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Unconventional methods to treat geotechnical uncertainty in slope design

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Abstract

The definition of the geotechnical model for slope design is based on the geological, structural, rock mass and hydrogeological models. Each model is described by different sets of information and parameters and is defined at a scale of interest for the purpose of the analysis of slope behaviour. However, no clear guidelines exist in terms of the appropriate statistical methods to manage this information. Probabilistic methods are traditionally used to account for the uncertainty in engineering design, however, the base assumptions of these methods are not always fully understood, resulting in misinterpretations of results. There are two main approaches of statistical analysis known as frequentist (classical) and Bayesian, which are based on different interpretations of probability. In the classical approach, probabilities are considered as frequencies in a series of similar trials, whereas in the Bayesian approach, probabilities correspond to degrees of belief. One of the main characteristics of the Bayesian approach is that makes use of both prior information on the hypothesis (or model) being examined and the likelihood of the available data, to provide a balanced answer to the probability of that hypothesis (or model). Another aspect of the uncertainty characterisation process is the understanding of the type of uncertainty present in the various components of the geotechnical model. At a broad level there are two main types of uncertainty in geotechnical engineering, one due to random variation of the aspect evaluated (aleatory) and the second due to lack of knowledge of the subject under analysis (epistemic). The uncertainty is considered epistemic if it can be reduced with the collection of additional data or by refining models, otherwise it is treated as natural variation. The majority of the uncertainty in the geotechnical model for slope design is epistemic, typically analysed with probabilistic methods. However, epistemic uncertainty has different aspects some of which (i.e. vagueness or non-specificity) can be represented more naturally with alternative approaches outside the field of probability (i.e. interval analysis, possibility and evidence theories). Simple examples will be included to illustrate the merits of treating uncertainty in the mine slope design process with unconventional methods such as Bayesian statistics and non-probabilistic based approaches.

1 Introduction

One of the major difficulties encountered by the geotechnical engineer is to deal with the uncertainty present in every aspect of the process of slope design. Uncertainty is associated with natural variation of parameters and properties, and the imprecision and unpredictability caused by insufficient information on parameters or models. Design strategies to deal with the problems associated with uncertainty include conservative design options with large factors of safety, adjustments during the implementation phases based on observations of performance, and the use of probabilistic methods that attempt to measure and account for the uncertainty in the design. However, one of the drawbacks of the probabilistic approach is related to the strong component of subjective information such as engineering judgement that is incorporated in the process without a formal framework to do so. Another weakness of the probabilistic approach is related to the misunderstanding of the basic assumptions of the classical statistical methods that commonly results in interpretations of statistical results that exceed the capabilities of these methods. Some examples that illustrate this point are the assignment of probability distributions derived from samples as unique representations of populations, or the use of confidence intervals (CIs) as a measure of data reliability. The Bayesian approach is based on a particular interpretation of probability and offers an adequate framework to treat uncertainty in the geotechnical model for slope design. It offers a formal way to combine hard data with subjective information, and provides the probability measures of the hypothesis, parameters or models given the data. These are the type of results needed by the geotechnical engineer, as opposed to the probability of data assuming that the hypothesis, parameters or models are true.

The epistemic uncertainty associated with lack of information has a multifaceted character, and there are situations where probabilistic methods are incapable of adequately representing aspects such as non-specificity, fuzziness or ambiguity. Non-probabilistic methods such as interval analysis, fuzzy set analysis and approaches based on possibility and evidence theories are indicated in these cases. The paper provides a brief description of some unconventional approaches to treat uncertainty that have the application potential during the construction of geotechnical models for slope design.

2 Uncertainty in the geotechnical model for slope design

The geotechnical model for slope design is particularly complex because it incorporates information from different already complex models. The slope design model is based on the geological, structural, rock mass and hydrogeological models (Stacey 2009). Each model is described by different sets of information and parameters and is defined at a scale of interest for the analysis of slope behaviour. Intuitively, it is clear that there is uncertainty in the geotechnical model, but to have a better understanding of how this uncertainty affects the design process, it is necessary to look in more detail at its characteristics.

2.1 Types of uncertainty

Uncertainty is associated with various concepts such as unpredictability, imprecision, variability and so forth. At a basic level, uncertainty can be categorised into aleatory and epistemic. Baecher and Christian (2003) discussed these types of uncertainty in detail, indicating that aleatory uncertainty is associated with random variations, natural variability, occurring in the world, of external character; whereas epistemic uncertainty is associated with the unknown, derived from lack of knowledge, occurring in the mind, of internal character. The epistemic uncertainty can be reduced with the collection of additional data or by refining models based on a better understanding of the entities represented. The natural variation on the other hand cannot be reduced with more information, which will only serve to have a better representation of this type of uncertainty.

The sketch in Figure 1 is adapted from Bedi and Harrison (2013) and shows the distinction between the two types of uncertainty in terms of the available information at a particular point in time. The limit state of precise information that defines the point of irreducible uncertainty, moves through time towards the end of complete certainty as a result of technological advances. This is a consequence of a better understanding of the processes perceived initially as random. An example of this situation is the distribution of fractures in a rock mass. Baecher and Christian (2003) indicated that the separation between epistemic and aleatory uncertainty in a model is the result of a trade-off defined by the geotechnical engineer to treat the uncertainty.

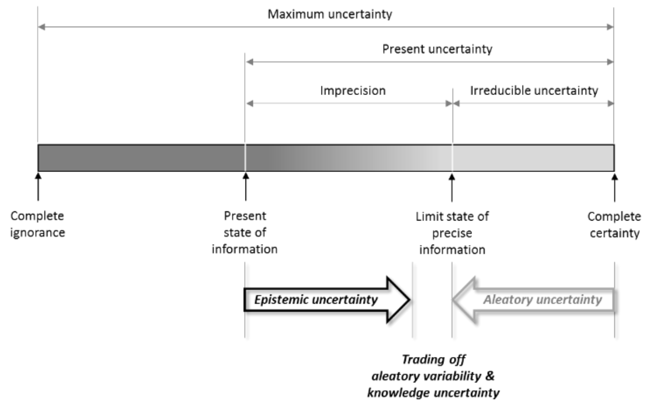


Figure 1 Relationship between types of uncertainty and information available (adapted from Bedi & Harrison 2013)

2.2 Uncertainty in the geotechnical model

The amount of geotechnical data typically available for slope design is small compared with that collected for mineral exploration and resource model estimation. Inferences on rock properties are based on limited data, uncertainty levels are perceived to be high, and the quantification of the confidence levels of model parameters is based on rudimentary methods or not evaluated at all. Moreover, the geotechnical model borrows information from other models with no measure of confidence, or with confidence levels assigned using rudimentary systems that cannot capture the complexities of spatial variations, and trends and cross correlations in addition to data characteristics. The transfer of information across models is done in an intuitive manner, with a strong judgemental basis. The end result is that the levels of confidence of the geotechnical model and its components are unknown or defined in a rudimentary way. The implications of the lack of a suitable approach to quantify the confidence of the geotechnical model are the inability to judge whether the available data is sufficient to support the design at the various stages of project development and the difficulty to define strategies for site characterisation on a rational basis.

