Probabilistic slope stability analysis as a tool to optimise a geotechnical site investigation program

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Abstract

Slope stability analysis is a branch of ground engineering science where there are a number of significant uncertainties. Although probabilistic slope stability analysis is an option in most commercial software, the use of this method is not common in practice. Apart from the ability of the probabilistic method to assess the impact of uncertainties on slope stability, it can also be used as a tool to optimise the geotechnical site investigation program. The first-order second-moment approximation of the Taylor series method is one of the probabilistic slope stability methods that determines the relative contribution of uncertainty projected by each component random variable. For a slope with a sequence of different geological units, each unit can be modelled as having several random variables such as cohesion and friction angle. A geological unit whose random variables are responsible for the greatest contribution to the uncertainty in the Factor of Safety (FS) will be the most controlling unit. This characteristic can be used to design geotechnical site investigation programs in order to minimise the uncertainties in these controlling units, which will enhance the Reliability Index of the computed FS. A code developed by the first author will be used in this paper to demonstrate the application of this method in to a real case study.

1   Introduction

Uncertainty is involved in many aspects of slope stability assessments. Geological uncertainties, spatial variability of ground materials, lack of sufficient data, unpredictable failure mechanisms, simplified geotechnical simulation techniques, human errors in characterisation of ground materials, and the modelling process are some of the main sources of uncertainties. Commonly used deterministic methods are not able to address the influence of these uncertainties on the overall Factor of Safety (FS). FS is calculated as the ratio of resisting forces to driving forces; is the most widely used engineering acceptance criteria for assessing the stability of rock slopes. In deterministic stability analyses, minimum, maximum or average values of input parameters are used in limit equilibrium or numerical modelling techniques to compensate the influence of uncertainties. These methods may result in either over-designed or under-designed slopes. Theoretically, a FS of over one represents stability conditions but because of the above-mentioned uncertainties, a range of 1.3–1.5 is often used in practice (Abramson et al. 2002). Two slopes with the same FS may have significantly different likelihoods of instability because of the different levels of uncertainty in their input geotechnical parameters.

Over the years, these uncertainties have been accounted for in some analyses by researchers and practitioners using methods such as the first–order second-moment (FOSM) expansion of Taylor series, Point Estimation Method, and Monte Carlo Simulations (Alonso 1976; Chowdhury 1978; Ang & Tang 1984; Wolff 1985; Christian 1996; Christain & Urzua 1998; Hassan & Wolff 1999; Malkawi et al. 2000; El-Ramly et al. 2002; Griffiths & Fenton 2004; Zoorabadi 2004; Fenton & Griffiths 2008; Suchomel & Mašín 2010; Li et al. 2015). In these so-called probabilistic analyses, the variability in geotechnical parameters is addressed by assigning a density distribution function to each parameter relevant to the failure mechanism being tested. The analysis is conducted using limit equilibrium and numerical models and results are interpreted as a Reliability Index (RI) and Probability of Failure (PF). These two parameters, along with the FS, provide additional acceptance criteria to assess the stability of slopes. Smith (1986), Santa Marina et al. (1992), Rettemeier et al. (2000),

Bridle (2000), El-Ramly (2001), Eddleston et al. (2004) provided acceptance criteria for slopes on the basis of a PF and RI.

In this paper, the unique capability of FOSM expansion of Taylor series to determine the relative influence of each input data on the calculated FS is used as a tool to optimise the geotechnical site investigation program by determining the relative influence of each data on the calculated FS. In fact this tool identifies the most controlling material unit and more influencing parameter. The geotechnical program can be redesigned to minimise the uncertainties of the controlling parameters. Therefore, the whole reliability of slope stability could be enhanced by applying this approach.

2 First-order second-moment expansion of Taylor series

In all existing limit equilibrium based slope stability analysis methods such as Bishop's method, FS is calculated as a function of input parameters (cohesion, friction angle etc. of ground materials). These functions are called performance functions in probabilistic stability analysis and are represented as follows:

$$FS = g(X_1, X_2, \ldots, X_n) \tag{1}$$

where:

$g(X)$ is the performance function or FS calculation function and X_i are the component input parameters.

Since the component input parameters are modelled as random variables with a certain probabilistic distribution function (PDF), then the FS will also be presented as a random variable with corresponding PDF. Similar to all other continuous PDFs, its mean, μ_{FS}, or expected value (E[X]) and variance (Var[X]), are calculated as follows:

$$E[X] = \int_{all\ c} xf(x)dx \tag{2}$$

$$Var[X] = \int_{all\ x}(x - \mu_x)f(x)dx \tag{3}$$

where:

$f(x)$ is PDF of FS.

