

Draw rate optimisation in block cave production scheduling using mathematical programming

F Nezhadshahmohammad *Urmia University, Iran*

F Khodayari *University of Alberta, Canada*

Y Pourrahimian *University of Alberta, Canada*

Abstract

Among the underground mining methods available, caving methods are favoured because of their low cost and high production rates. Block caving operations offer a much smaller environmental footprint compared to equivalent open pit operations due to the much smaller volume of waste to be moved and handled. In general, draw control is fundamental to success or failure of any block cave operation. Establishing relationships among draw columns to consider depletion rates of other draw columns is complex but essential to provide a reasonable solution for real block caving mines.

This paper presents a mixed-integer linear programming (MILP) model to optimise the extraction sequence of drawpoints over multiple time horizons of block cave mines with respect to draw control systems. A mathematical draw rate strategy is formulated in this paper to guarantee exact solutions. Draw control management provides optimal operating strategies while meeting practical, technical and environmental constraints. Furthermore, dilution and caving are improved indirectly, because the method considers the draw rate strategy according to geotechnical properties of the rock mass. Surface displacements are controlled by using the draw rate in all drawpoints during the life of the mine. Application and verification of the presented model for production scheduling based on the draw control system are presented using a case study.

Keywords: *block caving, draw rate, optimisation, mixed-integer linear programming*

1 Introduction

Block caving is generally a large-scale production technique applicable to low-grade, massive orebodies and the least expensive of all underground mining methods, and can in some cases compete with open pit mining in cost and production rate. It is a technique which relies on natural processes for its success, therefore, gravity is used in conjunction with internal rock stresses to fracture and break the rock mass into pieces that can be handled by miners. This method requires more detailed geotechnical investigation of the orebody than other methods in which conventional drilling and blasting are employed as part of the mine production (Rubio 2006). Draw control in caving operations involves a combination of scheduling and geomechanics (Smith & Rahal 2001).

Draw column and drawpoint are two basic components of planning in a block cave. Depletion control and implementation of an effective draw strategy, once production commences from drawpoints, needs a strict schedule planning system. The introduction of a draw control system based on mathematical programming that integrates constraints from other disciplines such as geology, mining and metallurgy into the system will become more acceptable as real business planning tools. Production scheduling in block caving is generally referred to as draw control. The objectives of draw control are normally separated into short and long-term scheduling (Diering 2004a).

Unbalanced cave subsidence by poor ground control over time, decreased recovery and productivity, premature waste ingress and recompaction of broken material in the draw columns are issues faced in any block caving operation if draw from the drawpoints is not controlled. Also, the lack of effective draw strategy results in infrastructure instability, more dilution, flow of muck at the drawpoints, and safety and financial concerns.

Caveability, in the context of draw control, is primarily concerned with balancing caving rates and production. If draw rates are not controlled, either air gaps or damaging stress concentrations may occur. Stress is important because undercut advance rates must be maintained to prevent stress damage to the production level. Draw must also be maintained across the production level to ensure that local stress concentrations and premature dilution entry do not occur (Rahal 2008).

The use of draw control to mitigate stress damage on the extraction level can be considered as one of the most important aspects of draw management. It is generally accepted that underdraw and overdraw behaviour leads to early dilution entry, excessive induced stresses, and loss of planning abilities (Heslop & Laubscher 1981). Consequently, a production planning program that does not incorporate the geotechnical properties of the rock mass within the block cave mining method will not be used for general purposes of production schedules. Careful control of the production in the long-term planning of a block cave mine will ensure that the schedule-drawing process within the cave moderates unwanted problems and preserves mining economics associated with production targets. Constraints must be appropriate with the mining method, objective function and real geotechnical condition of the rock mass. Maximising tonnage or mining reserves will not necessarily lead to maximum net present value (NPV) without aggregation reserve and grade constraint to time dynamic behaviour of the fundamental models in linking the mine planning parameters and draw rate curves.

In this paper, a mathematical formulation is developed to optimise the draw rate as a critical parameter of a block cave operation to maximise economic goals of companies by regarding the geotechnical production rate curves and allowable activity life of any drawpoints to model depletion rates. This paper considers the draw control system as an operational goal with respect to strategic decisions to achieve an exact solution through an operations research technique. Several mathematical methods are currently used to generate planning scheduling in block cave mines without considering a given draw control system. Although following the production rate curve (PRC) in mathematical formulation makes the problem more complex and increases the size of the problem, it is a more realistic method.

2 Literature review

Mining optimisation models presented in the literature have been developed for block cave mines, but they solve only a restricted type of planning problem. The literature on draw rate models based on the PRC for block caving operations is relatively new. Most of the models use simulation in consideration of PRC to evaluate production schedules. There is no clear example of block caving production scheduling using a mathematical approach in the literature that formulates draw control system in the block cave according to PRC. Most of the studies consider only the upper and lower boundary for draw rate in their modelling. Khodayari and Pourrahimian (2015b) presented a comprehensive review of operations research in block caving. They summarised several authors' attempts to use different methods to develop methodologies for optimising production scheduling in block caving operations.