The uncertainty in the geotechnical model for slope design is present in all the component models in different forms. The sources of uncertainty include:

- Inherent variability of the basic properties considered as random variables (i.e. structural features, Unconfined Compression Strength (UCS), Rock Quality Designation (RQD) etc.).
- Measurement errors of the properties.
- Estimation of the statistical parameters used to represent the variables (i.e. mean, standard deviation etc.).
- Approximations in the definition of sub-models to estimate derived variables (i.e. Hoek-Brown mi parameter from UCS, Brazilian Tensile Strength (BTS) and Triaxial Compression Strength (TCS) testing, Geological Strength Index (GSI) from joint structure and joint condition descriptors etc.).

A large part of the uncertainty present in the geotechnical model for slope design corresponds to epistemic uncertainty that would be susceptible to reduction with increased data collection, but this is rarely achieved due to the typical constrains in the mining environment.

3 Conventional ways to treat uncertainty in slope design

The situation in the geotechnical model for slope design is that the levels of information are relatively low and the range of the epistemic uncertainty as sketched in Figure 1 is wide, and commonly treated as aleatory uncertainty by means of assuming large variances and wide distributions of parameters. However, the statistical methods used in this process are inconsistent with these practices, as will be discussed hereafter. Common strategies to deal with uncertainty in geotechnical engineering were described by Christian (2003) and a brief description of the strategies relevant to the slope design process is presented next.

3.1 Conservative design

The simplest (although not the most efficient) way to account for the uncertainty in the geotechnical model is through the implementation of conservative designs. This is done by selecting higher factors of safety or low probabilities of failure in the acceptability criteria of the slope design. However, the difficulty of defining what are acceptable design values remains. Moreover, this strategy might not be effective in many mining projects where the steepest or highest slopes are often required to achieve the sought economic benefit of the project. A conservative design in this scenario likely would result in a financially unviable mine.

3.2 Observational method

The observational method is a common way to deal with uncertainty in geotechnical information in many types of engineering projects. The approach is part of the normal process of measuring performance of the works as the project progresses, to verify the original assumptions and models, and to implement the pertinent design adjustments to ensure design objectives are achieved. However, there are situations in mine slope projects where changes are difficult to implement at the time they are identified as necessary, reducing the space for this strategy. For example, this is the case when the flattening of a slope is required to prevent a ramp failure, but the implementation might be unfeasible at the time the need for this measure is identified.

3.3 Quantification of uncertainty

Uncertainties are quantified with probabilities, which in turn can be interpreted as frequencies in a series of similar trials, or as degrees of belief. Some aspects of geotechnical engineering can be treated as random entities represented by relative frequencies and others may correspond to unique unknown states of nature better considered as degrees of belief. An example of the former is a material property evaluated with data from laboratory testing, and the latter can be represented by any form of expert opinion, for example when a geological section is constructed from site investigation data. Baecher and Christian (2003) provide a detailed discussion on the topic of duality in the interpretation of uncertainty and probability in geotechnical engineering. They indicate that both types of probabilities are present in risk and reliability analysis, and point out that the separation between them is a modelling artefact rather than an immutable property of nature.

The two alternative interpretations of probability are at the base of the two approaches of statistical analysis known as frequentist (classical) and Bayesian. In mineral exploration, the approach to deal with uncertainty is based on classical statistics characterised by the systematic collection of data and the use of geostatistics to model spatial variation. In the oil and gas industries, uncertainty is evaluated through risk analysis methods based on decision theory and Bayesian concepts. In the geotechnical engineering field for slope design there is not a clear definition on the appropriate statistical approach to follow to quantify uncertainty. However, it is argued that Bayesian statistical methods are a better option to treat the geotechnical uncertainty in slope design, because they provide a formal framework to combine hard data, which is typically scarce, with other sources of information that may be available, including expert judgment.

4 Probabilistic methods to treat uncertainty

The basis of classical statistical methods is consistent with the concepts behind the aleatory type of uncertainty but less so with the epistemic uncertainty. The Bayesian statistical approach is well suited to deal with both types of uncertainty and will be of great benefit to treat the uncertainty in the geotechnical model for slope design. Unfortunately, its use in this particular area is rare, probably due to lack of understanding of its conceptual basis.

4.1 Frequentist versus Bayesian statistical methods

The more relevant points of contrast between the frequentist and Bayesian approaches are summarised in Table 1. The first aspect constitutes one of the more important advantages of the Bayesian approach as it addresses the question of interest to the geotechnical engineer. This aspect is also at the base of the misunderstanding on the type of answer that the classical methods provide. A simple way to present Bayes' equation, using the definition of terms in Table 1 is:

$$p(H|D) = p(D|H)\ p(H) / p(D) \tag{1}$$

which can also be interpreted in the following manner (Kruschke 2014):

$$\text{posterior} = \text{likelihood prior} / \text{evidence} \tag{2}$$

The 'posterior' is the answer of interest when defining the geotechnical model for design, the 'likelihood' of data is the answer given by classical statistical methods, the 'prior' represents the initial knowledge (or lack of it) on the hypothesis and the 'evidence' of data normally treated as a normalisation factor so that the posterior integrates to 1. When p(H) is set to a uniform distribution representing the case of no previous knowledge, the equation reduces to $p(H|D) \propto p(D|H)$ and the two approaches provide the same answer. For this reason, the frequentist method can often be viewed as a special case of the Bayesian approach for some (implicit) choice of the prior (VanderPlas 2014).

Table 1 Key aspects of contrast between the frequentist (classical) and Bayesian approaches of statistical analysis

Aspect	Frequentist approach	Bayesian approach
Question answered with the approach	What is the probability of the data if the hypothesis (parameter or model) examined is true ($p[D/H]$)	What is the probability of the hypothesis (parameter or model) examined given the data observed ($p[H\|D]$)
Information used	Only data collected with sampling ($p[D\|H]$)	Prior information of any type ($p[H]$) and data from sampling ($p[D\|H]$)
Characteristics of the result from the inference process	Point estimate (maximum likelihood) and standard error of the parameter (or model) evaluated	Probability distribution of the parameter (or model) evaluated
Assumptions regarding data and parameters (or models)	Data are random, parameters (or models) are fixed	Data are fixed, parameters (or models) are random
Inference method	Based on null hypothesis significance testing	Based on the updating of prior information by adding the effect of observed data to provide a posterior distribution reflecting a balance between the two inputs

The main criticism to the Bayesian approach is related to the use of prior information which in some cases could be subjective. However, this aspect is of little relevance in the area of mine slope design, where subjective information is important and continuously incorporated into the process, although in an intuitive and non-formal way. The Bayesian approach provides a framework to use this type of information in a formal and more rational way.