Considering the complexity of performance functions for FS in slope stability, the above direct integrations are not feasible. In this regards several methods such as FOSM expansion of Taylor series, Rosenblueth's point estimate methods, Monte Carlo simulation, and Fourier analysis were developed as tools to estimate the E[FS] and Var[FS] (Abramson et al. 2002).

Expansion of Taylor series is a common technique for estimating the moments of the performance function based on the moments of the input random variables. In this method, the Taylor series of the performance function is expanded about the expected values of random variable (Hahn & Shapiro 1967). When FOSM expansion of this series is applied, expected value and variance of performance function would be as in the following equations:

$$E[FS] = \mu_{fs} \approx g(E[X_1], E[X_1], \ldots, E[X_n] + \sum_{i=1}^{k}\sum_{j=1}^{k} \frac{\partial^2 FS}{\partial X_i\ \partial X_j}\ Cov\left(X_i, X_j\right) + e \qquad for\ i < j \tag{4}$$

$$Var[FS] = \sigma_{FS}^2 \sum_{i=1}^{k}\left[\left(\frac{\partial\ FS}{\partial\ X_i}\right)^2 Var[X_i]\right] + 2\sum_{i=1}^{k}\sum_{j=1}^{k}\left[\left(\frac{\partial\ FS}{\partial\ X_i}\frac{\partial\ FS}{\partial\ X_j}\right)\ Cov\ [X_i, X_j]\right] + Var[e]\ for\ i < J \tag{5}$$

where:

$Cov\ [X_i, X_j]$ is covariance of random variables when they have some dependency. In most cases the contribution of the summation term is negligible, e shows modelling error, and $Var[e]$ is the variance of the modelling error.

In the previous formulations, the partial derivative, $\partial\ FS/\partial\ X_i$, which represents the variation FS with respect to each variable should be calculated for each random variable.

Considering the nonlinearity of the performance function (common slope stability formulations such as Bishop), the partial derivatives should be solved numerically. US Army Corps of Engineers (1997) used finite difference approximation and obtained the following solutions:

$$\frac{\partial FS}{\partial X_1} = \frac{g(X_1+\sigma_1,X_2,X_3,...,X_n)-g(X_1-\sigma_1,X_2,X_3,...,X_n)}{2\sigma_1} = \frac{\Delta FS_1}{2\sigma_1}$$

$$\frac{\partial FS}{\partial X_2} = \frac{g(X_1,X_2+\sigma_2,X_3,...,X_n)-g(X_1,X_2-\sigma_2,X_3,...,X_n)}{2\sigma_2} = \frac{\Delta FS_2}{2\sigma_2}$$

$$\vdots$$

$$\frac{\partial FS}{\partial X_n} = \frac{g(X_1,X_2,X_3,...,X_n+\sigma_n)-g(X_1,X_2,X_3,...,X_n-\sigma_n)}{2\sigma_n} = \frac{\Delta FS_n}{2\sigma_n}$$

$$(6)$$

$$Var[FS] = \sigma_{FOS}^2 \approx \frac{1}{4}\sum_{i=1}^k[\Delta FS_i]^2 + \frac{1}{2}\sum_{i=1}^k\sum_{j=1}^k[\Delta FS_i\,\Delta FS_j\,\rho_{ij}] + Var[e] \qquad for\ i < j \qquad (7)$$

where:

σ_n represents the standard deviation of the nth component random variable, and ρ_{ij} shows the correlation coefficient between variables.

The reliability of a slope is the probability that the slope will remain stable under specific design conditions. As such, the RI is defined as the distance between average calculated FS and a critical FS, for example $FOC_c = 1$ for stability, normalised by the standard deviation of the FS. For FS with PDF of normal and lognormal, RI is calculated by the following equation:

$$\beta = \frac{|FS_c - \mu_{FS}|}{\sigma_{FS}} \qquad (8)$$

$$\beta = \frac{\left|\ln(FS_c) - \ln(\mu_{FS}) - \frac{1}{2}(\ln(1+(\frac{\sigma_{FS}}{\mu_{FS}})^2))\right|}{\sqrt{\ln(1+(\frac{\sigma_{FS}}{\mu_{FS}})^2)}} \qquad (9)$$

RI itself is used to evaluate the reliability of slope stability. The acceptable range for RI has were obtained by the back analysis of failed slopes. For example, US Army Corps of Engineers (1997) found RI<1 represents hazardous conditions for a slope.

