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Three-dimensional Voronoi-based distinct element model for simulation of hydraulic fracture propagation

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Abstract

Hydraulic fracturing is a complex physical process which involves the coupling of hydraulic and mechanical behaviours as well as, in some cases, temperature effects. The interdependence of these various factors is complex, and cannot be fully captured by the relatively simple state-of-practice tools employed in investigations of hydraulic fracturing. In this paper, a new approach is presented for fully coupled hydro-mechanical simulation of hydraulic fracture propagation by using three-dimensional Voronoi geometries within the context of distinct element formulation. The block boundaries formed by the Voronoi tessellation, providing a random flow pathway for the fluid and the contact breakage due to the increase of the fluid pressure acting on them, replicate the hydraulic fracture propagation. While the Voronoi approach for hydraulic fracturing simulation has been implemented previously in 2D models, this work puts forward a technique for extension of its application to 3D models. A series of verification tests are performed to investigate the suitability of the proposed approach. Finally, example applications are presented for simulation of single-stage and multi-stage hydraulic fracturing of intact rock by using the 3D Voronoi models at large scale.

Keywords: *numerical modelling, 3D Voronoi tessellation, hydraulic fracturing*

1 Introduction

The hydraulic fracturing treatment is a methodology that has been employed in the oil and gas industry over the past few decades with general success for enhancement of the formation permeability by increasing the interconnectivity of the fracture network and its permeability. Similar practice has also been applied to the mining industry for cave initiations and improving fragmentation (Jeffrey et al. 2009; Mills et al. 2001; van As & Jeffrey 2000; van As et al. 2004). Furthermore, with mines reaching greater depths with high stress conditions more recently, hydraulic fracturing is used for pre-conditioning of rock masses to manage its strength and stiffness to mitigate the geomechanical risks associated with high stress environments, such as rockbursting (Board et al. 1992; Kaiser et al. 2013).

Massive multi-stage hydraulic fracturing (MMHF) operations in oil and gas implemented repeated hydraulic fracturing processes is along the length of a well bore. The same approach applies to the mining industry, but at a smaller and more controlled scale. In general, the hydraulic fracturing process involves first fracture initiation, and second fracture growth/mobilisation and interaction with existing natural fractures. A primary goal of most mechanistically-focused models for hydraulic fracturing is to capture the fracture initiation and growth/mobilisation of fractures (and the associated changes in matrix stresses and overall reservoir permeability). Once a fracture has developed and the fluid volume pumped into this fracture continues to increase, there are several competing factors which determine where the fluid travels and how the energy which is input into the system is re-distributed. These factors include: matrix permeability, rock mass stiffness, fracture toughness and the most important, existing natural fractures.

Natural fractures have several key influences on system behaviour during hydraulic fracturing. Primarily, these fractures represent high permeability flow pathways relative to the rock matrix, meaning that they can act either to increase stimulated volumes (in cases where fracture growth is limited) and/or limit fracture pressurisation by acting as preferential leakoff pathways. At depth, natural in situ fractures typically have very small apertures, and so they require either the build-up of fluid pressure or stress-induced shear dilatancy to develop into significant flow pathways. Further, natural fractures can also redistribute the in situ state of stress, particularly when deformation occurs along these features; this impact is also critical given that, ultimately, it is the local stress state rather than the far-field stress state which determines the critical minimum pressure for fracture opening and extension. The combined stress re-distribution and flow re-direction effects influence the mode of interaction between new and pre-existing fractures, determining whether induced fractures cross the natural fracture, become redirected, or have their propagation arrested (Gale et al. 2007). All of the stress and flow related influences of natural fractures are highly dependent on the in situ fracture characteristics, including aperture, roughness, and orientation, which are often poorly understood.

The factors discussed above (in particular the effects of pre-existing natural fractures on the local stress state and rock mass permeability) increase the gap between the in situ conditions and the simplifying assumptions for analytical models for hydraulic fracturing (e.g. significant geometrical constraints and continuous isotropic and homogenous media) such as those described in Sneddon (1946), Sneddon and Elliot (1946), Khristianovich and Zheltov (1955), Geertsma and de Klerk (1969), Perkins and Kern (1961) and Nordgren (1972). This limits the validity of such conventional analytical solutions for simulation of hydraulic fracturing in naturally fractured formations. Nevertheless, analytical models provide a basis for understanding fundamental physical behaviours, and can serve as a check on more complex numerical approaches.

To address this limitation, a new approach is introduced in this paper for simulation of hydraulic fracturing process based on the synthetic rock mass (SRM) concept (Pierce et al. 2007) and the application of three dimensional Voronoi tessellation for fracturing of intact rock (Ghazvinian et al. 2014).

2 Numerical simulation of hydraulic fracturing

Numerical simulation of hydraulic fracturing provides a tool for prediction of ground response in terms of dimensions of induced fractures, their interaction with the pre-existing natural fractures, activation/slipping of the exiting discontinuities and the associated stress shadowing, etc. Therefore, allowing for optimisation of the design parameters and improved efficiencies during the field application of hydraulic fracture stimulations (oil and gas, e.g. Lee et al. 2016; Maxwell et al. 2016) and preconditioning (mining, e.g. Brzovic et al. 2015; Damjanac et al. 2015; Preisig et al. 2015).