4.2 Common misinterpretations of results from frequentist statistical methods

A consequence of the different interpretations of probability is the contrasting assumptions regarding data and parameters made by the approaches, which in turn affects how the boundaries of model parameters are determined. In the frequentist approach CIs from data are used to define meaningful parameter boundaries, whereas in the Bayesian approach this is done with credible regions of the posterior distribution.

The CI is defined by upper and lower bound values above and below the mean of a data sample, and is associated with good estimates of the unknown population parameter investigated. The CI is calculated from a particular sample and its width depends on the number of data points in the sample, and the chosen level of confidence for the estimation. For this reason, this result is commonly used as a measure of confidence of parameter estimates, without a full understanding of the meaning. A CI is specific to a data sample and its confidence level only has meaning in repeated sampling. For example, if the 95% CI for the mean UCS of a particular rock type is constructed, it either includes the true UCS value or it does not, but it is not possible to know the situation for that particular CI. The 95% confidence means that if the sampling process were repeated numerous times, and CI's calculated for those various samples, 95% of the sample sets will have CI's containing the true UCS value. However, because the true value is an unknown fixed parameter in the frequentist framework, it is not possible to identify the sample sets containing the true UCS. The uncertainty regarding the true UCS value remains.

Figure 2 shows an example of repeated sampling that allows an appreciation of the meaning of the CI in the frequentist approach. The values could represent UCS results for a particular rock type, but the data was randomly generated to illustrate the point. A total of 100 data sets of 15 values each were sampled from a normal distribution with a mean of 120 and standard deviation of 30, which represent the unknown fixed parameters of the population. Each data set has its own mean and standard deviation and the bars in Figure 2 correspond to the 95% CIs of the mean. However, for this particular group of data sets, 91 of the intervals contain the true mean. A larger number of data sets would be required to get a better approximation of the 95% level used for the construction of the intervals. Nevertheless, the important point with this example is that in terms of each individual data set, there is no probability associated with the inclusion of the true mean. The interval either includes it or does not, and in a real case situation, there would be only one data set and it would not be possible to estimate the true value.

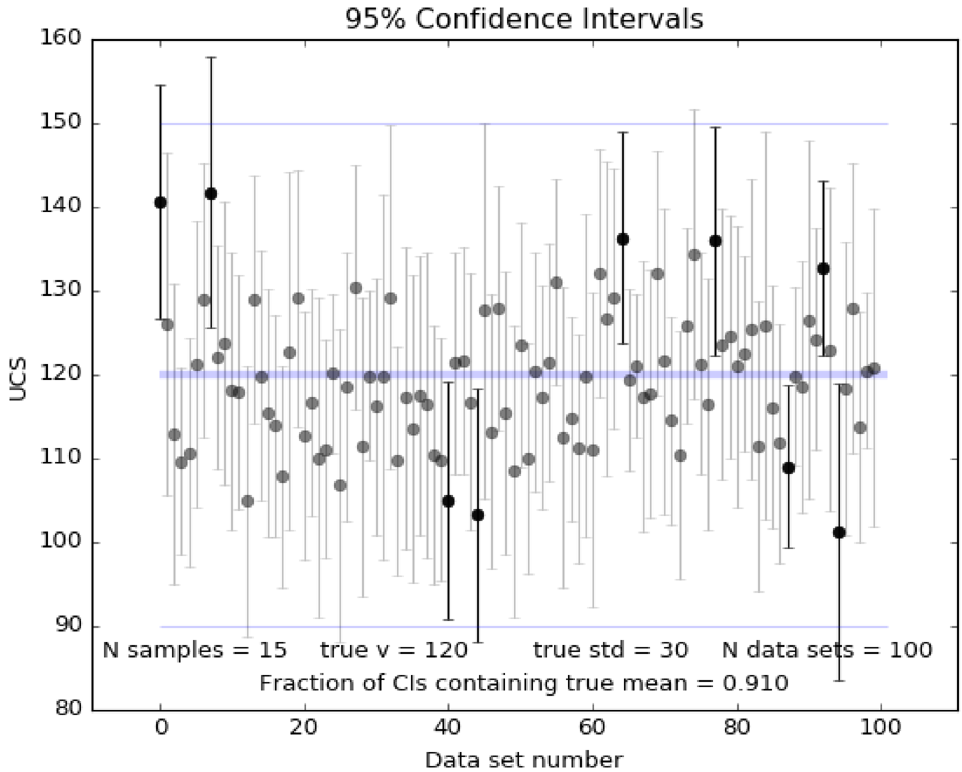


Figure 2 Frequentist interpretation of CIs for randomly generated UCS data sets of 15 values with a mean of 120 and standard deviation of 30

In the Bayesian approach the situation is different because the unknown parameter investigated is considered a random variable that is updated for every new data set. The posterior probability distribution resulting from the Bayesian updating process is used to define the highest density interval with a particular level of precision, and this interval defines the bounds of the credible region for the estimation of the parameter. In many simple situations the results from both approaches coincide, but the meaning of the result is different. The Bayesian result has a meaning consistent with the answer that is normally sought by the analyst, whereas the frequentist result responds to a different question that is of less interest to the analyst.

Figure 3 compares the frequentist 95% CI for data set 27 in Figure 2 with the credible interval corresponding to the 95% high density interval (HDI) of the posterior distribution. The posterior distribution is calculated with the Bayesian approach for the same data set, assuming a uniform prior distribution which is considered a non-informative prior in this case. The results show that the likelihood of the data is not affected by the prior, yielding a result that seems to coincide with the frequentist result, although with different meanings. In this case, the Bayesian interval indicates a range for the sought mean with a 95% credibility. This is possible because in the Bayesian framework, the parameter investigated is not fixed and changes as new data is available. The frequentist result corresponds to a point estimate of the mean and a measure of the error represented by the width of the CI, whereas the Bayesian results provides a full probability distribution for the mean based on the data used.

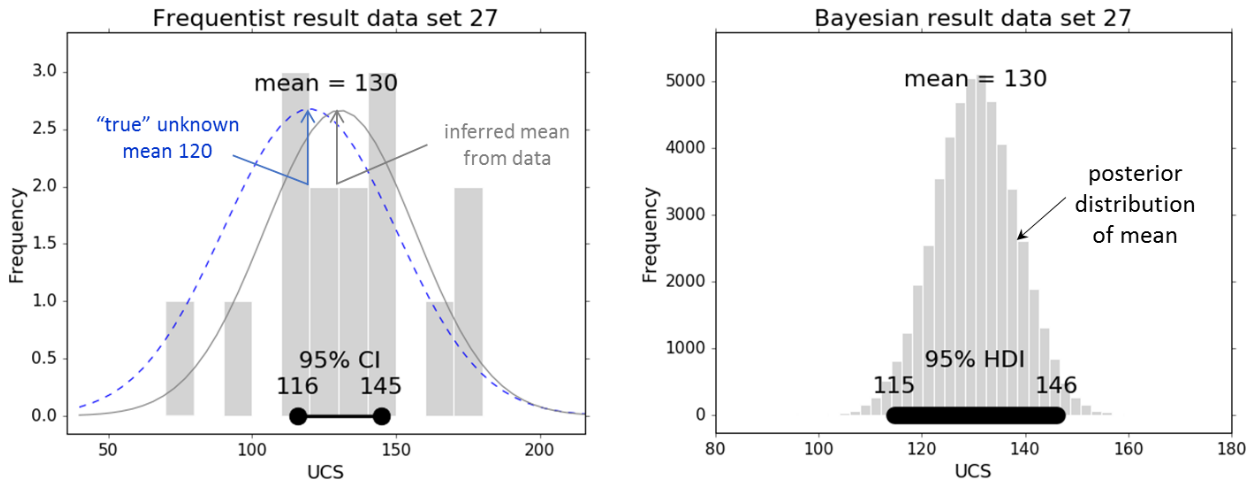


Figure 3 Comparison between the frequentist (left) and Bayesian (right) results for the inference of the mean UCS of data set 27 in Figure 2