In addition to the ability of calculating the RI, as can be seen from Equations (5) to (7), the terms are summed to calculate the variance of FS. Therefore, it is possible to determine the relative contribution of uncertainty projected by each component random variable. This advantage of Taylor series can be used as a tool to rank the ground materials in a slope stability problem on the basis of their contribution to the reliability of the calculated FS.

To demonstrate this advantage of FOSM expansion Taylor series, a code was developed using Visual Basic. This code uses the simplified Bishop method to analyse the stability of slopes for mass failure potential. The accuracy of the simplified Bishop method, which does not satisfy all conditions of equilibrium, was studied by Duncan and Wright (1980). They found that despite the simplicity of the Bishop method, for circular slip surfaces it is comparable in accuracy to the more complicated methods which satisfy all conditions of the equilibrium. The technique demonstrated in this paper is applicable to all existing limit equilibrium based methods. The code only considers the cohesion and friction angle as a random variable and the formulation of the Taylor series was also programed for probabilistic analysis. As a first step, the geometry of slopes and material properties are defined by entering them as a series of segments. The slope can be made up of as many as 20 different materials. Then the average, standard deviation and correlation coefficient for the cohesion and friction angle of each material are entered into the program. By defining the failure surface searching network, the code uses average values of input data and finds the average FS, i.e. deterministic FS, and corresponding failure surface. After this, a probabilistic analysis is conducted on the obtained failure surface as follows:

- From each material unit, only the average value of its cohesion (μ_c) is replaced by $\mu_c + \sigma_c$ (where σ_c is the standard deviation) and FS is determined for the previously obtained failure surface (FS_c^+).

- For each material unit, the previous calculation is repeated by replacing μ_c with $\mu_c - \sigma_c$; the results would be FS_c^-.

- Steps 1 and 2 are repeated for the friction angle of each unit and FS_φ^+ and FS_φ^- are calculated.

- For a slope with n units, the FS is calculated 5n times. Therefore it is possible to do this analysis even using existing commercial software.

- Now, $\Delta FS_c = FS_c^+ - FS_c^-$ and $\Delta FS_\varphi = FS_\varphi^+ - FS_\varphi^-$ are calculated for each unit. These two parameters represent the relative contribution of uncertainty projected by the cohesion and friction angle of each material units to the uncertainty of FS.

- Two types of PDF, normal and lognormal, were assumed for FS, then Equations (8) and (9) were used to calculate the RI of FS.

A case study is simulated by this code to illustrate the application of this technique. This slope comprises five distinctly different units, with a sequence that unit 1 is at the bottom unit and unit 5 is at top unit. Their properties and cross-section (Figures 1 and 2).

Material Property

	Material 1	Material 2	Material 3	Material 4	Material 5	Material 6	Mate
Unit Weight KN/m3	22	23.5	22	22	22		
Cohesion KN/m2	75	175	70	175	70		
Phi	25	30	22	30	22		
standard deviation of c	10	20	7	30	5		
standard deviation of phi	5	10	4	4	2		
correlation coefficient	0.3	0.3	0.3	0.3	0.3		

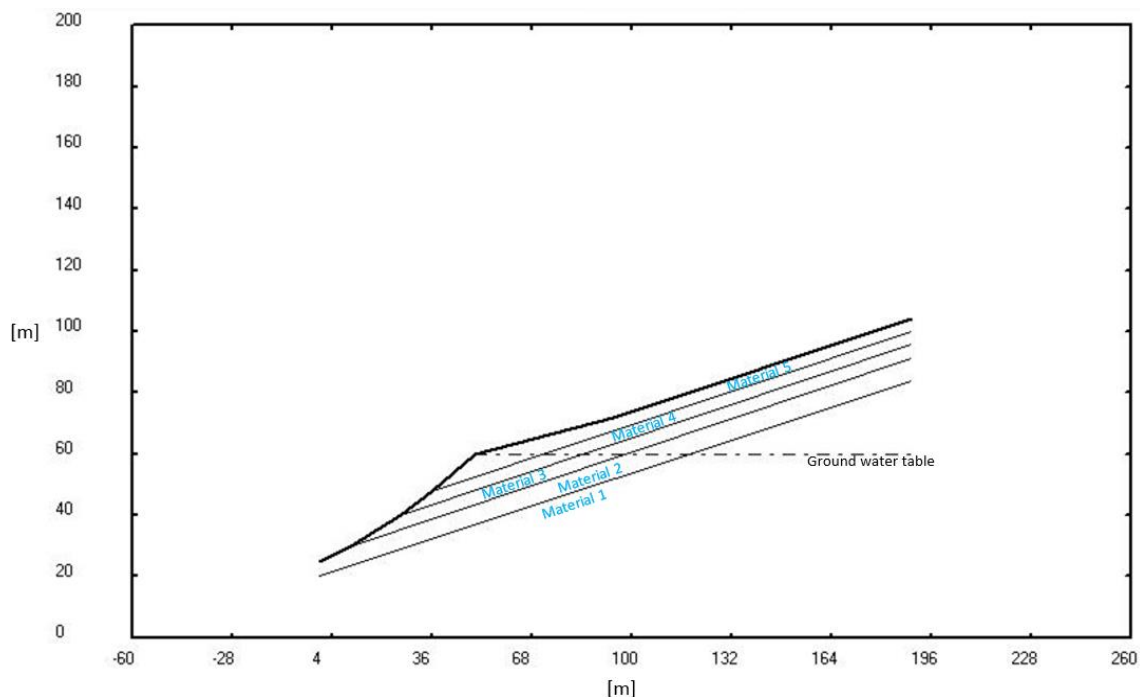**Figure 1   Material properties for each unit**