The multiphysics nature of hydraulic fracturing is a complex process. The hydraulic behaviour of the injection fluid as well as the mechanical response of the rock formation and the interdependence of these two factors assure the challenging task of numerical simulation of hydraulic fracturing, particularly at MMHF operation scale. In the simplest form, the propagation of hydraulic fractures (HFs) are calculated by means of analytical solutions in combination (may or may not be fully-coupled) with continuum numerical analysis of the mechanical component of the model. In a more advanced approach, which can also be referred to as the current state-of-practice in two and three-dimensional (3D), a distribution of natural fractures is introduced to the hydro-mechanically coupled distinct element model (DEM) of an intact rock, in the form of a discrete fracture network (DFN), whereby fluid flow is allowed on the fracture surfaces between the blocks (e.g. Katsaga et al. 2015; Nagel et al. 2013a; Riahi & Damjanac 2013). This approach can provide realistic information regarding fracture aperture and fluid flow within a fractured reservoir, however, it lacks the ability to propagate new hydraulic fractures through the intact rock.

The emerging codes based on finite-discrete element method (FDEM) provide a solution to a fully-coupled model with the ability to simulate initiation and growth of HFs as well as their interaction with the pre-existing natural fractures (Grasselli et al. 2015). However, the application of this method for simulation of hydraulic fracturing is currently limited to two-dimensional models and has non-trivial simulation time. The 3D nature of pre-existing fractures in unconventional reservoirs require the simulation of hydraulic fracturing to be performed in 3D (Damjanac et al. 2015). With the recent advances in formulation and efficiency of DEM-based numerical codes, the SRM concept has shown promising results for simulation of fully hydro-mechanically coupled, 3D HFs (Damjanac et al. 2015; Damjanac & Cundall 2016).

3 Voronoi tessellation approach

The SRM method (Pierce et al. 2007) was developed to capture a more realistic behaviour of jointed rock masses compared to the conventional numerical representations. The SRM models commonly consist of the bonded particle model (BPM) to represent the intact rock in terms of fracturing and its mechanical response as well as the smooth joint model (SJM) for BPM to mimic the mechanical behaviour of the pre-existing discontinuities. The SRM concept is not limited to the BPM-SJM methods and can be extended to any DEM formulation that is able to simulate fracturing for the intact rock and allows for the representation of the existing discontinuities and their interaction with the induced fractures.

Representation of intact rock in DEM with a tight assembly of blocks tessellated by the Voronoi scheme (random shape convex polygons), as shown in Figure 1, allows for simulation of induced fracturing through intact rock as the rupture of randomly oriented Voronoi blocks edges (in 2D) or faces (in 3D). The Voronoi assembly can be employed in combination with a DFN to represent SRM. Previous works in two dimensions, such as Itasca (2011) and Lan et al. (2010), and three dimensions by Ghazvinian et al. (2014) on the application of Voronoi tessellated domains for simulation of intact rocks, have shown the success of this approach in prediction of brittle rock behaviour.

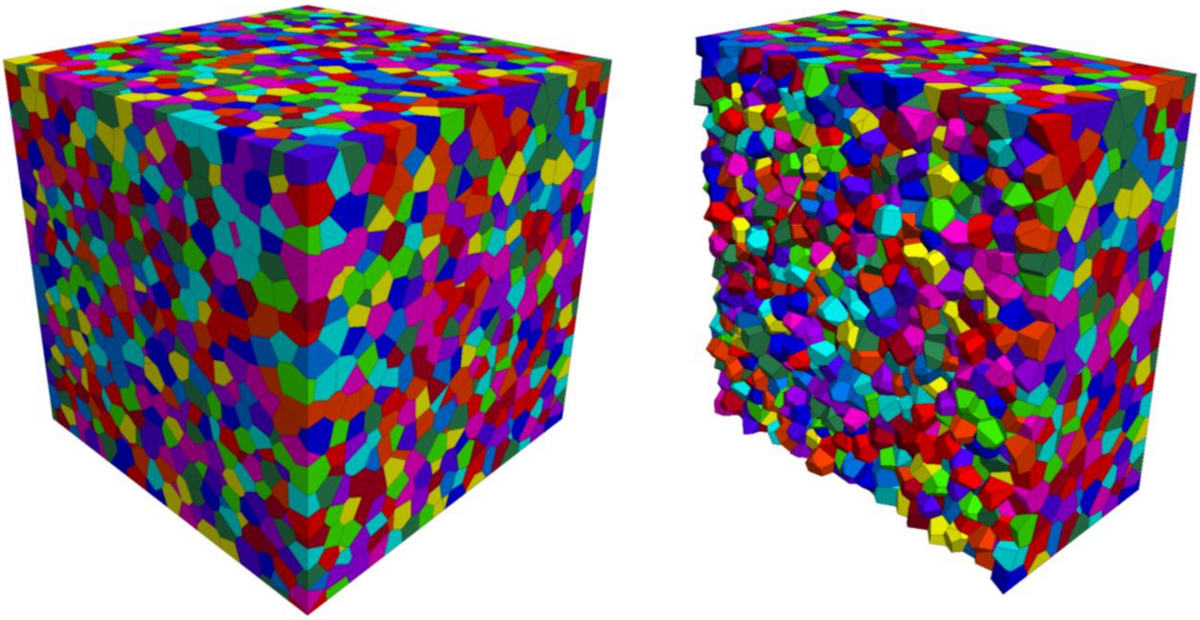

Figure 1 An example for a Voronoi tessellated domain and Voronoi cell geometries

Boundaries of the Voronoi blocks can act as potential random flow pathways for fluid in a similar fashion to simulation of stress-induced fracturing along the Voronoi edges or faces. Simulation of hydraulic fracturing by means of Voronoi blocks has been performed for research purposes in the past, for instance the work by Preisig et al. (2015), Zangeneh et al. (2014) and Pirayehgar and Dusseault (2014). However, the difficulty of generating 3D Voronoi geometries has limited the scope of these investigation to 2D cases.