4.3 Inability of the frequentist approach to represent epistemic uncertainty

The definition of probability within the frequentist approach is inconsistent with the definition of epistemic uncertainty. It is not possible to randomise epistemic uncertainty nor to model it by means of repetition of trials with a particular probability distribution. Some aspects of this type of uncertainty are closer to the interpretation of probability as a degree of belief that can be assigned directly to states of nature.

For this reason, the Bayesian approach seems better equipped to model uncertainty in general, including epistemic uncertainty. Subjective knowledge and expert opinion can formally be incorporated into this methodology through the selection of the appropriate priors. The frequentist approach does not allow the use of information that is not the result of a random sampling process. Nevertheless, at least within the geotechnical engineering field in open pit mining, it is not conceivable to have a slope design where some form of previous knowledge is not used in the process. However, a drawback from this practice is the difficulty to quantify the uncertainty of the design, because the inclusion of this information is based on the intuition of individuals and carried out in a rather arbitrary way.

4.4 Simple example of the Bayesian method

The Bayesian approach is not meant to be used in simple cases like the UCS analysis presented above, where apart from the subtle differences in their meaning, numerical results seem to coincide. The real strength of this approach is shown in situations where the models examined are multidimensional, with a multitude of parameters that need to be inferred, where the frequentist methods would be less efficient and produce results more difficult to interpret. A few recent examples of the application of Bayesian analysis in rock mechanics and slope problems include: the estimation of the rock mass deformation modulus based on model selection and Bayesian updating by Feng and Jimenez (2015), the characterisation of the UCS from the Bayesian selection of a site-specific model based on the Point Load Index (IS_{50}) by Wang and Aladejare (2015) and the back analysis of slope failure based on a Bayesian model solved with Markov Chain Monte Carlo (MCMC) analysis by Zhang et al. (2010).

The example of the Bayesian approach included in this paper to illustrate the method corresponds to a linear regression analysis to estimate the Hoek–Brown mi parameter for intact rock from UCS, TCS and BTS test results. The main advantages of the method compared with a conventional linear regression analysis are the proper handling of the outliers, with no requirement of judgments from the analyst, and the natural assessment of the confidence level of the estimation.

The estimation of mi as described by Hoek (2006) consists of fitting the test results on a graph of $(\sigma_1-\sigma_3)^2$ versus σ_3. The Hoek–Brown strength envelope is linear in this plot and a linear regression analysis provides the required values of UCS and mi. UCS is calculated as the square root of the intercept and mi as the slope divided by the calculated UCS. Hoek indicates that this method is robust, reliable and has the advantage that it gives a good visual impression of the distribution and scatter of the data.

The formula that supports this procedure is derived by rearranging the terms in the original expression of the Hoek–Brown failure criterion for rock masses, after incorporating the parameter values for the condition of intact rock. The H-B failure envelope is given by:

$$\sigma_1 = \sigma_3 + \sigma_{ci}\sqrt{\mathrm{m}\frac{\sigma_3}{\sigma_{ci}} + \mathrm{s}} \quad (3)$$

For intact rock, s = 1 and the equation can be rearranged such that it forms a straight line with coordinate axes σ_3 and $(\sigma_1-\sigma_3)^2$, as follows:

$$(\sigma_1 - \sigma_3)^2 = mi\ \sigma_{ci}\ \sigma_3 + \sigma_{ci}{}^2 \quad (4)$$

where:

σ_1, σ_3 = major and minor principal stresses.

σ_{ci} = unconfined compressive strength of intact rock.

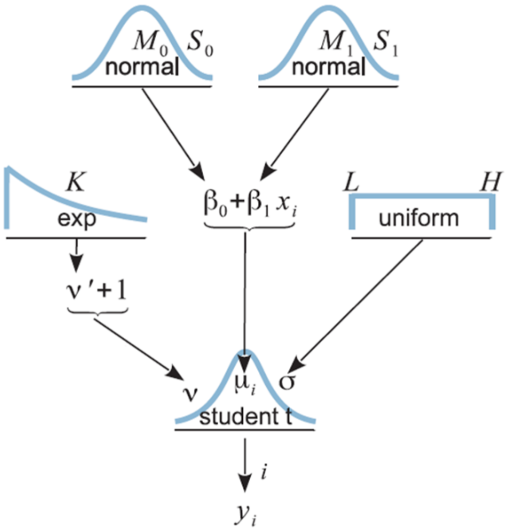
m, s = parameters of the Hoek–Brown strength criterion for rock masses.

mi = parameter m of the Hoek–Brown strength criterion for intact rock.

The method relies on estimation of the direct tensile strength (DTS) values from indirect measurements with BTS tests. Perras and Diederichs (2014) suggests the use of a factor of 0.9 for metamorphic rocks, 0.8 for igneous rocks and 0.7 for sedimentary rocks.

The main difficulty with the conventional (frequentist) linear regression analysis is that it is affected by the presence of outliers, requiring different sorts of manipulation of the data set to avoid the bias they cause in the estimation. In addition, the result corresponds to a point estimation based on the data considered without a proper measurement of the confidence of the estimated intercept and slope parameters.

The sketch in Figure 4 shows a description of the generic Bayesian model used for the linear regression analysis. The original model is described in detail by Kruschke (2014) and was implemented in a software code for statistical analysis named R. The example presented in this paper was implemented in the Python programming language (Python Software Foundation 2001) and was modified to account for the correct direction of measurement of errors in the tensile strength tests. The method is robust in the true statistical sense, because it uses a student (t) distribution to model the spread of the data points in the direction of measurement of errors. The t distribution is defined by three parameters which control the central value (mean μ), the width (scale σ) and the weight of the tails (normality ν). The possibility to set heavy tails with this distribution allows for accommodating outliers without shifting the mean. The model considers prior distributions on four parameters, the intercept (β0) and the slope (β1) of the regression line modelled with normal distributions, and the scale (σ) and normality (ν) parameters of the t distribution modelled with a uniform and exponential distributions respectively, as sketched in Figure 4. The specification of the parameters of the prior distributions is based on the characteristics of the data set and consists in setting up values sufficiently vague to avoid constraining the result. The justification for the selection of these distributions as well as the selection of the prior constants is described by Kruschke (2014) and is not presented here. The Bayesian posterior distribution of the parameters sought with the regression analysis is shown at the bottom of Figure 4. However, the equation does not need to be expanded on further, as the various components can be incorporated into specialised packages used to sample the distribution and get credible estimates of these parameters. The sampling process is carried out with a methodology known as MCMC, which in turn can be implemented with different algorithms. The example in this paper was solved with the affine-invariant ensemble sampler algorithm implemented in the emcee Python package developed by Foreman-Mackey et al. (2013).



