from 106 to 214 MPa for σ_{ci} and from 12 to 23 for m_i. As the data spread is very close compared to that of less carefully selected specimens and still exhibit such variability, it can be concluded that the Hoek–Brown intact rock failure criterion parameters are very sensitive to data variations and cannot be used independently. As a result, it is better to consider σ_{ci} and m_i as parameter pairs that should always be derived and used together.

Table 2 Single stage versus multistage analysis result comparison

Test	σ_{ci} (MPa)	m_i
Single stage (25 specimens)	**154**	**19**
Multistage specimen 1	214	12
Multistage specimen 2	106	21
Multistage specimen 3	145	23
Multistage specimen 4	146	21
Multistage specimen 5	160	20
Multistage average	**154**	**19**

The last result to be presented is that of Experiment 5. The results show that the deviator stress at a Poisson of 0.4 is not perfectly constant but shows a slight increase for successive stages with the results being: 109, 112, 113, 114 and 116 MPa. Nevertheless, given the lack of precision apparent in testing single stage triaxial tests, such small errors may still provide useful results.

5 Conclusion

The results of the analysis presented in this paper showed that the intact Hoek–Brown parameters σ_{ci} and m_i can be estimated using multistage testing. This requires automated selection of the stage termination deviator stress for each stage in combination with an adjustment factor based on the ratio of the peak deviator stress to the deviator stress at a Poisson's ratio of 0.4. The comparison presented in this paper was based on a total of 25 single stage triaxial specimens compared to only five multistage triaxial specimens. This represents a saving in number of specimens of 80%. In addition, as five estimates of σ_{ci} and m_i were obtained, probabilistic analysis is possible with multistage testing but not single stage testing. The paper also found that the values of the intact Hoek–Brown parameters σ_{ci} and m_i are very sensitive to changes in input values, and as they are determined using regression, should rather be derived and used as parameters pairs.

As the paper is only based on a single, highly homogenous rock type, the reader must be cautious not to assume the relationships discovered are universally applicable. It is suggested that the methodology be repeated for different rock types until such time as the relationships can be universally demonstrated. In particular, the validity of the ratio between the peak strength and the deviator stress at a Poisson of 0.4 is not generally established, even though it may appear to be valid in this case.

Acknowledgement

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