Riddle (1976) and Song (1989) described a primary structure of planning in block caves. They did not give any indication of drawing constraint in their work. Chanda (1990) concentrated on a mixed-integer programming (MIP) short-term planning problem that covers a time horizon of a few weeks to a few months, applying single step optimisation rather than multi-period optimisation. Lack of draw control planning constraint in the Chanda model (Chanda 1990) had upsets in variability and operational problems.

Rubio and Diering (2004) noted that Chanda (1990) has not recognised the fact that the set of constraints is a function of the planning horizon under study. Guest et al. (2000) assumed that by following a set of surfaces that conceptually define a draw control strategy, dilution can be minimised and therefore the NPV can be maximised. Guest did not apply a PRC in the presented model and stated the importance of the draw strategy on dilution control as part of the production scheduling process. Smith and Rahal (2001) noted that none of the available scheduling methodologies fully address complications associated with caving, in particular, ore mixing and frequent loss of drawpoints.

Rahal et al. (2003) and Rahal (2008) described mixed-integer linear goal programming (MILGP) models. The models had the dual objectives of minimising deviation from the ideal draw profile and achieving the production target. This algorithm assumes that the optimal draw strategy is known. The authors developed life-of-mine draw profiles for notional scenarios and showed that by using the results from their integer program, they greatly reduced deviation from ideal drawpoint depletion rates while adhering to a production target. They consider constraints of capacity, precedence, material handling, and maximum and minimum levels of draw rates. They emphasised that the major outcomes from their research were a preliminary optimised life-of-mine production plan and the identification of areas where additional work can refine the parameters used in the optimisation.

Diering (2004a, 2004b) presented a non-linear programming optimisation method to maximise NPV and minimise the deviation between a current draw profile and the target defined by the mine planner. Diering emphasises that this algorithm could also be used to link the short-term with the long-term plan. The long-term plan is represented by a set of surfaces used as a target to be achieved based on the current extraction profile when running the short-term plans. Diering used only two boundaries for the draw rate constraint in the mathematical method. Weintraub et al. (2008) presented a MIP model to maximise the profit of El Teniente and just focused on size reduction methods, and they had no reference to issues of draw control rules.

Queyranne et al. (2008) presented a MIP model for block caving that maximises the NPV and uses the capacity constraints of mine production, maximum opened and active drawpoints, and neighbouring drawpoints. The model presents a good method for the optimal solution, but it does not consider the geotechnical and other significance constraints. Smoljanovic et al. (2011) presented an MILP model to optimise the NPV value in a panel cave mine to study the drawpoints' opening sequence. The emphasis is in the precedence, geometrical, and production constraints. He did not consider PRC for draw rate constraint. Parkinson (2012) developed three integer programming (IP) models for finding the optimal opening sequence in an automated manner. All of the models share three basic constraints. The start-once constraint ensures that each drawpoint is opened once and only once. The global capacity constraint ensures that the number of active drawpoints does not exceed the downstream processing capacity. The big disadvantage of this research was the lack of draw rate constraint. Parkinson assumed a constant draw rate for the life of the mine. Epstein et al. (2012) presented a methodology for long-term production plans considering underground and open pit deposits sharing multiple downstream processing plants. The method was successfully used in Chilean copper mines by Codelco for both underground and open pit extraction. As in Parkinson, the extraction rate of the Epstein method had a constant value in the life of the mine.

Pourrahimian et al. (2012, 2013, 2014) made other applications of MILP to develop a practical optimisation framework for caving production scheduling by maximising NPV. Pourrahimian attempted to find an optimal schedule for the life of the entire mine, solving simultaneously for all periods by considering all required constraints, but he did not consider geotechnical properties of rock mass through the draw rate constraint. Pourrahimian (2014) mentioned that the formulation tries to extract material from drawpoints with a draw rate within the acceptable range without considering a specific shape. Alonso-Ayuso et al. (2014) considered a planning MIP medium range problem for the El Teniente mine in Chile to maximise NPV by introducing explicitly the issue of uncertainty. They used only some operational constraint without taking into account the draw control mechanism.

Literature shows that some of the models did not incorporate, on a routine basis, operational performance to adjust the medium and the long-term plans because of loss of geotechnical rules in the modelling of actual draw management systems. Determining the optimum configuration of block caving operation was the first goal of all mentioned studies.

During the production, the only control is through the drawpoints. The rate at which material can be drawn from an individual drawpoint depends on several rock mass and design parameters such as equipment size, layout configurations, and stress transfer on the extraction level, haulage infrastructure, and seismic activity (Rubio & Diering 2004). Considering a series of simple definitions of the process of moving and

drawing caved material from drawpoints is not good planning for caving operations. The researchers have modelled the draw rate constraints regardless of the production rate curves and only by defining lower and upper bounds. Deciding minimum and maximum draw rates of drawpoints is appropriate according to principles of geotechnical rules in all previous studies. However, during the entire life of any drawpoints, draw rate varies and the draw control system is a critical key in accurate designing and operation of block cave mines.