Robust Bayesian Linear Regression:
$y = \beta_0 + \beta_1 x$

Errors in y modelled with a Student dist. with parameters:

μ (mean):	central value
σ (scale) :	width
ν (normality) :	weight of tails

Prior distributions on four parameters:

Intercept β_0 :	normal distribution (M_0, S_0)
Slope β_1 :	normal distribution (M_1, S_1)
Scale σ :	uniform distribution (L, H)
Normality ν :	exponential distribution (K=1/29) shifted+1

Posterior distribution equation sampled with a MCMC process to get credible estimates of β_0, β_1, σ, ν:

$$p(\beta_0,\beta_1,\sigma,\nu\,|D) = \frac{p(D\,|\beta_0,\beta_1,\sigma,\nu)\;\; p(\beta_0,\beta_1,\sigma,\nu)}{\iint\iint p(D\,|\beta_0,\beta_1,\sigma,\nu)\;\; p(\beta_0,\beta_1,\sigma,\nu)\;\; d\beta_0\, d\beta_1\, d\sigma\, d\nu}$$

Figure 4 Conceptual basis of the robust Bayesian linear regression model used for the estimation of credible UCS and mi values from UCS, TCS and tensile strength test results (generic model from Kruschke 2014)

The mi estimation analysis was carried out with a reduced data set of 31 points (8 UCS, 8 DTS and 15 TCS) without outliers and with the extended data set of 60 points (15 UCS, 15 DTS and 30 TCS) including a few outliers. The results of the analysis using a conventional least squares regression method (frequentist result) and the Bayesian approach are shown in Figure 5. The mi results for the case with 31 data points are similar (frequentist 15.4, Bayesian 16.6); however, they differ for the case of 60 data points with a difference of 5.3 points in the value of mi (frequentist 11.9, Bayesian 17.2) and a flatter line with the conventional regression method caused by the outliers. The Bayesian result on the other hand, appears less affected by the outliers, showing the robustness of the method with estimated mi values of 16.6 and 17.2 for the two data sets.

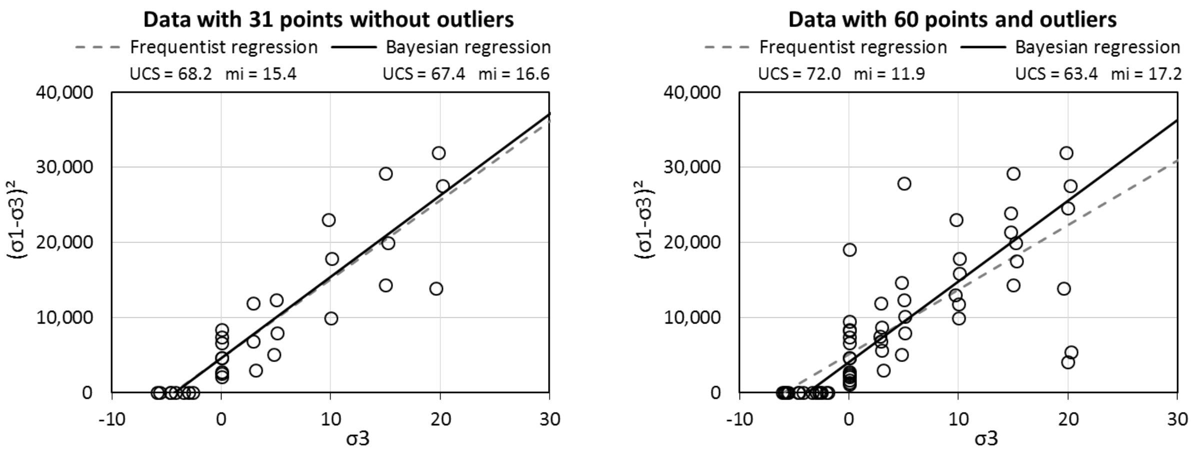


Figure 5 Comparison of results between frequentist and Bayesian linear regression analysis for data sets of 31 points without outliers (left) and 60 points with outliers (right)

The result of the Bayesian analysis is richer than just the regression line; it includes various diagnostic graphs, probability distributions and scatter plots of the four parameters investigated. The diagnostic graphs are intended to ensure that a proper stable solution has been obtained, the probability distributions serve to define the ranges of credible values defined by the 95% HDI and the scatter plots facilitate the identification of correlations between parameters. Due to space limitations not all of these results are included and discussed in this paper, and only a selection of them are shown in Figures 6 and 7.

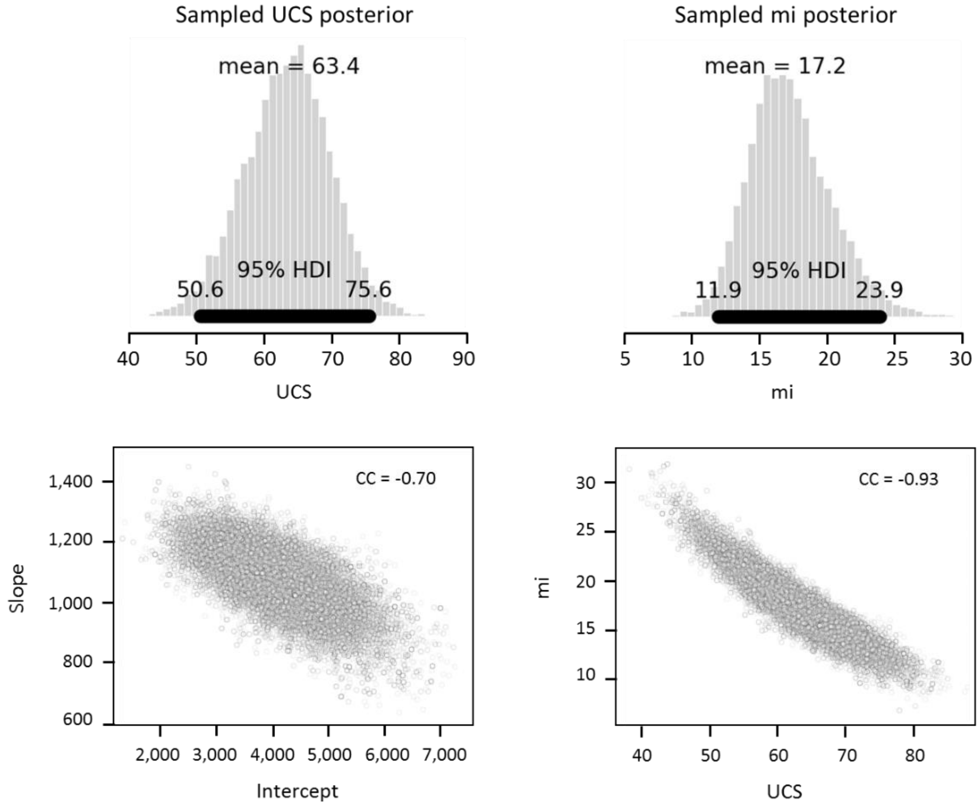


Figure 6 Posterior distributions of UCS and mi with mean and 95% HDI ranges indicated (top) and scatter plots of sampled values of intercept versus slope and corresponding values of UCS versus mi (bottom)

