

# Seismic hazard estimation using databases with bimodal frequency–magnitude behaviour

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## Abstract

*Many underground mines experience seismic events associated with rock mass failure which can be of sufficient magnitude to pose a significant hazard to operations. Probabilistic seismic hazard assessments are typically performed assuming a Gutenberg–Richter distribution for the frequency–magnitude relation for which the parameters are obtained from a best fit to the data. This distribution assumes self-similar data above the magnitude of completeness but this is not always valid. The breakdown in self-similarity can occur when there are multiple superimposed seismic sources, or when there are artificial noise sources such as orepasses and underground crushers.*

*This paper introduces an alternative parametric technique to decompose a bimodal frequency–magnitude relation into two sub-distributions. The composite distribution method assumes that two separate distributions are underlying the observed frequency–magnitude behaviour. This assumption was tested with respect to a single Gutenberg–Richter model to describe frequency–magnitude behaviour. The hypercube optimisation algorithm was used to solve the parameters of the two superimposed distributions while minimising the residual sum of squares for the fit compared to the observed data.*

*The mXrap software was used to implement the method at multiple underground mines for specific volumes and for grid-based analysis. The results show that locally, the seismic hazard can be severely underestimated if a single Gutenberg–Richter model is assumed but this can be improved with the composite distribution method.*

**Keywords:** *seismic hazard, seismic sources, seismic noise, frequency–magnitude relation*

## 1 Introduction

Large seismic events result from dynamic rock mass deformation and are a major threat to the safety and sustainability of deep, underground mines (Potvin 2009). Geotechnical engineers are charged with estimating the seismic hazard and introducing appropriate tactical and strategic controls to manage this hazard. An underestimated seismic hazard may result in insufficient controls and an increased risk to personnel. This paper focuses on the two main cases where seismic hazard can be underestimated:

- Multiple, spatially superimposed sources of seismicity.
- Sources of seismic noise that are spatially superimposed with real seismicity.

Given that the current state of knowledge of seismicity is incomplete, we must treat seismic events as partially random phenomena and therefore, a probabilistic approach is required to quantify seismic hazard (Lasocki 2005). The frequency–magnitude chart is a cornerstone of probabilistic seismic hazard assessment. The most commonly used distributions to describe the frequency–magnitude behaviour in mines are the open-ended Gutenberg–Richter distribution (GR) (Gutenberg & Richter 1944) and the upper-truncated Gutenberg–Richter distribution (TGR) (Page 1968). The cumulative density functions for the GR and TGR are given in Equations 1 and 2 respectively.

$$F(m) = 1 - 10^{-b(m-m_{\min})}; m_{\min} < m \quad (1)$$

$$F(m) = \frac{1 - \exp[-b \ln 10(m - m_{\min})]}{1 - \exp[-b \ln 10(m_{\max} - m_{\min})]}; m_{\min} \leq m \leq m_{\max} \quad (2)$$

These equations are valid for event magnitudes greater than  $m_{min}$ , which is the minimum magnitude event that can be reliably observed by a seismic monitoring system. Compared to the original GR distribution, the truncated version differs by introducing a maximum possible event magnitude for a source of seismicity,  $m_{max}$ . The b-value describes the relative number of small and large events in the analysis period and is an important parameter as it provides insight into underlying seismic source mechanisms (Wesseloo 2014). The identification and parameterisation of individual seismic sources is an essential component for the probabilistic assessment of seismic hazard (Kijko 2011). Furthermore, an objective approach to spatial seismic hazard assessment is highly desirable as manual involvement introduces a source of error due to various forms of cognitive and motivational biases (Montibeller & Winterfeldt 2015).

The seismic hazard can be defined as the probability that a design event magnitude,  $M_{design}$ , will be exceeded in the next time increment, T. This was defined by Kijko et al. (2001) as per Equation 3 where  $\lambda$  is the mean activity rate for volume under consideration. An alternative (but similar) definition of seismic hazard is the maximum event size that can be expected to occur within a volume under consideration, with a confidence of P, within period T (Wesseloo 2013). While a continuation of current conditions is assumed in the hazard assessment, it is important to note that this approach does not forecast the future seismic hazard but quantifies the current seismic hazard state from recent data.

$$P(M_{design}, T) = 1 - \exp\{-\lambda T[1 - F(M_{design})]\} \quad (3)$$

Wesseloo et al. (2014) presented a grid-based analysis techniques with the general principle to:

*“Assign a representative seismic parameter value to grid points in space based on the events in its neighbourhood in order to extract information from the variation of these different parameter values in space.”*

Grid-based analysis offers an approach to identify seismic sources and the assessment of seismic hazard. Figure 1 shows the results from a grid-based assessment which spatially evaluates the b-value for seismicity within an Australian mine. Lower b-values indicate there are proportionally more large events on the stopping abutment. This volume of rock mass has proportionally higher seismic hazard for an equivalent number of seismic events.

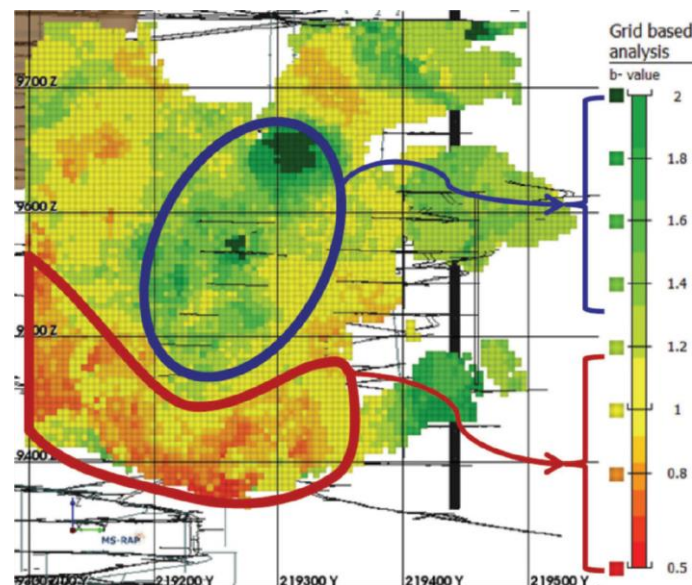


Figure 1 Grid-based analysis of the spatial variation of the b-value at an Australian mine. The blue zone indicates a higher b-value associated with the stopping area. The red zone indicates lower b-values on the stopping abutment (Wesseloo et al. 2014)