A consequence of disregarding the effect of PRC on the extraction of a drawpoint is shown in Figure 1. It can be seen that the draw rate from a drawpoint is in the defined range, but this kind of extraction is not practical. The reason for this dissimilar form of draw rate is related to the aim of the models to solve problems with the fastest and the most convenient method. However, the lack of consideration given to the draw rate curve according to geotechnical rules, in addition to abuse of the objective function and non-optimised scheduling, causes a problematic caving progress.

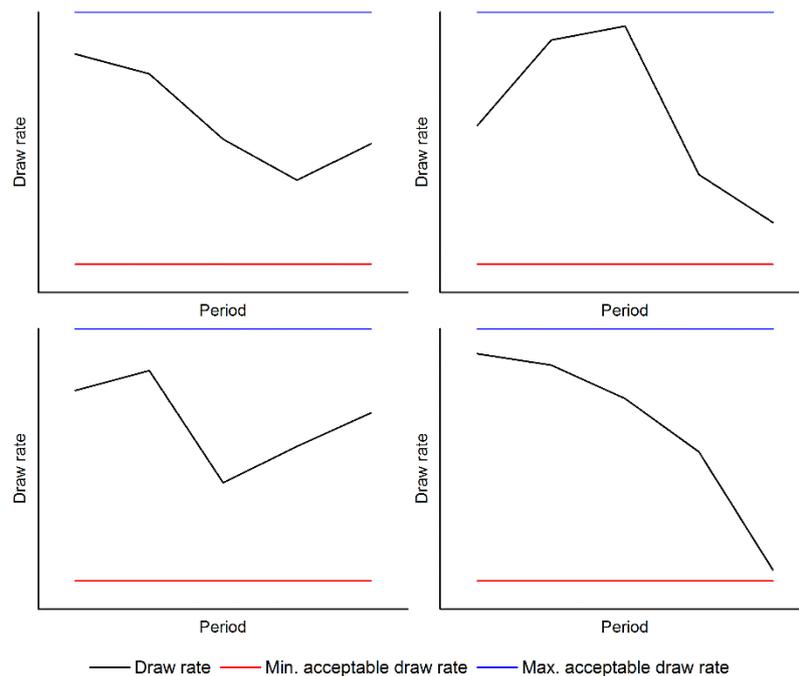


Figure 1 Draw rate variation without considering the production rate curve

Figure 2 shows a schematic view of a production rate curve. This profile is usually provided by the geotechnical team to consider many factors such as the engineering and geotechnical properties of the rock mass and safety issues in extraction. For instance, the draw rates established for Cadia East during the cave initiation (up to 30% of the block height) vary from 115 to 280 mm/day with an average of 190 mm/day. Higher than 30% to the top of the block, the draw rates vary from 280 to 400 mm/day with an average of 320 mm/day (Flores 2014).

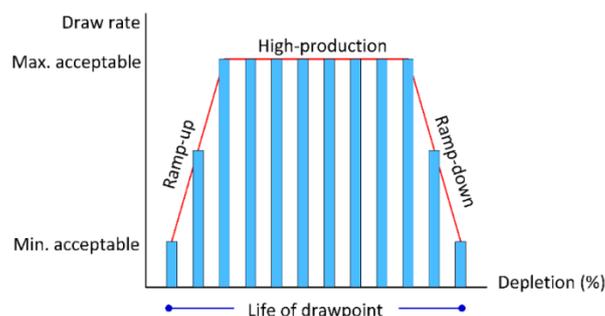


Figure 2 Production rate curve (ramp-up, high-production, ramp-down)

3 Problem definition

Mixed-integer linear programming (MILP) formulation presented in this paper maximises the NPV. The presented model is coded in MATLAB (R2016a) (The MathWorks, Inc. 2016), and CPLEX/IBM (IBM ILOG 2015) is used as the optimisation engine. Presented draw rate algorithms allow the use of the MILP formulation for planning large and more complex real block cave mines. Solving any mathematical model requires some decision variables, sets, indices, and parameters that correspond to a scheduling program. The following items are introduced according to the current model (Table 1).

Table 1 Decision variables, set, indices, and parameters of the MILP model

Indices	
$d \in \{1, \dots, D\}$	Index for drawpoints.
$t \in \{1, \dots, T\}$	Index for scheduling periods.
l	Index for a drawpoint belonging to set S^d .
Set	
S^d	For each drawpoint, d , there is a set S^d defining the predecessor drawpoints that must be started prior to extraction of drawpoint d .
Decision variables	
$U_{d,t} \in [0, 1]$	Continuous decision variable, representing the portion of draw column d to be extracted in period t .
$A_{d,t} \in \{0, 1\}$	Binary decision variable equal to 1 if drawpoint d is active in period t ; otherwise it is 0.
$Z_{d,t} \in \{0, 1\}$	Binary decision variable controlling the precedence of extraction of drawpoints. It is equal to 1 if extraction from drawpoint d is started in period t ; otherwise, it is 0.
Parameters	
D	Maximum number of drawpoints in the model.
$EVDC_d$	Economic value of the draw column associated with drawpoint d .
$DR_{l,d,t}$	Minimum possible draw rate of drawpoint d in period t .
$DR_{u,d,t}$	Maximum possible draw rate of drawpoint d in period t .
$N_{Ad,t}$	Maximum allowable number of active drawpoints in period t .
$N_{Nl,d,t}$	Lower limit for the number of new drawpoints, the extraction from which can start in period t .
$N_{Nu,d,t}$	Upper limit for the number of new drawpoints, the extraction from which can start in period t .
Ton_d	Total tonnage of material within the draw column associated with drawpoint d .
k	Small number equal to fraction of minimum draw rate on $\max\{Ton_d\}$.
L	Large number equal to fraction of $\max\{Ton_d\}$ on minimum draw rate.
i	Discount rate.
M_u	Upper limit of mining capacity in period t .
M_l	Lower limit of mining capacity in period t .
$Max_{Activity}$	Maximum allowable periods that any drawpoints can be active.
$G_{u,d,t}$	Upper limit of the acceptable average head grade of drawpoint d in period t .
$G_{l,d,t}$	Lower limit of the acceptable average head grade of drawpoint d in period t .
$\bar{G}_{d,t}$	Average grade of drawpoint d .