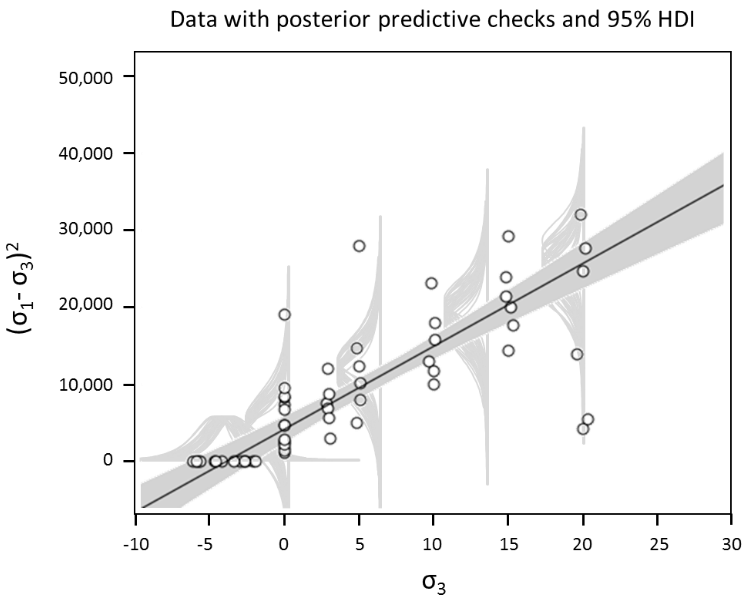


Figure 7 Data points with a selection of credible regression lines including the mean and t-noise distributions superimposed

Figure 6 shows the inferred posterior distributions for UCS and mi with the respective 95% HDIs which define the ranges of credible values for these parameters. Figure 6 also includes the scatter plots of sampled values of intercept versus slope, showing low correlation between these parameters, and the respective plot of UCS versus mi showing a marked inverse correlation between these variables. Figure 7 shows a plot with the 95% confidence band of the regression lines, which considers the correlation between UCS and mi indicated in Figure 6. The plot also includes the data points and a selection of the t distributions used to model the scatter (noise) in the directions of measurement of errors, depicting how they can include the outliers without shifting the mean.

5 Non-probabilistic methods for special cases of epistemic uncertainty

Although the Bayesian probabilistic methods are capable of dealing with the general aspects of epistemic uncertainty, there are uncertainty sub-classes whose representation would be incompatible with the principles of probability theory. A probability assignment somehow implies a sharp definition of the element assessed. This is a consequence of the probability axiom that indicates that once the probability of occurrence of an event p is defined, its probability of no occurrence is automatically stated as equal to 1-p. Alternative approaches based on theories that some authors (Klir 1989; Halpern & Fagin 1992) see as generalisations of the probability theory, have been developed to deal with these situations as described hereafter.

5.1 The multifaceted character of epistemic uncertainty

A description of various aspects associated with imprecision in uncertainty-based information such as vagueness and ambiguity of various classes (for example non-specificity, dissonance and confusion) was given by Klir (1989). He stated mathematical arguments for the suitability of various theories available at the time to treat uncertainty. More recently, the same author (Klir & Wierman 1999; Klir 2004) provided a more detailed taxonomy of the existing theories to treat uncertainty related to information within the framework of the generalised information theory. Zimmermann (2000) provides a less formal and more practical classification of uncertainty properties in terms of four aspects: its causes, the type of available information, the type of numerical data and the requirements of the model output. Blockley (2013) argues that any type of uncertainty can be defined in terms of three basic aspects i.e. fuzziness, incompleteness (epistemic) and randomness (aleatory), which can be represented in a tridimensional space (Fuzziness, Incompleteness and Randomness space or FIR space). Other attributes of uncertainty such as ambiguity, dubiety and conflict, can be interpreted as complex mixes of interactions in the FIR space. Figure 8 shows a representation of the FIR space as presented by Blockley (2013) with the interpretation of some uncertainty attributes.

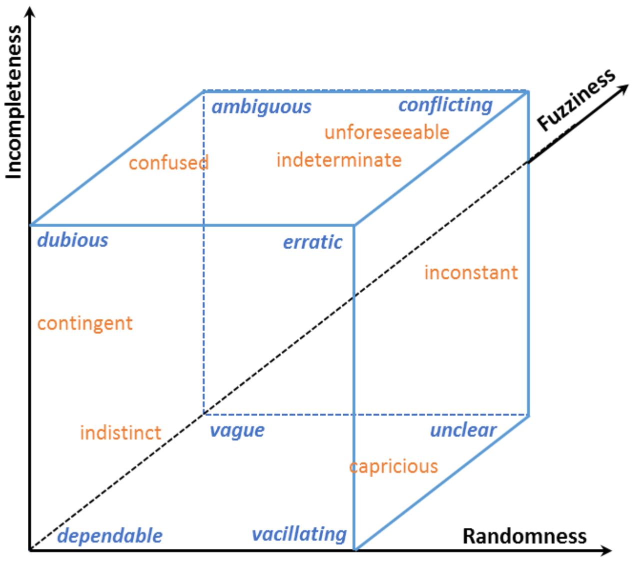


Figure 8 Interpretations of uncertainty attributes in the FIR space (Blockley 2013)

5.2 Description of non-probabilistic approaches

Some of the more common alternative approaches to represent epistemic uncertainty include interval analysis (Moore et al. 2009), evidence theory also known as Dempster-Shafer theory (Halpern & Fagin 1992) and possibility theory (Dubois & Prade 2009). A comparison of these approaches is presented by Helton et al. (2004) with some hypothetical simple problems to illustrate the main aspects of each methodology. Uncertainty characterised by fuzziness is treated with a branch of methodologies based on fuzzy representation of uncertain variables, which is not included in this paper. However, to illustrate the group of non-probabilistic approaches to treat uncertainty, a simple hypothetical example is used to show the main features of the interval, possibility and evidence theory approaches, which are compared with the traditional probabilistic result.