A major challenge of using a 3D spatial grid-based assessment is that computation requirements increase exponentially with decreasing grid spacing. For this reason, methods of fitting the Gutenberg–Richter

distribution tend to be computationally constrained and do not consider more complex frequency–magnitude relations. In the case of grid-based analysis, spatially superimposed sources of seismicity with different frequency–magnitude characteristics will invalidate the assumption of a single Gutenberg–Richter model. Figure 2 shows an example at an Australian mine where the grid-based b-value calculation using a method of maximum likelihood has resulted in an underestimated hazard due to a bimodal frequency–magnitude relation. In this example, seismic noise associated with tipping material into an orepass has contributed to the bimodal frequency–magnitude relation.

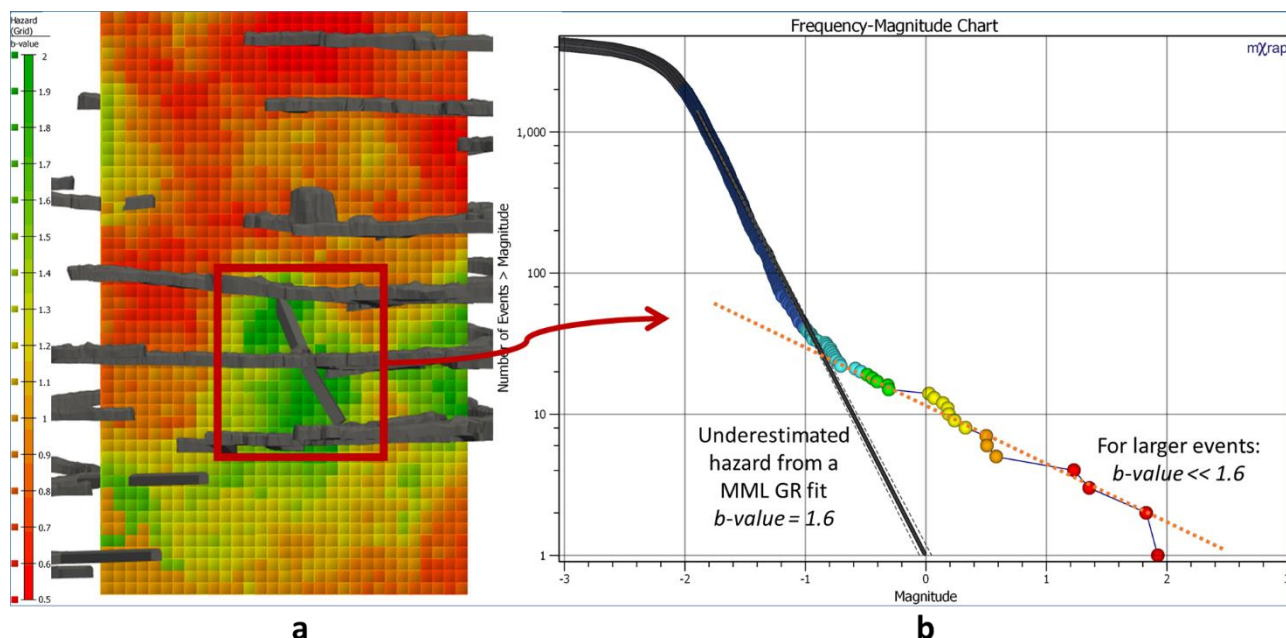


Figure 2 (a) Long section of grid-based b-value calculation at an Australian mine; (b) Frequency–magnitude relation for the highlighted zone. In this case, the automated b-value calculation (black line) has led to an underestimated hazard

## 2 Background

Implicit in traditional approaches to assessing seismic hazard is the assumption that event magnitudes greater than  $m_{min}$  can be reasonably estimated with the modified Gutenberg–Richter model. This model has been widely accepted in crustal and mining seismology.

A common feature of mining seismicity databases is the occurrence of seismic noise (e.g. orepasses, underground crushers, and blasting) which causes multimodality in frequency–magnitude relations. There has also been speculation that the frequency–magnitude relations have not been well described with the Gutenberg–Richter distribution when there is an accumulation of multiple seismic sources (Kijko et al. 1987; Finnie 1999; Richardson & Jordan 2002; Kwiitek et al. 2010). Amidzic (2001) explained multimodal relations at various sites in South Africa as separate failure mechanisms and physical processes. Morrison et al. (1993) suggested that at Creighton Mine, there were at least two distinct groups of seismic events that could be separately modelled by different Gutenberg–Richter relations. Hudyma (2008) summarised the extensive literature on the breakdown in self-similar behaviour and concluded that larger datasets with multimodal behaviour contain spatially constrained subsets of seismicity that can be modelled individually by the Gutenberg–Richter model.

The foremost concern of seismic hazard assessment is the estimation of large event occurrence and hence the statistical estimation of the relevant underlying frequency–magnitude distribution. Existing approaches aiming to improve hazard estimation for bimodal frequency–magnitude distributions have focused on ‘per event’ discriminators or non-parametric approaches which are briefly discussed in Section 2.1 and 2.2.

## 2.1 Discriminators

There have been numerous attempts to discriminate between different natural and artificial sources of recorded seismicity. Discriminators assume that there is an observable difference when comparing the waveforms and subsequent source parameters of real seismic events to noise. Discrimination methodologies are ‘trained’ using a dataset of exclusively noise or exclusively real seismicity. Lessons from these areas are then used to discriminate between seismic noise and real seismic events. This concept is often extended to discriminate between real seismic events that are generated by different rock mass failure processes.