3.1 Objective function

The objective function of the presented model is to maximise the overall discounted profit, including the cost of the mining operation. The profit from mining a drawpoint depends on the economic value of the draw column and the costs incurred in mining. The maximisation of NPV is closely associated with maximising ore tonnes, as the ore tonnes generate revenue. The objective function, Equation 1, is composed of the economic value of the draw column and a continuous decision variable $U_{d,t}$ that indicates the portion of a draw column, which is extracted in each period. The most profitable draw columns will be chosen as part of the production to optimise the NPV.

$$\text{Maximize } \sum_{t=1}^T \left[\frac{EVDC_d}{(1+i)^t} \right] \times U_{d,t} \quad (1)$$

3.2 Constraints

Obtaining the best solution using an operations research technique forces the mine planner to limit objective function by some constraints, which appear in several different forms: geotechnical, grades, period, advancement direction, and priority of productive units, productivity, production rates, and many others that depend on the mining method. Knowledge, experiments of mine planners, and corresponding planning horizons have a critical role in the process of assigning constraints to the optimisation problems. The following constraints are part of the problem in deriving the formulation:

Mining capacity

$$M_l \leq \sum_{d=1}^D (\text{Ton}_d) \times U_{d,t} \leq M_u \quad (2)$$

Production grade

$$\sum_{d=1}^D (\text{Ton}_d \times (G_{l,d,t} - \bar{G}_{d,t})) \times U_{d,t} \leq 0 \quad (3)$$

$$\sum_{d=1}^D (\text{Ton}_d \times (\bar{G}_{d,t} - G_{u,d,t})) \times U_{d,t} \leq 0 \quad (4)$$

Maximum number of active drawpoints

$$A_{d,t} \leq LU_{d,t} \quad (5)$$

$$U_{d,t} \leq A_{d,t} \quad (6)$$

$$\sum_{d=1}^D A_{d,t} \leq N_{Ad,t} \quad (7)$$

Precedence of drawpoints

$$Z_{d,t} - \sum_{j=1}^t Z_{l,j} \leq 0 \quad (8)$$

Continuous extraction

$$\sum_{t=1}^T Z_{d,t} = 1 \quad (9)$$

$$A_{d,t} - A_{d,(t-1)} \leq Z_{d,t} \quad (10)$$

$$A_{d,1} = Z_{d,1} \quad (11)$$

Number of new drawpoints

$$N_{Ni,d,t} \leq \sum_{d=1}^D Z_{d,t} \leq N_{Nu,d,t} \quad (12)$$

$$\sum_{d=1}^D Z_{d,1} \leq N_{Ad,1} \quad (13)$$

Reserve

$$\sum_{t=1}^T U_{d,t} \leq 1 \quad (14)$$

Equation 2 ensures that the total tonnage of material extracted from drawpoints in each period is within the acceptable range that allows flexibility for potential operational variations. The constraint is controlled by the continuous variable $U_{d,t}$. There is one constraint per period. Equations 3 and 4 force the mining system to achieve the desired grade. The average grade of the element of interest has to be within the acceptable range and between the certain values.

According to Equations 5, 6 and 7 in each period, the number of active drawpoints must not exceed the allowable number and has to be constrained according to the size of the orebody, available infrastructure, and equipment availability. A large number of active drawpoints might lead to serious operational problems. According to the determined advancement direction, for each drawpoint d there is a set S^d , which defines the predecessor drawpoints among adjacent drawpoints that must be started before drawpoint d is extracted. To control the precedence of extraction, a binary decision variable $Z_{d,t}$ is employed in Equation 8.

The constraint introduced by Equations 9 and 10 forces the mining system to extract material from drawpoints continuously after opening until closing. Equation 11 is only used for period one. Based on the footprint geometry, the geotechnical behaviour of the rock mass, and the existing infrastructure of the mine, the maximum feasible number of new drawpoints to be opened at any given time within the scheduled horizon must be defined on the basis of Equations 12 and 13. Equation 14 ensures that the sum of fractions of the draw column that are extracted over the scheduling periods in maximum value is one, which means there is selective mining, and hence all the material in the draw column may not be extracted.