This study is examining the hydro-mechanical response of 3D Voronoi tessellated intact rock model to fluid injection to propose the Voronoi-based SRM as a simulation tool for in situ scale multi-stage hydraulic fracturing. The three-dimensional DEM numerical code, 3DEC version 5 (Itasca 2013) was chosen for the modelling in this study. 3DEC is fully coupled hydro-mechanically where the flow is allowed through fractures as well as the rock matrix, whereby the fluid flow in fractures (joints) is governed by the lubrication equation (Batchelor 1967). The fractures can deform elastically or inelastically in association with the fluid pressure change. Initiation and propagation of HFs in intact rock in this study is approximated by progressive failure and opening of Voronoi contacts in response to fluid injection and consequent pressurisation of trapped fluid at the contacts at the HF front.

4 Verification tests

To demonstrate the suitability of the formulation implemented in 3DEC for coupled hydro-mechanical simulations, as well as the possibility of using 3D Voronoi geometries for guiding the fluid flow within the intact rock, a series of verification tests are performed and reported in this section. The fluid flow in a penny-shaped crack for both planar (Figure 1(a)) and Voronoi (rough) surfaces (Figure 1(b)) was simulated to mimic the propagation of induced hydraulic fracture in a viscosity-dominated system.

The asymptotic analytical solution that was proposed by Savitski and Detournay (2002) for propagation of a penny-shaped hydraulic fracture in an impermeable elastic rock with zero fracture toughness was used for verification of the numerical results. This solution assumes fluid injection at a point source (on a flat surface at a global scale), no-lag case and the validity of the lubrication theory.

To satisfy the assumptions of the employed analytical solution, the models with zero in situ stresses were generated with 50 m cubes of elastic rock characterised by the Young's modulus of 70 GPa and Poisson's ratio of 0.22. The fracture surfaces cut through the blocks at mid-height with the penny-shaped crack at their centre with an initial hydraulic aperture of 1×10^{-4} m and zero normal and shear strengths. The model geometries are shown in Figure 2. The fluid with the viscosity of 0.001 Pa·s was injected to the centre of the penny-shaped crack with the constant rate of 0.01 m^3/s. The results are discussed in Sections 4.1 and 4.2.

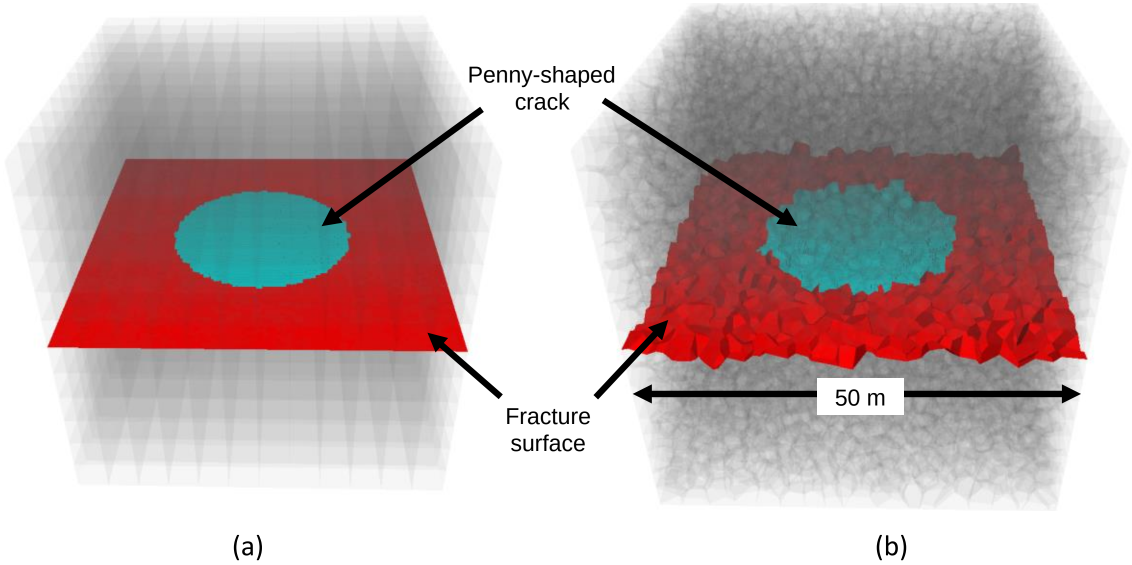

Figure 2 Model set-up and penny-shaped cracks with (a) planar; and (b) Voronoi (rough) surfaces for verification tests

4.1 Planar fracture

Aperture and pore pressure evolution contours for the planar penny-shaped crack at 10 s of elapsed injection time are shown in Figure 3. The aperture and pore pressure along the fracture radius as predicted by the model for three injection times of 5, 10 and 15 s are compared to the exact analytical solution in Figure 4.