Discriminators have been used in crustal seismology to separate earthquakes from man-made explosions (e.g. nuclear tests, quarry blasts) (Del Pezzo et al. 2003) and in mine seismology to clean seismic event databases of blasts (Malovichko 2012).

A number of discriminators are based on waveform spectrum analysis (Ford & Walter 2010). These techniques require original seismic waveforms which are not routinely available for retrospective analysis. More commonly for mining seismology, discriminators use multivariate statistical methods (Taylor 1996; Malovichko 2012) or soft computing techniques (Finnie 1999; Kuyuk et al. 2011). These methods rely on the availability, accuracy, and consistency of discriminant parameters, e.g. the time of occurrence, event magnitude, S-wave and P-wave energy, static and dynamic stress drop moment, and corner frequency.

Finnie (1999) used a test database for discriminating between ‘genuine’ and ‘spurious’ seismic events. These two distinct modes were assumed to arise from geological features (genuine) and from the fracturing in extreme stress concentrations around excavations (spurious). Kwiitek et al. (2010), Amidzic (2001) and Richardson & Jordan (2002) have also classified separate modes of seismicity in a similar way. Finnie (1999) used a trained artificial neural network to separate these seismic sources based on several source parameters. Figure 3 shows the results of the discrimination with respect to the frequency–magnitude behaviour of the full dataset and the separated seismic modes.

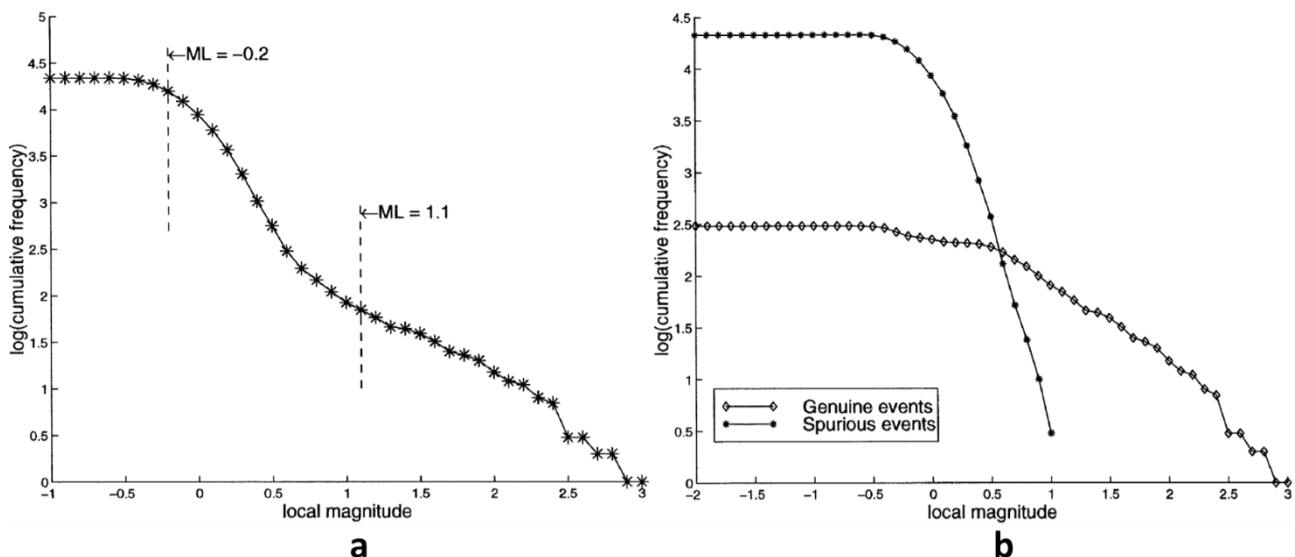


Figure 3 (a) Frequency–magnitude relation for the combined dataset of 21,833 events; (b) Separate sources after discriminating with an artificial neural network (Finnie 1999)

The drawback of discriminators is the lengthy set up and calibration procedure. Each discriminator must be tailored to a specific site once a large enough test database has been obtained and hence may not be suitable for younger mines. Discrimination methods rely on the accurate and consistent determination of seismic source parameters (Morkel et al. 2015). This can be challenging in mines which have suboptimal seismic monitoring arrays, e.g. planar arrays, few triaxial sensors, and a small range of monitoring frequencies (Morkel & Wesseloo 2017).

## 2.2 Non-parametric hazard estimation

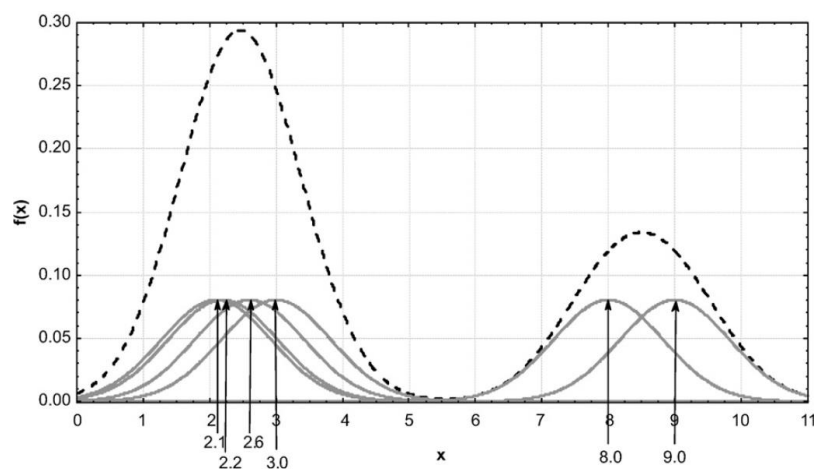
Given the potential for multiple sources influencing the frequency–magnitude relation in the same dataset (orepass or crusher noise, blasts and multiple seismic mechanisms), isolating each source into self-similar populations can become challenging. A non-parametric approach to estimating seismic hazard does not require the assumption of an underlying distribution model (such as the Gutenberg–Richter distribution). This approach for estimating seismic hazard is referred to as a data-driven or model-free technique and has been described by Kijko et al. (2001), Lasocki (2001), Lasocki and Orlecka-Sikora (2006), and Orlecka-Sikora and Lasocki (2017).