The draw rate needs to be fast enough to avoid recompaction and slow enough to avoid air gaps and dilution. The activity period of the drawpoint, in the context of draw control, is mainly concerned with the assessment of draw rates to adjust extraction tonnage of any drawpoint and prevent any recompaction or dilution in the activity life of drawpoints. This constraint causes the maximum activity life of any drawpoint to be limited to a deterministic value, so the draw rate of drawpoint must be large enough to maximise the NPV yet must be small enough to prevent over dilution.

3.2.1 A comprehensive draw control constraint

Increasing the depletion volume from drawpoints is the ideal demand of all mining companies. But the geotechnical limitation of the rock mass does not allow mines to draw material with an extreme velocity. Consequently, according to the market assessment and many limitations related to the extraction of mines, companies have different draw strategies to maximise profit, safety and minimise loss of time, financial cost, and fatalities in the mine. Depletion rates are specified as tonnes per square metre per day per drawpoint. This calculation is often linked to a production rate measured in metres per day.

It is important to ensure that draw management system is in place before production begins, to prevent resource loss due to production pressures during cave initiation (Preece & Liebenberg 2007).

To establish a comprehensive draw control system, it is essential to control the depletion velocity of broken materials from the drawpoints. For this purpose, the combination of some controlling constraints is used to manage draw rate response to geotechnical problems.

3.2.1.1 Draw rate constraint

Selecting the production rate is one of the primary mine design variables required early in the process capacity (Charles et al. 2011). The production rate is directly calculated from the draw rates of drawpoints in the activity life of any drawpoints. The changes in draw rate are normally classified as a drawpoint opening to ramp-up, high-production, and ramp-down to drawpoint closure. In this research, the PRC in a general form is to be modelled and practiced by all mines according to their draw requirements. This general form includes (i) ramp-up, (ii) steady, and (iii) ramp-down.

In the case of mines that consider geotechnical and economic issues simultaneously, use of general form is proposed. The general form is best because the low draw rate in the early years of the life of drawpoints is caused by the geotechnical characteristics of the surrounding environment aligned with the caved material, and in fact, increasing stress caused by draw and caving process relaxes with a relatively soft tendency. Therefore appropriate management of stress relaxation can be done. Mines which exclusively persist on geotechnical rules must use a minimum draw rate in the first years of extraction from drawpoints.

Development of an overlap and disjunctive (OD) system for regulating drawpoint production begins by breaking the production profile into a number of regions where each has a binary indicator variable. When the binary variable for each level takes a value of 0, the draw constraint for that level is relaxed; otherwise, that region binds production. Each region is, in fact, a new sub-constraints. Any of these regions can be available for draw, but their activity returns to a depletion percentage.

In the general form (see Figure 3), there are three regions and it will have three new sub-constraints. For example, the second region is active if the depletion is between M and F, and thus its binary variable, $A_{d,t}$, takes 1 and other regions' binary variables take 0. The extraction begins from drawpoint d with a minimum acceptable draw rate in the starting period. Then the draw rate increases gradually (blue) to reach the maximum acceptable draw rate. In the steady region (green), the draw rate follows the maximum acceptable draw rate; therefore the draw rate has a zero slope. By extraction of material from drawpoints and reduction of the amount of remaining materials in the draw columns and reaching to depletion F (%), the draw rate must be reduced with the defined slope. This is the ramp-down region (red). The number of periods in each region depends on M and F values and is obtained as a result of optimisation. Equations 15, 16 and 17 are related to the ramp-up, steady, and ramp-down regions, respectively.

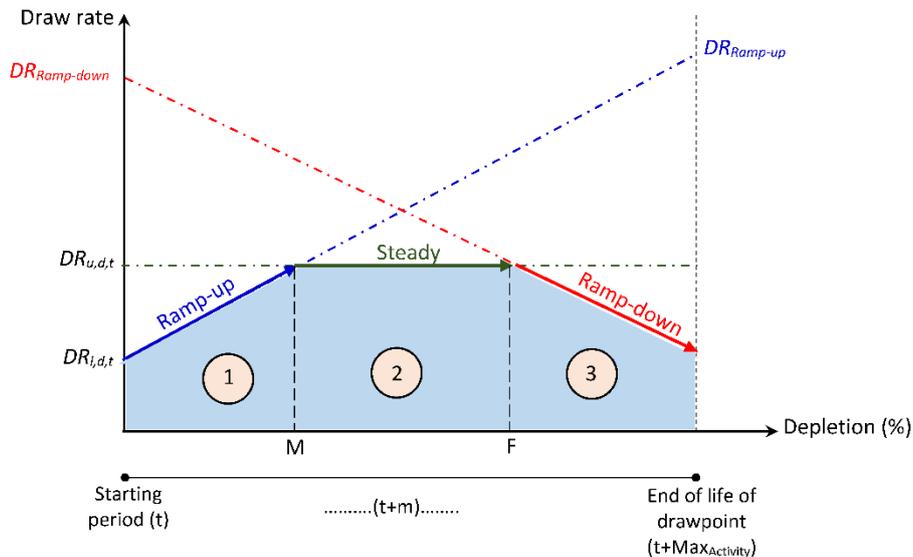


Figure 3 Ramp-up, steady, and ramp-down regions

$$U_{d,(t+m)} \times Ton_d - \left(\frac{DR_{Ramp-up} - DR_{l,d,t}}{1 - \frac{DR_{l,d,t}}{Ton_d}} \times \sum_{t=1}^{t+m} U_{d,(t+m)} \right) \leq DR_{l,d,t} - \left(\frac{DR_{Ramp-up} - DR_{l,d,t}}{1 - \frac{DR_{l,d,t}}{Ton_d}} \times \frac{DR_{l,d,t}}{Ton_d} \right) \quad (15)$$