The overall predictions of the numerical model were in good agreement with the analytical results, particularly near the injection point. The increasing gap between the numerical results and the analytical solution with moving towards the crack tip can be attributed to the finite aperture of the fracture surrounding the penny-shaped crack. While in the numerical model the extension of the penny-shaped crack is restricted, the finite initial aperture of the surrounding fracture surface allows for leak-off at the crack tip which deviates from the no leak-off assumption in the analytical solution.

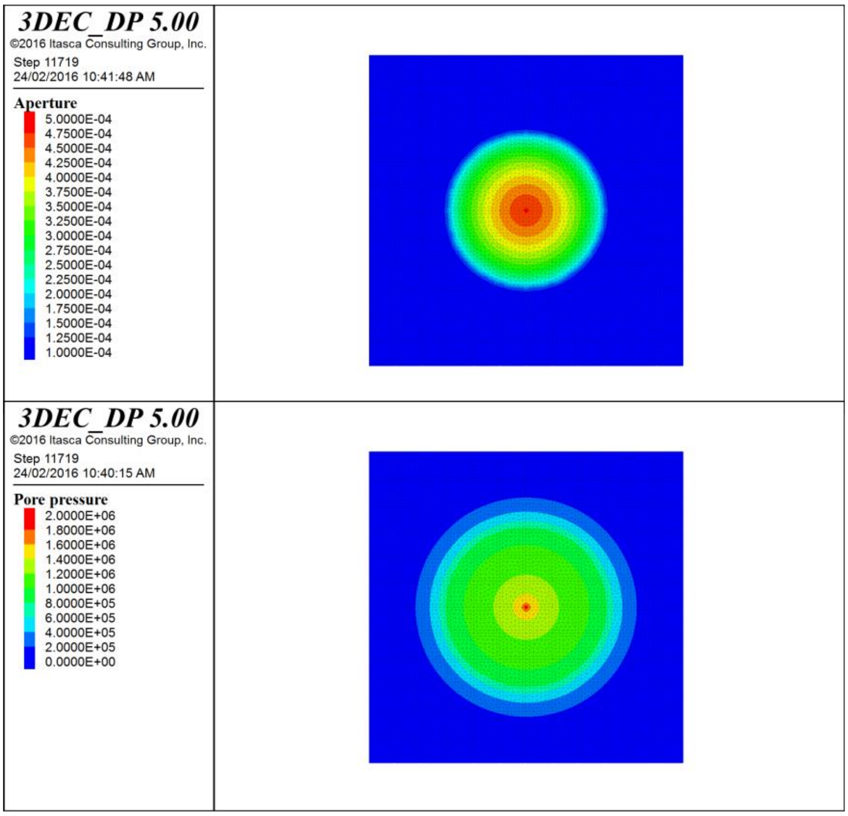

Figure 3 Aperture and pore pressure contours in planar penny-shaped crack after 10 s of fluid injection

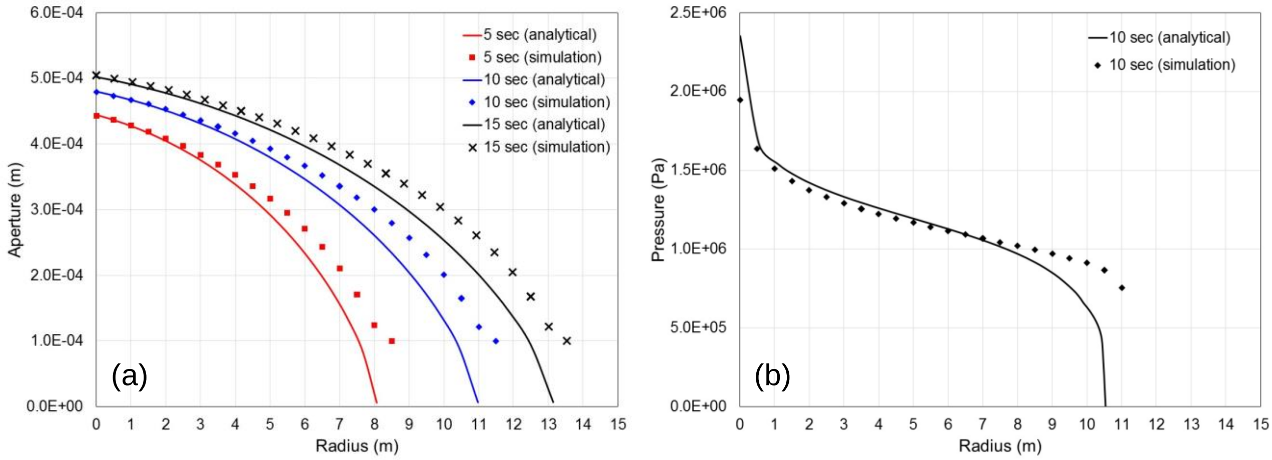

Figure 4 Comparison of the analytical solution and simulation results for (a) aperture; and (b) pressure along the planar fracture radius for different injection times

4.2 Voronoi (rough surface) fracture

The model with the Voronoi surface fracture was generated by tessellating the 50 m cube with 10,000 Voronoi blocks. The Voronoi blocks on the top and bottom of the fracture surface were glued separately to only allow for fluid flow along the penny-shaped crack. The aperture and pore pressure contours for the fracture after 10 s of injection are shown in Figure 5. It is interesting to notice the continuous change in pore pressure as a function of the distance from the injection point compared to the discrete aperture profile. It can be observed in Figure 5 that the aperture is mainly controlled by the dipping angle of each comprising facet of the fracture. The dipping angle of each plane controls deformation mode for that plane whether opening or slipping and therefore the fracture width (aperture).