The probability density function,  $f(x)$ , of event magnitude occurrence is estimated by accumulating the contribution of each data point according to the kernel function. Figure 4 illustrates this concept where each data point (event magnitude) has been replaced with a Gaussian kernel function. Equations 4 and 5 describe the probability density function and the Gaussian kernel function used by Kijko et al. (2001) for univariate data  $x_i, i = 1, \dots, n$ .

$$f(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x-x_i}{h}\right) \quad (4)$$

$$K(\xi) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\xi^2}{2}\right) \quad (5)$$

The bandwidth factor,  $h$ , is an important parameter to define the level of smoothing required for the population. This has a similar effect as the bar width of a standard histogram plot. An additional weighting factor,  $w_i$ , is applied to the kernel bandwidth due to the tendency of constant kernels to generate spurious noise at the end of long-tail distributions (Silverman 1986). This effectively applies an adaptive kernel that is wider for larger, less frequently occurring events. Once the cumulative distribution function had been estimated, Kijko et al. (2001) calculated the corresponding seismic hazard as per Equation 3 and in terms of the mean return period for an event greater than  $M_{design}$ . Orlecka-Sikora and Lasocki (2017) described an iterated, bias-corrected, and accelerated approach to calculate the uncertainty in the non-parametric kernel estimation of seismic hazard.



**Figure 4** Illustration of non-parametric, kernel estimation of the probability density function of a univariate data series (such as event magnitude). Each data point is replaced with a kernel function (Gaussian function in this case) and the probability density is estimated by accumulating the respective contribution of each kernel function (Lasocki & Orlecka-Sikora 2006)

The non-parametric approach is extremely computationally intensive and therefore poorly suited for grid-based analysis where the procedure must be repeated for many grid points. Specifically, computational times are a major constraint for finding the optimum bandwidth factor using the least-squares cross-validation technique described by Silverman (1986). Furthermore, non-parametric approaches are only valid if the whole frequency–magnitude distribution remains consistent. Hence, when sources of seismic noise are no longer present (e.g. no longer using an orepass) the non-parametric estimate of seismic hazard is no longer applicable. Aside from an estimate of seismic hazard, non-parametric approaches do not provide additional information, such as the b-value, which carries interpretive value concerning seismic source mechanisms of their underlying rock mass failure processes.

### 3 Composite distribution method

Given the current limitations of discriminators and non-parametric approaches, this paper presents a method for the parametric estimation of seismic hazard when the frequency–magnitude relation is bimodal. This method is computationally efficient which allows for the implementation into a grid-based analysis framework.

A simple parametric approach is to assume that the observed bimodal frequency–magnitude distribution is a result of two superimposed independent distributions. The first distribution models the frequency–magnitude distribution of frequently occurring smaller events (FS) which dominate the total event count. The second distribution models the frequency–magnitude distribution which is consistent with the observed occurrence of the infrequent larger events (IL).

Where two spatially superimposed rock mass failure processes generate very different frequency–magnitude distributions (e.g. a seismically active fault running through a zone experiencing stress fracturing), the IL frequency–magnitude distribution can be modelled by a GR distribution (i.e. Equation 1) and the observed FS frequency–magnitude distribution can be modelled by a TGR distribution (i.e. Equation 2). Figure 5 provides a conceptual example of modelling an observed bimodal frequency–magnitude using two superimposed Gutenberg–Richter models. The composite distribution method allows for more accurate seismic hazard estimation from a bimodal dataset and estimates the contribution of each source.

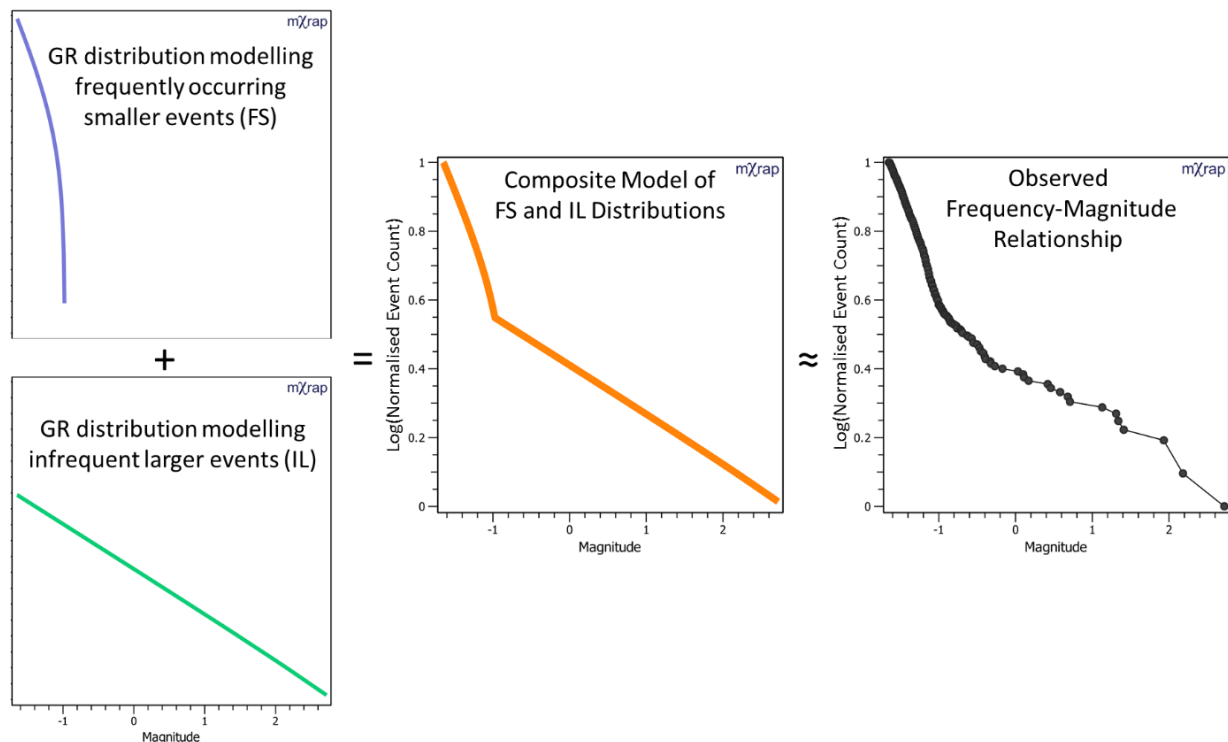


Figure 5 Illustration of the composite distribution method whereby two Gutenberg–Richter distributions are used to approximate the observed bimodal frequency–magnitude relation