A complete description of these approaches is outside the scope of this paper and the reader is referred to the documents cited above for more information on the mathematical formulations and procedures. A non-mathematical simple description of each approach is given with the aim of getting some intuition on the meaning of the results of the example included. The motivation to present these methods is to highlight certain situations where the representation of epistemic uncertainty might require techniques outside the conventional probability theory, and to provide a brief description of three techniques typically used to deal with imprecision due to lack of information.

5.2.1 Interval analysis

This is the simplest approach, consisting of the evaluation of the propagation of the bounding values of the input parameters, with no attempt to infer the uncertainty of the result based on any assumption of the uncertainty of the input variables within the known boundary values (Helton et al. 2004).

5.2.2 Possibility theory approach

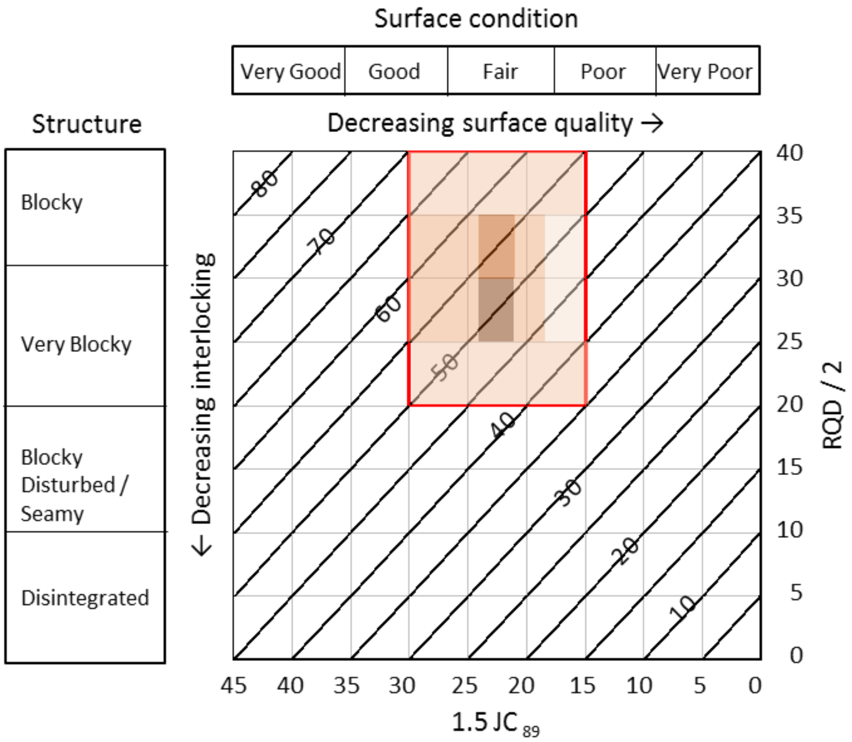
Possibility theory is defined by Dubois and Prade (2009, p. 6927) as "the simplest uncertainty theory devoted to the modelling of incomplete information. It is characterised by the use of two dual set functions that respectively grade the possibility and the necessity of events." If A represents a particular set of information regarding an unknown value x, a qualitative description of these attributes would indicate that the necessity of A, Nec(A), is a measure of the amount of uncontradicted information that supports the proposition that A contains the correct value for x; and the possibility of A, Pos(A), is a measure of the amount of information that does not refute the proposition that A contains the correct value for x (Helton & Sallaberry 2008). A key element of the possibility theory approach is the possibility measure (r), which is a function associated with the amount of likelihood that can be assigned to each element of a set.

5.2.3 Evidence theory approach

Helton et al. (2004, p. 42) indicates that "Evidence theory provides an alternative to the traditional manner in which probability theory is used to represent uncertainty by allowing less restrictive statements about likelihood than is the case with a full probabilistic specification of uncertainty." In this case the two specifications of likelihood are represented by the belief and plausibility attributes of sets of information. Again, if A represents a particular set of information regarding an unknown value x, a qualitative description of these attributes would indicate that the belief of A, Bel(A), corresponds to the likelihood that must be associated with A regarding the value of x; and the plausibility of A, Pla(A), corresponds to the likelihood that could potentially be associated with A (Helton & Sallaberry 2008). In this case the function associated with the amount of likelihood that can be assigned to each element of a set is the basic probability assignment (m). Although there are similarities between the concepts of necessity and belief, and possibility and plausibility, they are defined by different mathematical descriptions.

5.3 Example of non-probabilistic approaches

The example corresponds to the numerical estimation of GSI based on uncertain inputs of RQD and joint condition (JC) rating, using the relationship proposed by Hoek at al. (2013). The condition of epistemic uncertainty in the RQD and JC values is represented in this example by assuming that only ranges of values from different sources are known with insufficient information on how these values may vary within the boundaries given. Three possible intervals for RQD and four for JC values are considered as listed at the right of Figure 9. Examples of sources supporting the various sets of data might include records from borehole logs, data from face mapping, back analysis of slopes performance, judgements from experts etc. Figure 9 also shows the chart used for the calculation of GSI from RQD and JC values, with the shaded area indicating the range of possible GSI values associated with the input intervals.



RQD and JC parameters with epistemic uncertainty:

RQD_1 = [50 , 60]
RQD_2 = [50 , 80]
RQD_3 = [40 , 70]

JC_1 = [14 , 16]
JC_2 = [14 , 20]
JC_3 = [12 , 16]
JC_4 = [10 , 20]

Figure 9 Example of treatment of epistemic uncertainty. Chart for the calculation of GSI from RQD and JC_{89} values (left). The shaded areas represent the likely GSI values proportionally to the support from the imprecise information according to the possibility theory. Uncertain information at the right in the form of ranges of RQD and JC values assumed to be originated from different sources

The conventional probabilistic approach to define GSI would assume a uniform distribution of the property for each interval and calculate the joint probability distribution for each parameter (RQD and JC). The density of the resulting distributions will reflect the relative support of the values within the range from the various input sets. A Monte Carlo simulation of the GSI calculation, based on sampling the input parameters from these distributions, produce a distribution of GSI values. This result can be presented in the form of a reverse cumulative distribution to express the probability of exceeding a particular value, P(>GSI), as shown in the graphs of Figure 10. These graphs indicate probabilities of 100%, 50% and 0% of exceeding GSI values of 36, 52 and 69, respectively. The criticism of this approach is that any type of assumption on the values of the input parameters within the boundaries provided, are not supported and effectively means adding information that does not exist. In other words, the existence of epistemic uncertainty (lack of information) is being neglected and replaced with added data to enable a randomised simulation with the model.