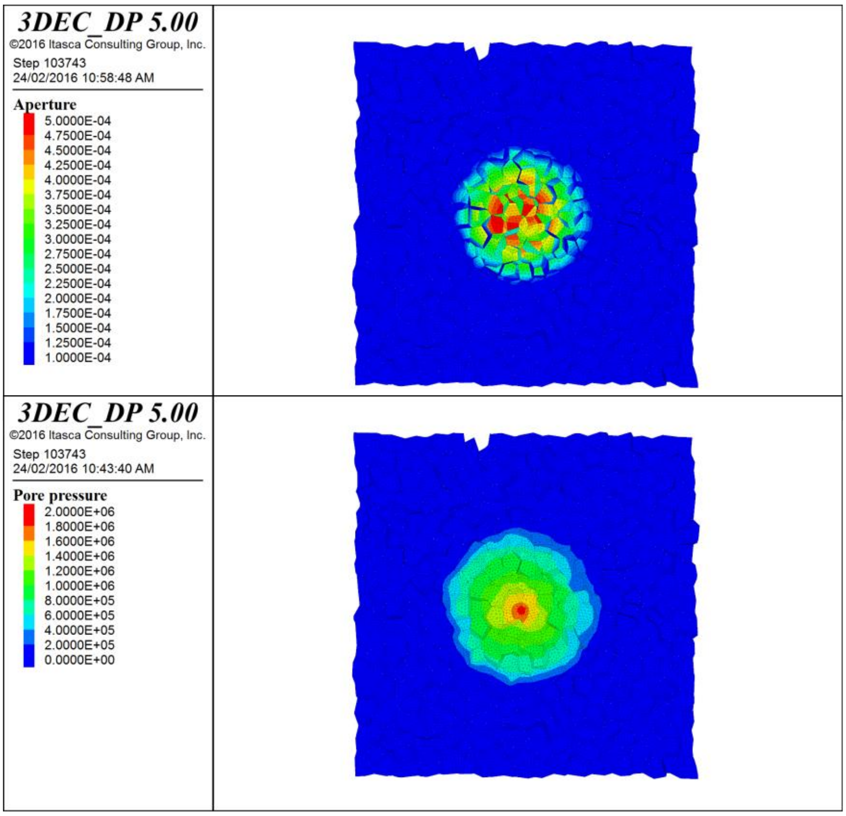

Figure 5 Aperture and pore pressure contours in rough penny-shaped crack (10,000 blocks) after 10 s of fluid injection

The aperture (opening of the fracture perpendicular to the direction of fracture extension) and pore pressure predictions of the 3DEC model with the Voronoi fracture surface for 5, 10 and 15 s elapsed injection times are shown in Figure 6. The values at each radius are the average of that parameter for 36 samplings with 10° angular interval at that given radius. A good agreement exists between the simulated and analytical pore pressure. It should be noted that the accuracy of the 3DEC solution for Voronoi fracture was verified in Section 4.1 and also in the work by Damjanac and Cundall (2016) for hydro-mechanical response of each single facet and in combination.

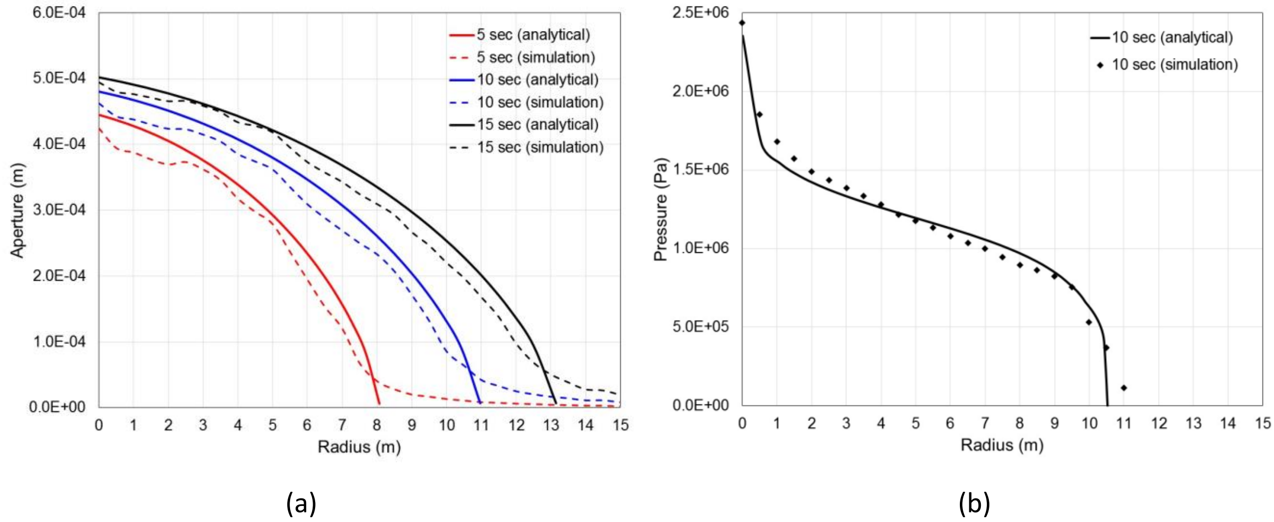

Figure 6 Comparison of the analytical solution and simulation results for (a) aperture; and (b) pressure along the rough fracture radius for different injection times

5 Example applications

Two example applications are discussed in this section to demonstrate the application of the implementation of Voronoi tessellation in 3D DEM for simulation of hydraulic fracturing of intact rocks.