While the IL range should always be real seismic events that can be reasonably modelled by a Gutenberg–Richter distribution, the FS range may be generated by several artificial sources and therefore, may not be well modelled by a GR or TGR distribution. Additional distributions were considered as replacements. Table 1 summarises the distributions that were investigated to potentially model the FS frequency–magnitude distributions. The frequency–magnitude distribution may not be bimodal and hence, we also use a single GR distribution to model observations (GR<sub>RSS</sub>). A summary of each parametric approach can be found in Table 2. These combinations of distributions were simultaneously assessed so fitness comparisons could be made. Probabilistic seismic hazard results are similar for truncated and open-ended Gutenberg–Richter distributions for smaller datasets where the observed maximum magnitude event is much less than  $m_{max}$ . The use of an open-ended Gutenberg–Richter model is preferable to fit the IL distribution as it helps to constrain the range of free parameters for the different combinations of distributions.

**Table 1** Summary of distributions used to model FS frequency–magnitude relation

Distribution	Parameters	Probability density function
TGR	$m_{min}$ $m_{max}$ b-value	$f(x) = \frac{b \ln(10) 10^{-b(x-m_{min})}}{1 - 10^{-b(m_{max}-m_{min})}}$
Gumbel	$\mu$ = mean $\beta$ = scale constant	$f(x) = \frac{1}{\beta} e^{-(z+e^{-z})}, \text{ where } z = \frac{x - \mu}{\beta}$
Weibull	$\lambda$ = scale constant $k$ = shape constant	$f(x) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1}, \text{ where } x \geq 0$
Normal	$\mu$ = mean $\sigma$ = std. deviation	$f(x) = \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

**Table 2** The distribution combinations that are considered by hypercube optimisation

Abbreviation	FS distribution	IL distribution
GR <sub>RSS</sub>	GR	N.A.
TGR–GR	TGR	GR
GB–GR	Gumbel	GR
WB–GR	Weibull	GR
NL–GR	Normal	GR

Before applying a fitting procedure for combinations of distributions, the  $m_{min}$  and b-value are calculated using the method described by Wesseloo (2014). This approach to fitting the GR uses the method of maximum likelihood (GR<sub>MML</sub>) and is insensitive to the tail (i.e. IL portion) of the frequency–magnitude distribution). The resultant b-values are expected to be erroneous for bimodal frequency–magnitude distributions but provide an informative comparison when considered with respect to b-values determined from a composite distribution. Since this approach is insensitive to IL events, the  $m_{min}$  determined by the GR<sub>MML</sub> approach is reliable.

The fitting procedure minimises the residual sum of least-squares (RSS) and considers all event magnitudes greater than  $m_{\min}$  found by the GR<sub>MML</sub> approach. The seismic events are discretised into magnitude bins and weighted equally across all magnitude ranges. This procedure effectively increases the weighting of larger events which prevents the high frequency distribution from dominating the fitting procedure. Discretising event magnitudes also allows computational times to be greatly reduced and is pragmatically preferred when implementing this methodology into a framework for grid-based analysis.

Given the high number of parameters that influence the optimisation, the computation time with brute-force optimisation is not suitable. The hypercube optimisation algorithm is used to find the optimum parameters. This method is described in detail by Abiyev and Tunay (2015) who demonstrated that it offers improved performance compared with the other commonly used techniques for high-dimensional global optimisation. The hypercube algorithm optimised the RSS for the parameter ranges listed in Table 3.

**Table 3** Input parameter ranges for the hypercube optimisation algorithm

Distribution	Parameter	Value range	
		Min	Max
GR (IL)	b-value	0.0	2.5
TGR (FS)	b-value	0.0	4.0
	$m_{\max}$	-2.0	0.0
Gumbel (FS)	$\mu$ (mean)	-2.0	0.0
	$\beta$ (scale)	0.0	0.5
Weibull (FS)	$\lambda$ (scale)	0.5	3.5
	k (shape)	2.5	35
Normal (FS)	$\mu$ (mean)	-2.0	-1.0
	$\sigma$ (std. dev.)	0.1	0.5

## 4 Case studies

### 4.1 Grid-based analysis: orepass noise

The spatial variation of seismic hazard can be assessed using a grid-based assessment. This case study uses grid-based assessment to examine seismicity and noise surrounding orepasses. This seismicity has been heavily contaminated with tipping noise although it is also likely to include a portion of smaller events associated with stress fracturing. Irrespective of whether these smaller events are real or not, the events result in the underestimation of local seismic hazard.

Figure 6 shows the number of events neighbouring each grid point (a) and the resultant probabilistic seismic hazard for the best fit distribution (b). Seismic hazard is expressed as the annualised probability of an event greater than  $M_L$  1.5 within 100 m with a continuation of current rates.

There is no apparent difference in seismic event density surrounding orepass A in comparison to orepass B, although there is a significant and distinct difference in the relative seismic hazard surrounding each orepass. While both orepass A and B exhibit bimodal frequency–magnitude distributions, the seismic hazard surrounding orepass A is negligible when compared to orepass B which is approximately 200 m away. Figure 7 illustrates typical frequency–magnitude distributions and parameterisation for the spatial volumes surrounding orepasses A and B. These results indicate that while noise and real seismic events are observable surrounding both orepasses, the seismic response surrounding orepass B is far more likely to



generate larger events. Identifying areas of high seismic hazard is critical when considering the risk of failure of potentially unsupported capital infrastructure such as orepasses.