$$U_{d,t} \times Ton_d \leq DR_{u,d,t} \quad (16)$$

$$U_{d,(t+m)} \times Ton_d - \left(\frac{DR_{Ramp-down} - DR_{u,d,t}}{\frac{DR_{l,d,t}}{Ton_d} - F} \times \sum_{t=1}^{t+m} U_{d,(t+m)} \right) \leq DR_{Ramp-down} - \left(\frac{DR_{Ramp-down} - DR_{u,d,t}}{\frac{DR_{l,d,t}}{Ton_d} - F} \times \frac{DR_{l,d,t}}{Ton_d} \right) \quad (17)$$

Determining M and F is a specific effort and depends on the skill of the geotechnical group of any mines. The number of periods in the steady region decreases by increasing the value of M and decreasing the value of F. If the value of M is too small, it is obvious in the initial period of extraction with rapid depletion of material that hints of the separation of the huge broken rock from the cave back and, consequently, other phenomena such as air blast and closure of drawpoint are inevitable. On the other hand, if the value of F is too large, rapid rates in the last periods lead to inappropriate control of grade and mixing and reduction in recovery. Therefore, it is clear that the values of M and F have a critical role in the draw control system. Adopting a respectable depletion for these values is crucial to avoid many undesirable problems.

3.2.1.2 Maximum activity period

Uneven draw within an active production area creates zones of low or underdraw which allow compaction to occur within the fragmented ore overlying the production level (Rubio 2006). Draw rate will define the capacity of the drawpoint and it needs to be fast enough to avoid compaction and slow enough to avoid air gaps. Equation 18 indirectly affects the draw rate by controlling the number of activity periods of any drawpoint. Increasing the drawpoint activity life increases the probability of recompaction and dilution.

$$\sum_{t=1}^T A_{d,t} \leq Max_{Activity} \quad (18)$$

3.2.1.3 Minimum draw rate

In each period that drawpoint depletion is started, the first step of the draw rate must be minimum. Equation 19 forces models to start depletion with a minimum acceptable draw rate.

$$DR_{l,d,t} \times Z_{d,t} - Ton_d \times U_{d,t} \leq 0 \quad (19)$$

4 Case study

The considered dataset contains 298 drawpoints. The total tonnage of material is 36.7 M tonne with an average grade of 1.12% Cu. Figure 4 illustrates the drawpoints' configuration and the grade and tonnage distribution of the draw columns. The performance of the proposed MILP models is analysed based on the maximising NPV at a discount rate of 12%. The draw control system, by enrolling an exact production rate curve, seeks to optimise and present a practical block cave planning. This model assures that all constraints are satisfied during the mine life.

The model was tested on a Dell Precision T7600 computer with Intel® Xeon® at 2.3 GHz, with 64 GB of RAM. The scheduling parameters have been summarised in Table 2. The model was verified over 15 periods in the south to north (SN) advancement directions. This advancement direction was selected using the methodology presented by Khodayari and Pourrahimian (2015a). Table 3 summarises the achieved numerical results.