5.1 Single-stage hydraulic fracture simulation

This example involves injection of a fluid to the centre of an intact block of rock at the depth of 3,000 m. The block of rock is a 300 m cube (2.7×10^7 m^3) tessellated with 2,000 Voronoi blocks. The elastic Voronoi block material was characterised with the Young's modulus of 20 GPa and Poisson's ratio of 0.25. Vertical in situ stress associated with 3,000 m of overburden and horizontal stresses corresponding to vertical/horizontal stress ratios of 1.0 (k_H) and 0.5 (k_h) were applied to the model as shown in Figure 7. Pore pressure equivalent to 3,000 m column of water was also initiated in the model.

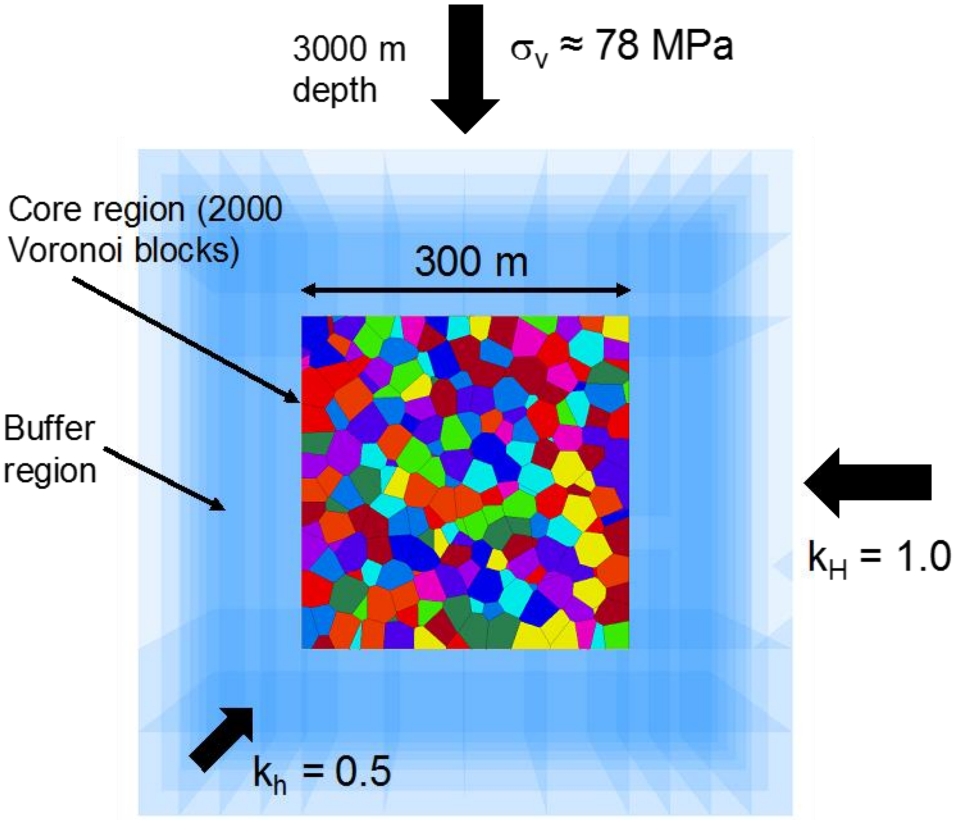

Figure 7 Model set-up for the single-stage HF simulation

A fluid with the viscosity of 0.0015 Pa × s was injected to the centre of the intact rock block with an injection rate of 0.05 m^3/s. Before injection, the contacts in the vicinity of the injection point, within the radius of 15 m, were fractured to allow for propagation of the HF. The pressure build-up due to the injection was allowed to break the bonded Voronoi contacts. The fluid was only allowed to flow within the fractured (broken) contacts.

State of the model at the elapsed injection time of approximately 90 min is shown in Figure 8. It should be noted that this model was not calibrated for flow time and the model parameters were chosen merely to provide an understanding of the numerical approach and its potentials. The propagation of the induced HF is identified in the form of increase in the contacts pore pressure above the background values (Figure 7 top) or the contacts with near zero normal stresses (Figure 7 bottom). It is interesting to observe propagation of the HF in the expected direction, normal to the orientation of smaller horizontal stress while having a characteristic pattern similar to what is usually seen in rock masses.