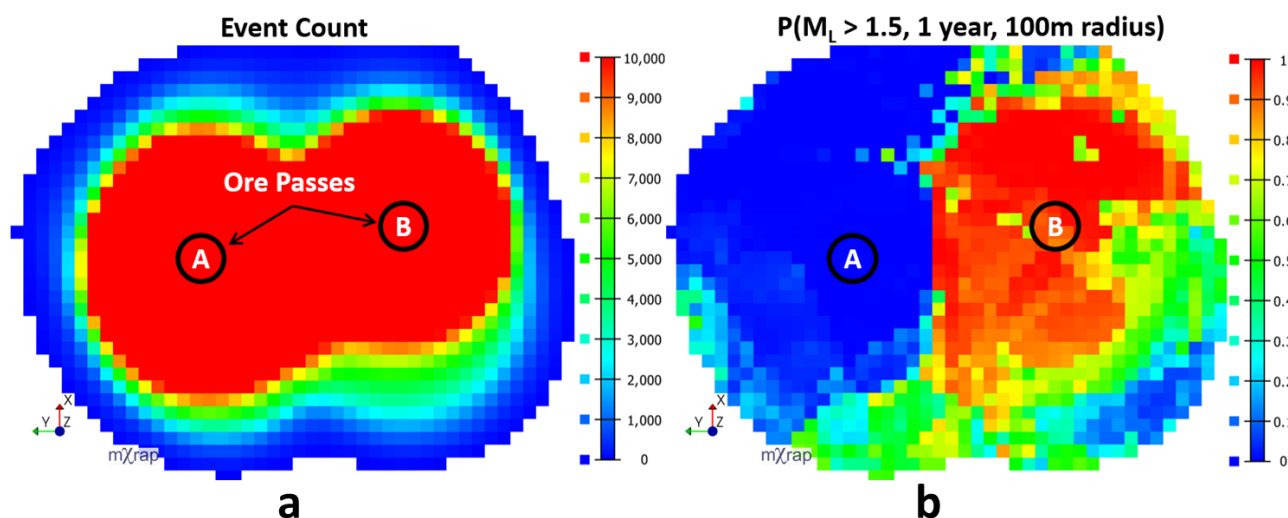


Figure 6 (a) Number of events surrounding each grid point; (b) Probabilistic seismic hazard for the best fit distribution

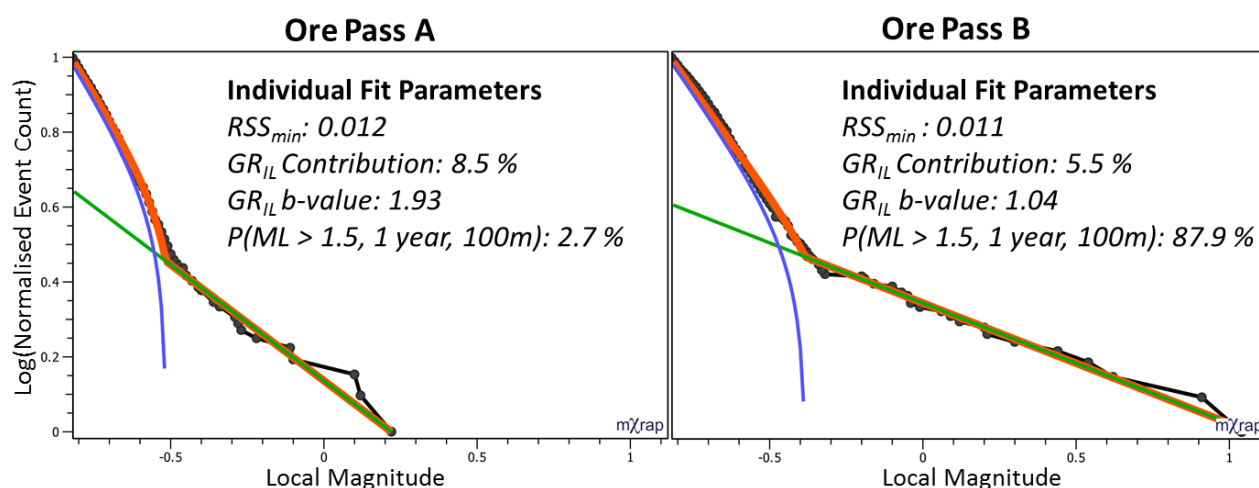


Figure 7 Typical frequency-magnitude distributions and parameterisation for the spatial volumes surrounding orepass A and B

## 4.2 Volume-specific analysis

The composite distribution modelling approach can be applied to any specific volume within a mine to estimate the seismic hazard where a bimodal frequency-magnitude relation exists. The case presented here considers 4,158 events over two years of mining for an Australian underground mine. The volume selected was not close to any artificial noise sources but there is a strong bimodal frequency-magnitude behaviour observed. Figure 8 illustrates the optimised modelling results for five distribution models that were considered and the specific volume used for assessment (Figure 8(a), red box).

Table 4 summarises the RSS and a probabilistic expression of hazard for each distribution model. These results detail the b-value, and percentage contribution for the low frequency distribution of seismicity, along with a probabilistic estimate of the seismic hazard using Equation 2. Probabilistic seismic hazard is expressed as the likelihood of experiencing an event greater than  $M_L 2.5$  within the specified volume, given one year of seismicity at the currently observed event rate. There is a negligible contribution of the FS distribution to seismic hazard and it is not considered in probabilistic calculations.

Aside from slight differences in the NL–GR model (Figure 8(f)), there are no significant differences between the composite distribution methods used to model the frequency–magnitude distributions of seismicity. These models result in an approximate  $b$ -value of 0.5, 8% contribution, and a  $P[M_L > 2.5 \mid 1 \text{ year}] = 45\%$  (Figure 8(c), (d) and (e)). These results suggest that the FS distribution of events can be reasonably well represented by the alternative distributions and that results are reasonably similar.