Figure 2   Cross-section of the modelled slope

As initial results, the failure surface and corresponding average FS are calculated using Bishop's method (deterministic results), (Figure 3).
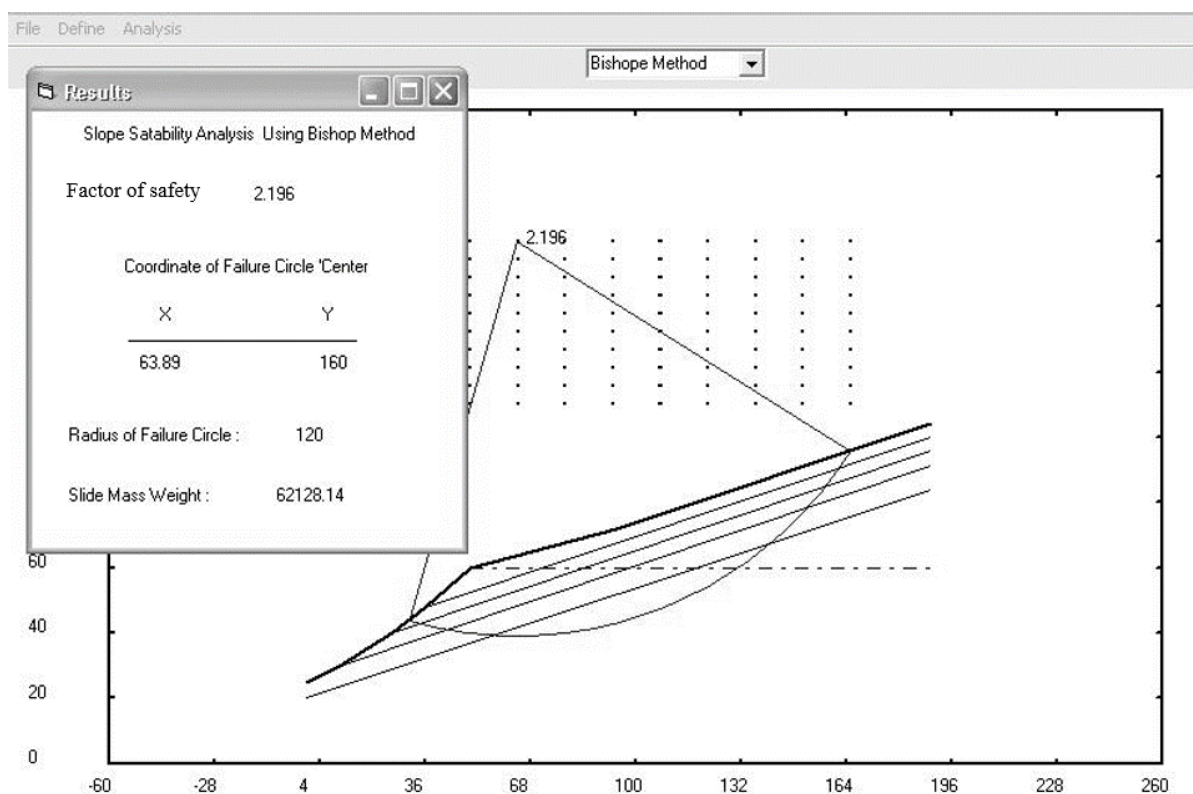