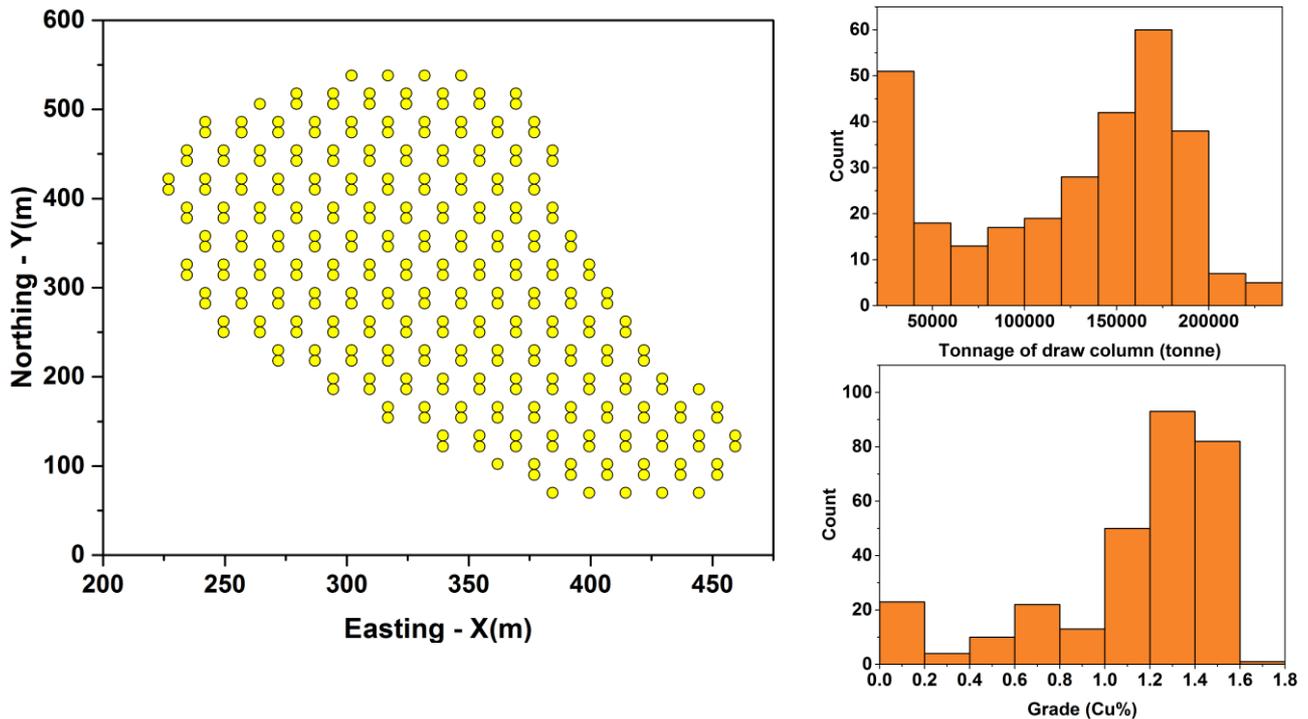


Figure 4 Drawpoints' configuration (circles represent drawpoints), grade and tonnage distribution of draw columns

Table 2 Scheduling parameters

Parameters	Value
Maximum mining capacity (tonne/period)	2,450,000
Number of periods	15
Max. number of activity period of drawpoint (period)	5
Draw rate (tonne/period)	Min: 11,000 Max: 40,000 M (%): 40 F (%): 90
Maximum number of active drawpoints per period	Period 1: 100 Periods 2–15: 75
Number of new drawpoints per period	Min: 5 Max: 35
Grade (%Cu)	Min: 1.0 Max: 1.5

Table 3 Numerical results

CPU time	Total extraction (Mt)	NPV (M USD)	Constraint	Binary variation
01:25:29	30.1	47	13,410	8,940

Figure 5 shows the average grade of production and the production tonnage in each period. It can be seen that the model attempts to deplete the drawpoints with a higher grade earlier. The average grade of production has a descending trend during the mine life. During the later periods, because of reaching the top of the draw column and of the probability of dilution, the average grade of production is less than in previous years. But it must also be noted that dilution should be reduced, because of the controlling feature of the production rate curve in the draw control system. In period 1, because of the minimum draw rate and total allowable active drawpoints, production is less than the maximum mining capacity. After period 8, production decreases gradually.

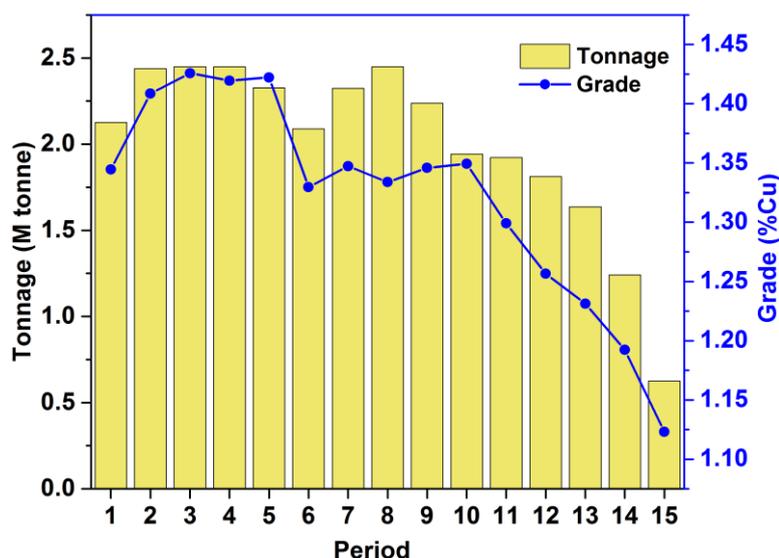


Figure 5 Average grade and production tonnage in each period

Figure 6 shows the number of active and new drawpoints in each period. The number of active drawpoints in period 1 is higher than in other periods because drawpoints are depleted with minimum draw rate in the first period. Afterwards, during the next 10 years, the mine works with maximum allowable active drawpoints. From period 12 to period 15, the number of active drawpoints reduces gradually.