The difference in  $GR_{MML}$  and  $GR_{RSS}$  models (Figure 8(b)) shows the effect of discretising the data and differing fitting methodologies. The model fit to discretised data (green line) is influenced relatively more by the larger magnitude events when compared to the original fit which uses the method of maximum likelihood (blue line).

The results from composite distributions significantly differ from both the  $GR_{MML}$  and  $GR_{RSS}$  models. The seismic hazard is severely underestimated due to the higher  $b$ -values found by the single GR models (1.25 versus 0.5). Such a significant difference in seismic hazard ( $P[M_L > 2.5 \mid 1 \text{ year}] = 0.6\%$  and  $45\%$  respectively) represents a major difference in the level of strategic and tactical methods required for the management of seismic hazard. In practice, seismic hazards surrounding orepasses are typically considered as erroneous. The lack of knowledge concerning frequency–magnitude distributions may be compensated for by a conservative seismic hazard management response and/or the interpolation of seismic hazard from surrounding areas.

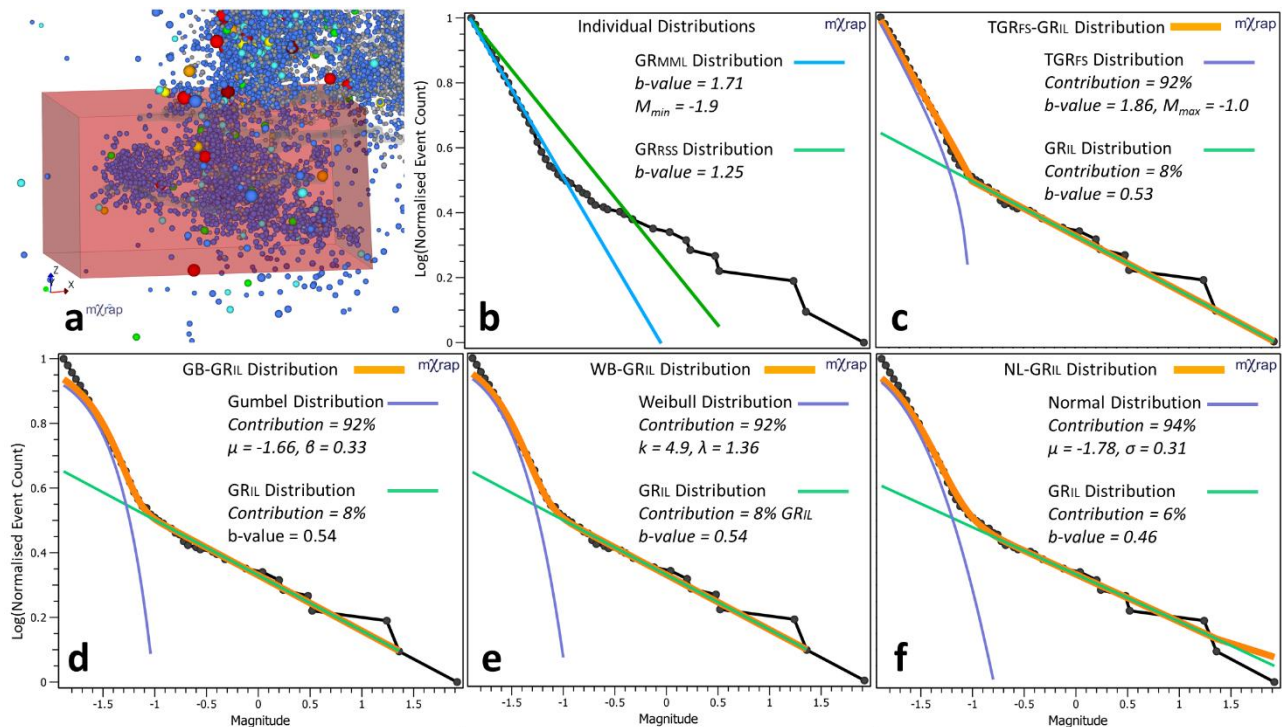


Figure 8 Example of volume-specific analysis at an underground mine in Australia

Table 4 Summary of results from distribution models

Figure 8	Distribution	RSS	$b$ -value	Contribution	$P(M_L > 2.5, 1 \text{ year})$
b	$GR_{RSS}$	0.108	1.25	100%	0.6%
c	TGR–GR	0.011 (best)	0.53	8%	47.9%
d	GB–GR	0.013	0.54	8%	44.7%
e	WB–GR	0.011	0.54	8%	44.7%
f	NL–GR	0.015	0.46	6%	61.1%

### 4.3 Grid-based analysis: bimodal seismicity

This case study considers the same dataset as previously presented in Figure 8. A dense grid was generated for this mine and distribution models were optimised for each grid point by considering all events within a Euclidean search distance of 100 m. The grid spacing ( $5 \times 5$  m) is significantly less than the search distance (100 m) and hence, adjacent grid points have overlapping search radii and similar subsets of seismic events. Grid points with less than 300 events within the search radius were not analysed.

To estimate seismic hazard, the distribution model with the best (lowest) RSS was considered. For this case study, the TGR–GR model had the lowest RSS for 99.5% of all the grid points in the study area. The remaining 0.5% of the grid points in the study area were best modelled with the composite WB–GR distribution while the  $GR_{RSS}$ , GB–GR and NL–GR models did not have the lowest RSS for any of the grid points.

Figure 9 compares the spatial variation in b-value found by the  $GR_{RSS}$  model (a) and the composite model with the lowest RSS (b). There are large errors associated with the  $GR_{RSS}$  model with an average RSS 10 times greater than the RSS of a composite distribution. Like the volumetric case study, these grid points showed strong bimodal frequency–magnitude behaviour. Figure 9(c) shows that the b-value results from using a single distribution (green line) which tend to be less consistent and larger in comparison to the composite distribution results (red line).