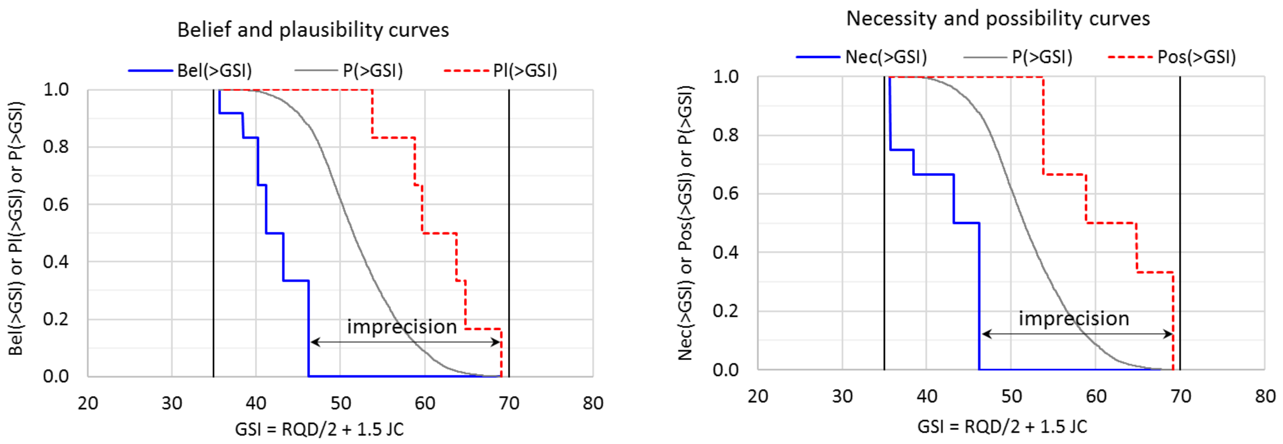


Figure 10 Comparison of GSI results using a conventional probabilistic analysis with belief and plausibility curves from evidence theory (left) and with necessity and possibility curves from possibility theory (right). The wider bounds from interval analysis are also indicated in both graphs

Figure 11 shows, in a simplified manner, the way in which the likelihood functions m (evidence theory) and r (possibility theory) are calculated for the variables RQD and JC from the input data. When these likelihood functions are incorporated into the GSI calculation model, they define distinct regions of likelihood of GSI represented by the shaded areas in the GSI space. In the evidence theory approach, a product of the likelihoods of the input parameters is used to estimate the GSI likelihood, whereas in the possibility theory approach, this operation is based on the minimum logic operator. A Monte Carlo simulation was used to generate the GSI likelihood functions with either approach and to define belief and plausibility (evidence theory), and necessity and possibility (possibility theory) curves, which are presented in the form of reverse cumulative distributions in the graphs of Figure 10.

70	80		1		0.33
60	70		1	1	0.67
50	60	1	1	1	1.00
40	50			1	0.33
RQD		RQD1	RQD2	RQD3	mRQD

Functions associated with the amount of likelihood of variable:

Evidence theory:
m: basic probability assignment.

Possibility theory:
r: possibility measure.

Evidence theory

mGSI = mRQD * mJC

	0.08	0.17	0.33	0.17	0.17
	0.17	0.33	0.67	0.33	0.33
	0.25	0.50	1.00	0.50	0.50
	0.08	0.17	0.33	0.17	0.17
mJC	0.25	0.50	1.00	0.50	0.50
JC1			1		
JC2			1	1	1
JC3		1	1		
JC4	1	1	1	1	1
JC	10	12	14	16	18
	12	14	16	18	20

Possibility theory

rGSI = MIN(rRQD, rJC)

0.33		0.25	0.33	0.33	0.33	0.33
0.67		0.25	0.50	0.67	0.50	0.50
1.00		0.25	0.50	1.00	0.50	0.50
0.33		0.25	0.33	0.33	0.33	0.33
rRQD						
	rJC	0.25	0.50	1.00	0.50	0.50
	JC1			1		
	JC2			1	1	1
	JC3		1	1		
	JC4	1	1	1	1	1
	JC	10	12	14	16	18
		12	14	16	18	20

Figure 11 Likelihood of GSI values derived from imprecise information in the input parameters RQD and JC, according to evidence (centre) and possibility (right) theory approaches

The results of Figure 10 allow an appreciation of the concept of imprecision associated to epistemic uncertainty reflected in the gap between the two envelopes either side of the conventional probability result. For reference, Figure 10 also includes the result of the interval analysis, which consist in the definition of the maximum interval defined by the propagation of the bounding values of the input parameters through the GSI calculation model. The results of the interval analysis are conservative and might be unjustified in many situations. On the other hand, the probabilistic result might be inappropriate in many risk based analysis, where an explicit separation between the aleatory and epistemic components of uncertainty are required to interpret results and to identify mitigation measures.

6 Summary and conclusion

Uncertainty is a common occurrence in geotechnical engineering and two main types of uncertainty are normally identified. These are, the irreducible aleatory uncertainty associated with the natural variation of parameters, and the epistemic uncertainty related to lack of knowledge on parameters and models that can be reduced with the collection of information. The geotechnical model for slope design takes information from different complex models and typically contains a large proportion of epistemic uncertainty due to the relative scarcity of data available for design.

There are two interpretations of probability for the frequentist and Bayesian approaches of statistical analysis. Probabilistic methods are commonly used to represent and quantify uncertainty in the slope design process. However, there are no clear guidelines with regard to the appropriate methods to use in specific situations, and most of the techniques of analysis used correspond to frequentist methods. Nevertheless, the adopted methods are not always fully understood and their results are commonly misinterpreted. Common misuses of frequentist methods include the characterisation of population parameters based on reduced sampling, and the use of CIs from single data sets to measure reliability of data. Bayesian methods can be used to represent both types of uncertainty and are especially suited for situations where data is scarce and previous knowledge exist. However, they are rarely used in the mine slope design process where they could be of great benefit. Some aspects of the epistemic uncertainty cannot be represented with probabilistic methods and alternative approaches are required in those cases. Interval analysis, and methods based on evidence theory and possibility theory can provide the tools required to deal with situations where imprecision due to incomplete information exists.

Two examples of unconventional methods to treat uncertainty in the slope design process were presented. The first example corresponded to the Bayesian estimation of the mi parameter of the H-B strength criterion using a robust linear regression method for UCS, TCS and tensile strength data plotted in a $(\sigma_1\text{-}\sigma_3)^2$ versus σ_3 space. A generic model implemented in Python code and solved with a MCMC methodology based on the affine-invariant ensemble sampler algorithm using the emcee Python package was used for this purpose. The results were useful to highlight the benefits of the method over a traditional frequentist regression method. The benefits are related to the adequate handling of the outliers in the data and the proper quantification of the confidence of the estimates. Further work will be carried out to improve the method using real data sets to validate results.

The second example consisted in the use of three non-probabilistic approaches to deal with epistemic uncertainty related to incompleteness of information represented by sets of intervals of input parameters. The estimation of GSI values from RQD and JC parameters using the model by Hoek et al. 2013, was carried out with interval analysis, and procedures based on the evidence and possibility theories and included the assessment of the likelihood of the estimates. These results were compared with the conventional probability distribution curve to highlight the implications of the incompleteness aspect of the uncertainty. The results showed the importance of having a separation between the aleatory and epistemic components of uncertainty, which are of relevance for risk based design procedures.

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