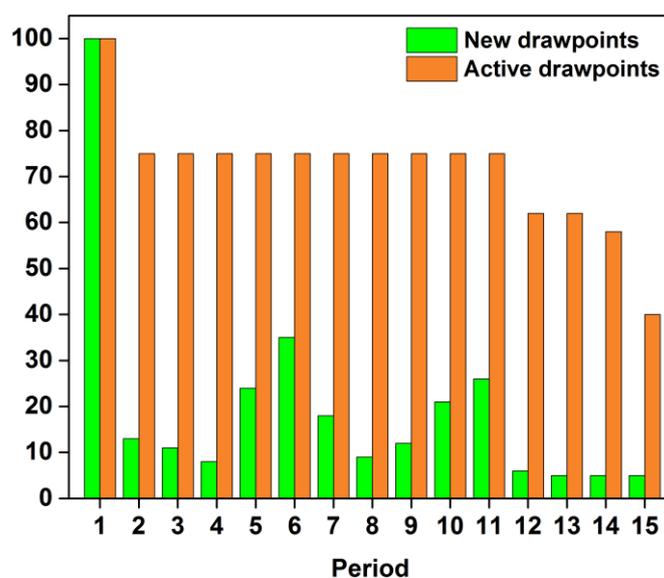


Figure 6 Number of active and new drawpoints in each period

According to the defined draw rate strategy, drawpoints cannot be depleted arbitrarily, but it is possible because of the objective function and the constraints, the material with lower economic value remains in the number of drawpoints. Figure 7 illustrates the draw rate changes for different drawpoints in the SN

direction. It is clear that the defined lower bound and the PRC for selected drawpoints are satisfied. For instance, extraction from DP 278 is started in period 1 with the minimum acceptable draw rate (11,000 tonne), then it increases gradually to reach the maximum acceptable draw rate (40 k tonne) in period 3. After a steady extraction with the maximum draw rate for two periods, the draw rate drops. It can be seen that the tonnage of extraction from drawpoints varies, based on the drawpoints' economic values because the objective function maximises the NPV and the tonnage of extraction from each draw column is a result of optimisation.

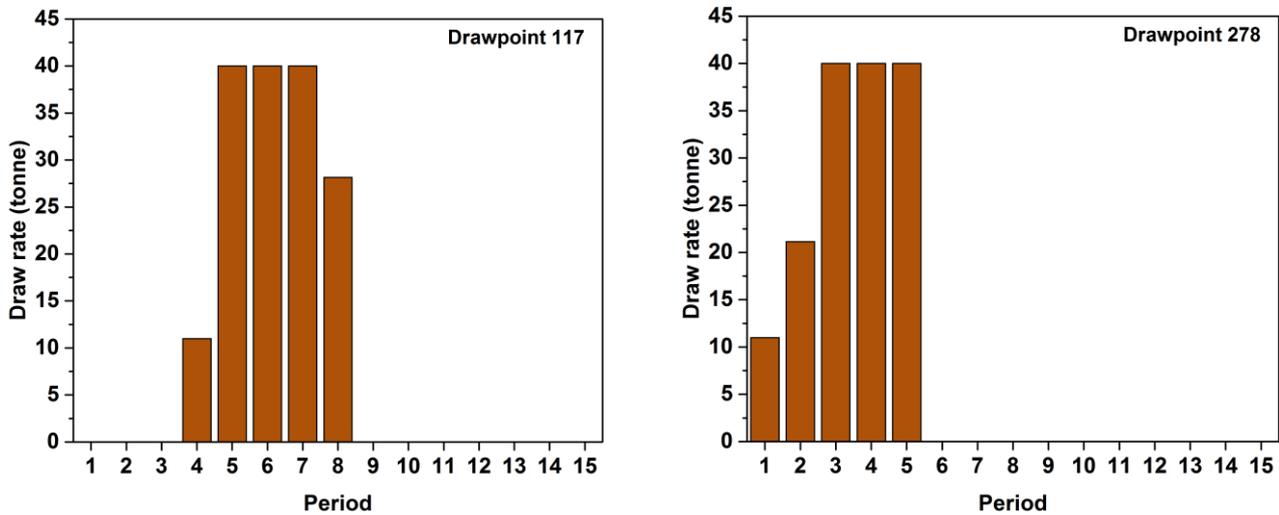


Figure 7 Obtained draw rate as result of optimisation for different drawpoints

One of the advantages of the draw control system is controlling the surface displacement by using the draw rate in all drawpoints during the mine life. The results show that all defined constraints have been satisfied. The draw rate amount for each drawpoint and starting and finishing periods are obtained as a result of optimisation. The model extracts the material from each draw column based on the defined draw rate model while maximising the NPV of the operation.

5 Conclusion

The objective of this study was to develop, implement, and verify a realistic optimisation MILP framework for block cave long-term production scheduling, whereby a mineral is extracted and prepared at a desired market specification, with the maximum economic return measured by NPV, and within acceptable operational constraints with respect to geotechnical rules and draw control management. This paper presented an exact draw controlling system for block cave production scheduling optimisation.

The comprehensive PRC model causes, to a defined draw rate strategy, a depletion of drawpoints based on the geotechnical properties of the rock mass. Time dynamic behaviour of the model forced a depletion of drawpoints with a minimum draw rate in the starting period. Two allowable depletion ranges were assumed in the model to define steady and ramp-down regions after ramp-up region. Results showed that draw rate completely follows the given PRC, therefore draw controlling system can overcome sudden surface displacement and reduce dilution over the life-of-mine.

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