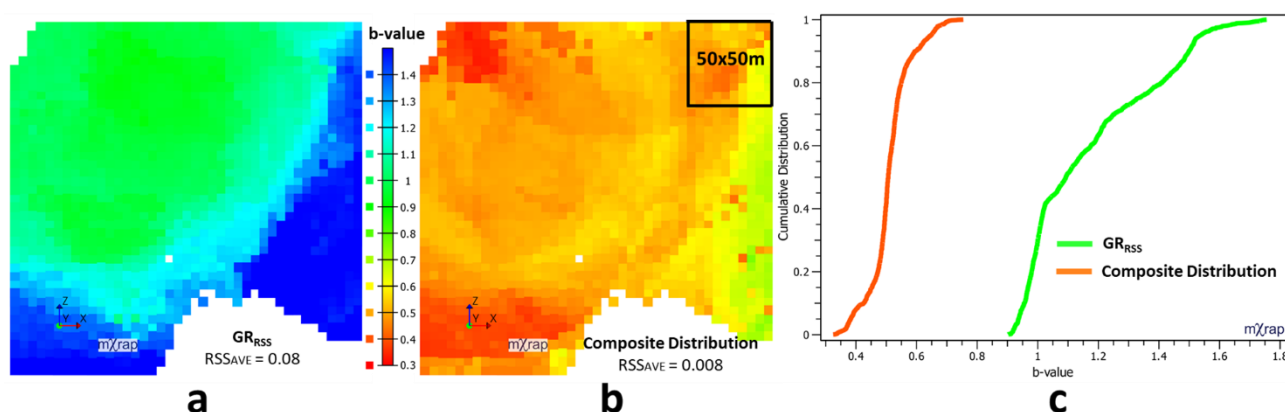


Figure 9 Grid-based analyses for a long section through a mine in Australia. (a) The b-value for the single Gutenberg–Richter distribution; (b) The b-value for the composite distribution with the lowest RSS; (c) Cumulative distribution of b-value within the grid

The corresponding probabilistic seismic hazard is plotted in Figure 10, showing the annualised probability of an event greater than  $M_L$  2.5 within 100 m. The assumption of a single Gutenberg–Richter distribution (Figure 10(a)) results in a relatively low probabilistic hazard in the grid volume ( $< 20\%$ ). In comparison, the probabilistic hazard determined by the composite distribution (Figure 10(c)) indicates a volume of relatively high seismic hazard ( $> 90\%$ ) and an intermediate zone of relatively moderate seismic hazard ( $\approx 50\%$ ). In comparison to the assumption of a single distribution, the composite distribution provides a measure of seismic hazard that is more representative of the frequency of larger seismic events observed, and a clearer spatial understanding of relative seismic hazard levels. This information translates to making more informed and objective seismic hazard management decisions, e.g. quantitative justification for optimising ground support design volumes for higher seismic hazard areas.

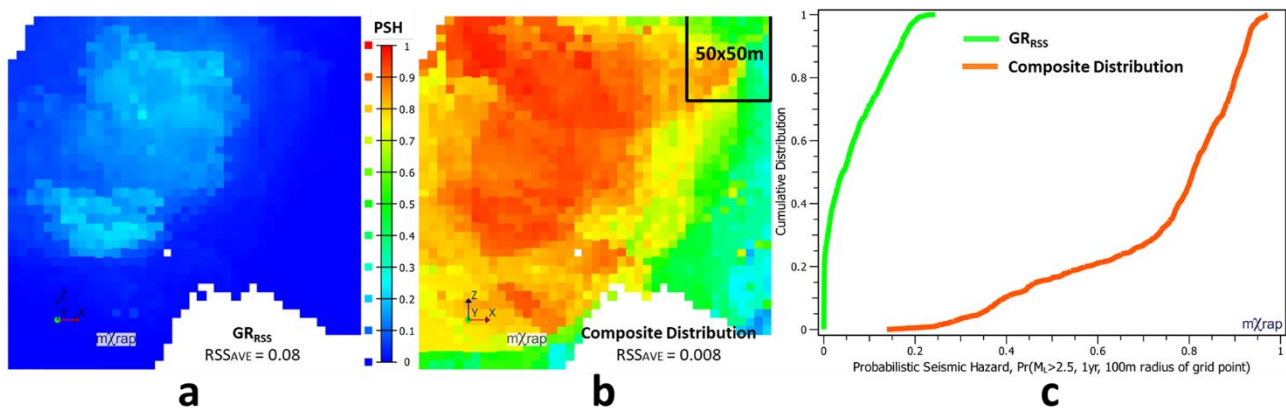


Figure 10 (a) Probabilistic seismic hazard  $P(M_L > 2.5, 1 \text{ year}, 100 \text{ m})$  calculated for the  $GR_{RSS}$  model; (b) Composite distribution with the lowest RSS; (c) Cumulative distribution of probabilistic seismic hazard within the grid

## 5 Areas for further improvements and research

There is a wide scope of further research given the preliminary nature of this work. Further work will likely include the assessment of synthetically generated data to fully investigate the numerical properties and limitations of the composite distribution method.

This further work can be summarised by three main categories:

- Assessing the influence of assumptions and simplifications made prior to the composite fitting procedure:
  - Size of magnitude discretisation.
  - Representativeness of magnitude discretisation.
- Development of the fitting procedure:
  - Investigation of parametric interdependency.
  - Improved constraint of free parameters.
  - Improved computational times.
- Assessment of results following the composite fitting procedure, specifically, the absolute quality of composite fits.

The misidentification of blasting as large real seismic events is common for datasets of mine seismicity. This problem is ideally solved by revisiting the original seismograms associated with suspicious events, although this manual approach is not always possible due to time and practical constraints. There is scope for future research to use a composite distribution model of frequency–magnitude to firstly assess if blast contamination is likely, and secondly to model blast contribution and improve estimates of seismic hazard.

## 6 Conclusions

This investigation has shown that there are some cases where the assumption of a single underlying Gutenberg–Richter distribution is inappropriate and results in a significant underestimation of local seismic hazard. The composite distribution approach results in the improved quantification of bimodal frequency–magnitude relations, and subsequently more realistic probabilistic seismic hazard assessments. This approach can be applied to specific volumes and is sufficiently computationally efficient to allow for routine grid-based spatial assessment.

## Acknowledgement

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