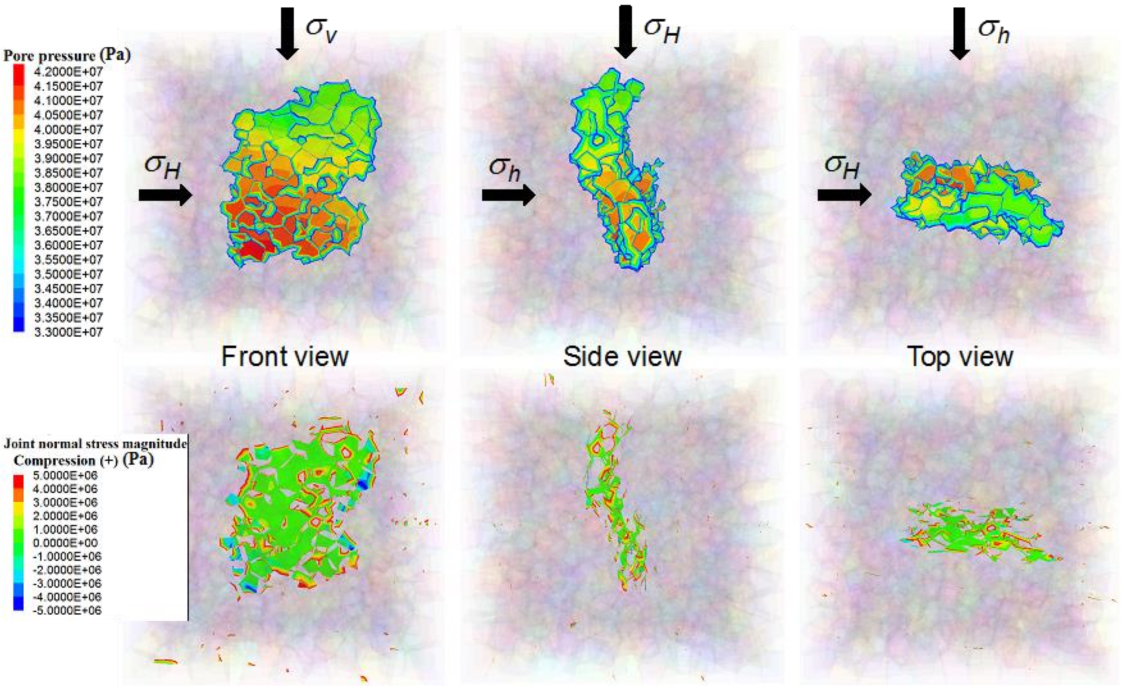

Figure 8 State of the single-stage HF model at the injection time of approximately 90 min; (top) pore pressure increase above the background values; and (bottom) contacts with near-zero normal stresses

5.2 Multi-stage hydraulic fracture simulation

The second example application involves injection of fluid to the intact rock in three stages for simulation of multi-stage hydraulic fracturing. The model parameters and in situ stress conditions are identical to those described in Section 5.1 with the only difference of the tessellated core region size. The model geometry for the three-stage hydraulic fracturing is shown in Figure 9.

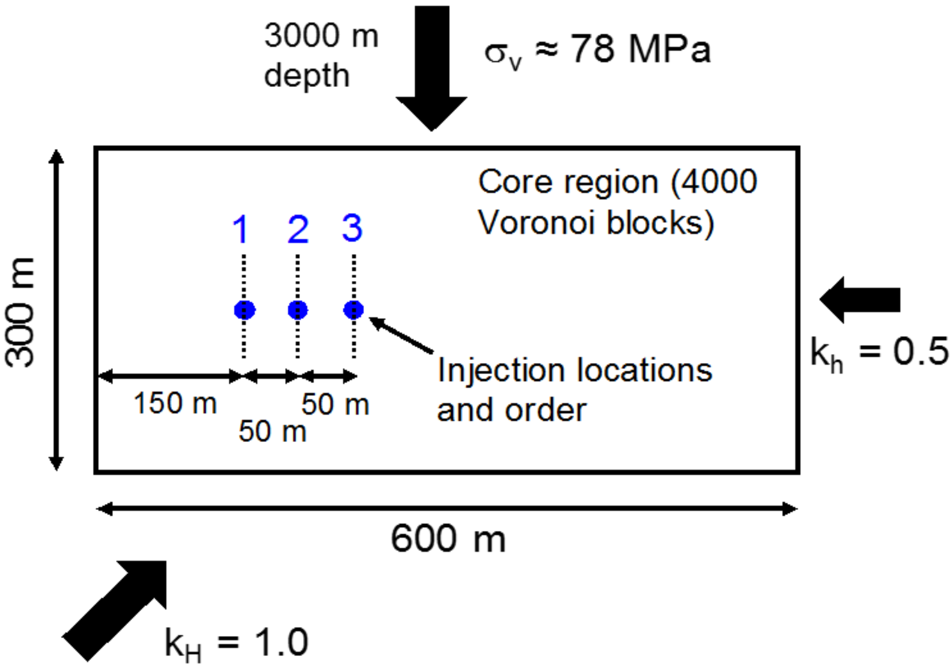

Figure 9 Model set-up for the multi-stage HF simulation

The tessellation resolution was kept identical to the single-stage HF simulation example. The injection locations were 50 m apart along the length of the tessellated core region. The fluid was first injected to location 1 in Figure 9 for a duration of approximately 60 min, followed by consecutive 30 min injections at each of the locations 2 and 3, respectively.