Figure 3   Deterministic results of slope stability using mean parameter values

In order to calculate the RI, a critical FS value needs to be chosen. A value of $FS_c = 1.5$ is entered in this example. Therefore, the probabilistic analysis will quantify whether or not the overall FS is higher than the chosen critical FS ($FS_c = 1.5$) for the existing level of the uncertainties listed in Figure 1. Probabilistic analysis of this slope was performed and the results are shown in Figures 4 and 5.
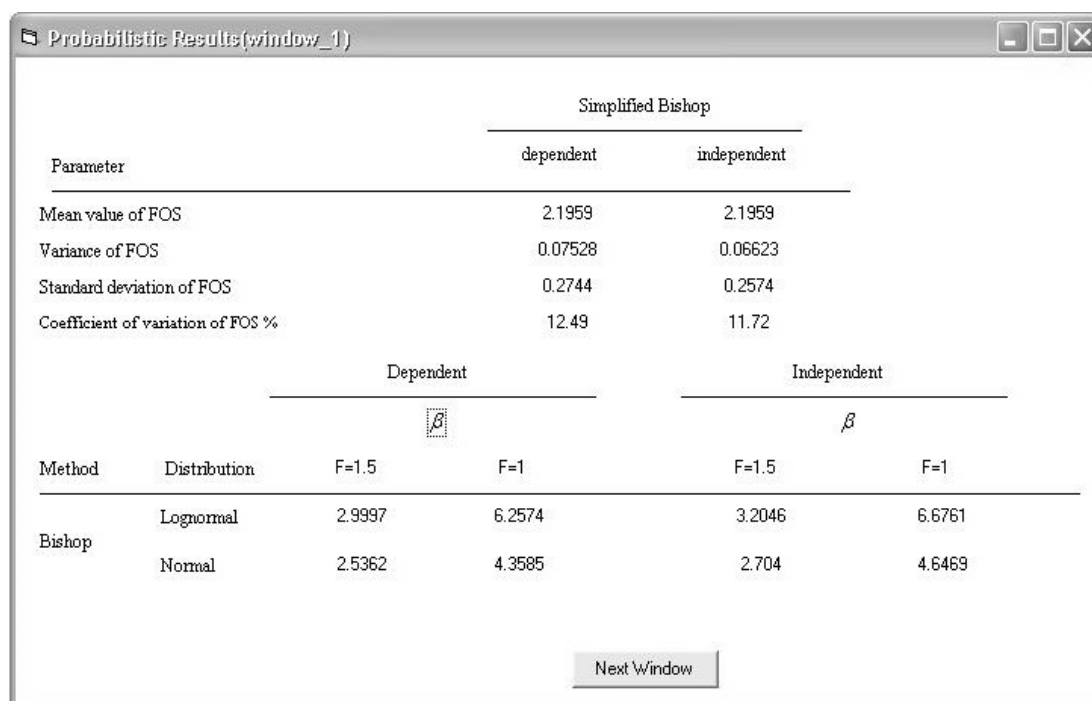

		Simplified Bishop	
Parameter		dependent	independent
Mean value of FOS		2.1959	2.1959
Variance of FOS		0.07528	0.06623
Standard deviation of FOS		0.2744	0.2574
Coefficient of variation of FOS %		12.49	11.72

		Dependent		Independent	
		β		β	
Method	Distribution	F=1.5	F=1	F=1.5	F=1
Bishop	Lognormal	2.9997	6.2574	3.2046	6.6761
	Normal	2.5362	4.3585	2.704	4.6469

Next Window

Figure 4   Probabilistic results of slope stability

Figure 5   Uncertainties caused by variability of cohesion (DSF-cohesion) and friction angle (DSF-friction)

As can be seen from Figure 4, the RI was calculated for two PDFs (normal and lognormal), for a dependent variable and two critical FS of 1.5 and 1. As all calculated RIs are higher than 1, it is concluded that the average FS of 2.196 has an acceptable reliability. The second set of probabilistic analysis results show ΔFS_c and ΔFS_φ, for each material unit. ΔFS_c and ΔFS_φ are higher for the material unit of 1, which is located at the bottom of the model. Additionally, it was found that uncertainties caused by the friction angle of the material have a higher impact on the reliability of the FS. The geotechnical study program should be redesigned to characterise the material unit number 1 more precisely. This optimisation on the geotechnical study program could increase the reliability of stability analysis with lower cost.

3   Conclusion

Probabilistic slope stability analysis quantifies the impact of uncertainties on the reliability of the slope stability analysis. It can also be used to plan and execute appropriate exploration programs. An approach to this method is presented in this paper. The advantage of the Taylor series expansion method to determine the relative contribution of uncertainty projected by each component random variable was demonstrated by a simple example. Although a code has been written for this method, the principles behind it could be integrated into existing commercial slope stability software to determine the influence of each geological unit on the reliability of the design. This capability can be used at early stages of geotechnical studies or during mining to optimise the designs.

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