Simulation results for the multi-stage hydraulic fracturing at the end of each injection cycle are shown in Figure 10. After completion of the first stage, the second fracture is observed to grow discretely, mainly vertically apart from the first fracture. Conversely, the third fracture is attracted towards the second fracture due to the stress shadowing effect, a mechanical phenomenon that is widely accepted (Nagel et al. 2013b). The third fracture ultimately interacts with and contributes to further extension of the second fracture.

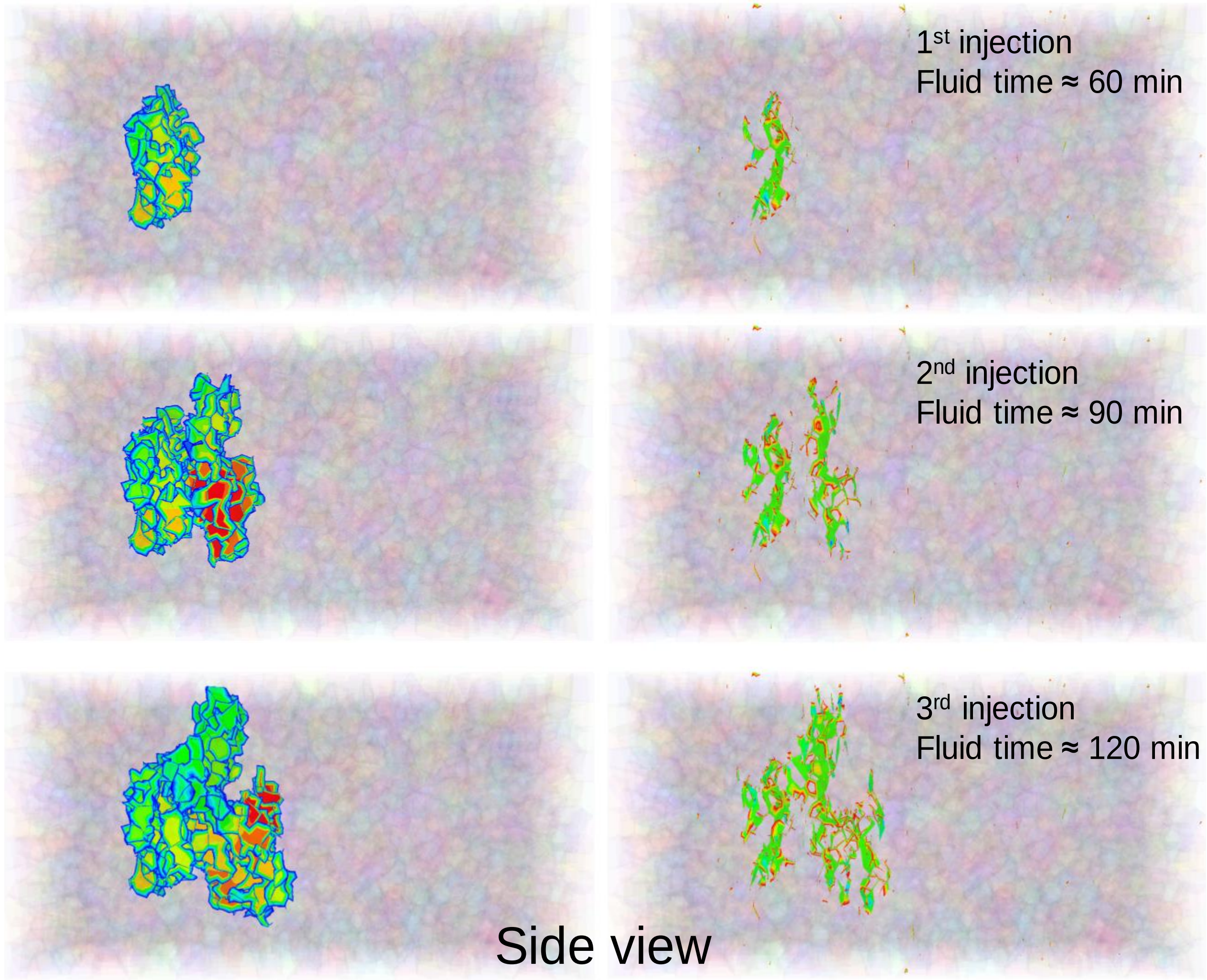

Figure 10 State of the 3-stage HF simulation model at the end of each injection cycle; (left) contact pore pressure increase above the background values; (right) contacts with near-zero normal stresses

6 Concluding remarks

Implementation of 3D Voronoi tessellation in the DEM-based code 3DEC was shown in this study to be a promising tool for simulation of hydraulic fracturing. The hydro-mechanically coupled interaction of fluid in fractures was shown in the verification tests to be predicted precisely for the Voronoi surfaces. Furthermore, the feasibility of simulating large-scale hydraulic fractures in multiple stages was confirmed in the example applications. It was also observed that the model is able to capture the correct mechanics of HFs interaction, such as stress shadowing effects.

Representation of natural fractures within Voronoi tessellated DEM models by means of DFNs would allow for initiation and propagation of new fractures as well as their interaction with the existing natural fractures for simulation of hydraulic fracturing and hydraulic shearing in fractured rock masses.

While the deficiencies of the DEM method in terms of computation time remains a challenge for hydro-mechanically coupled models, the authors are exploring for solutions to this problem with innovative modelling approaches.

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