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Using the single index model to create a short-term mine plan

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Abstract

How to effectively plan, sequence and schedule for extraction the areas within an underground mining operation is a financial reward-maximising issue for the mining community. The mining industry is one that has a heightened level of uncertainty due to the fact that the product is not known with certainty until after it has been produced. As a result, decisions on how to allocate the limited resources that are available in both the long-term and short-term must be made with less than perfect information. If the underground mine is divided into distinct areas, and each area is scheduled independently as a discrete, standalone area, the characteristics of the individual areas can be likened to those of a financial asset, having an expected return over a period of time, as well as risk associated with that return based on the uncertainty surrounding the underlying factors that contribute to its value.

There are limited theoretical tools that allow mine planning professionals to confidently plan their extraction sequences in a manner that minimises the risk of stochastic changes in industry forces. As a result, this study proposes the use of the single index model (SIM) as part of the mine scheduling process, providing a tool to assist decision-makers in selecting optimum mine areas for inclusion in the short-term plan so that proper mining sequences can be established with the most effective allocation of resources. This process is similar to that of an investor considering a number of alternative assets for investment, and using the SIM to identify the assets to be included in their portfolio.

Keywords: *mine planning, single index model, portfolio optimisation, decision support*

1 Introduction

How to effectively plan the sequence of extraction within a mine is a financial reward-maximising issue for the mining community. The information required for mine planning decisions goes beyond the external sources of uncertainty that are recognised by the typical valuation techniques used in the mining industry, to include technical factors (e.g. mine development layout) and the ability of a mineral extraction project to achieve production levels. Unfortunately, there are limited theoretical tools that assist mine planning professionals with the creation of extraction sequences that minimise risk due to stochastic changes in industry forces, such as grade changes and price movements.

While advancements in the valuation techniques used in the mining industry do offer planners more information and some guidance, with respect to how different options will react to the different ways that the uncertainties can resolve, they normally require an iterative approach to evaluate many discrete alternatives, and do not offer guidance on how limited resources (financial and human capital) can be invested/used to minimise cash flow risk in the mining operation. Considering the additional sources of uncertainty from scheduling mine sequences, together with external factors, will provide mine planners with better methodologies and tools to evaluate and maximise the value of a project. This paper borrows theories from the portfolio management sector to build a strategic mine extraction plan in the situation of limited available capital. Specifically, the single index model (SIM) is used early in the mine scheduling process to provide a tool for selecting optimum mine areas based on their risk–return characteristics. This allows mining sequences to be established with the most effective allocation of investment resources.

2 Background

Valuing projects in the mining industry is a challenging task. As noted by Snowden et al. (2002) the mining industry differs from most other industries in that the true characteristics of the product are never known exactly, and are thus based to a large extent on estimates derived from sample data. Also, the profitability of mining firms is controlled to a large extent by world commodity prices that are partially stochastic in nature and variable over time. The result is an industry where decisions must be made under situations having increased levels of uncertainty.

The valuation methodologies employed by the mining industry have undergone numerous changes due to the specific nature of the characteristics that affect individual projects. Like many other industries, the mining industry faces numerous sources of uncertainty resulting from such drivers as the market price of the commodity being extracted to the exploration project that was undertaken to delineate the orebody. However, unlike many other industries, it also faces the challenge of individuality amongst the projects being evaluated. Lee and Strang (2003) note that the most widely accepted and utilised valuation technique in the mining industry is discounted cash flows (DCF), which has been the standard evaluation technique in the industry for decades, used as both a method to place a value on a project, as well as a tool for managers and decision-makers to use as a guide in choosing among different alternatives.

Due to the inherent inflexibility of DCF many practitioners have attempted, and been successful in applying more advanced valuation techniques to mining projects that recognise additional levels of risk. For example, Samis and Poulin (1998) used decision trees in combination with DCF to incorporate the heterogeneous nature of a mineral deposit into a valuation model. Kazakidis (2001) used multiple European put and call barrier options to value flexible operating alternatives within an underground mine that are associated with ground-related problems. In this work, Kazakidis (2001) used a Monte Carlo simulation to consider two alternative mining sequences during the planning for the extension of a mine, and found that the inclusion of flexibility into the mine plan can alter the decision. Rodriguez and Padua (2005) applied portfolio optimisation techniques to exploration and production projects in the petroleum industry, with the aim of maximising the worth of the company. All of these applications showed that a better understanding and evaluation of a mining project can be obtained through the incorporation of more advanced valuation techniques.

The ability to meet production levels is also a crucial element of mining operations, and a crucial element of the mine planning process. The effects can be as influential on the value created by a project as the external sources of uncertainty that are recognised through the discounting of cash flows. Maybee et al. (2009) noted that even the smallest change in the rate of advance for development activities in an underground mining operation can have a significant effect on the value produced by a mining project.

There are numerous reasons why a mine may fail to meet its production plan; for example, poor ground conditions, a changing labour force, etc. (Kazakidis & Scoble 2003). The result is that the value expected through the implementation of a specific mine plan may not be realised. Thus, Maybee et al. (2010) developed a methodology that addresses the complexities associated with underground mine scheduling by combining techniques used to optimise the scheduling of a mining project with financial evaluation tools. The risk-based evaluation methodology (RbEM) addresses the frequent necessity to revise a schedule as circumstances change. It identifies a mining strategy and accounts for the uncertainty surrounding the ability to achieve a specific mine schedule as a complement to the standard recognition of external sources of uncertainty (e.g. market prices) that are taken into account within typical valuations through the use of a discount rate. Having this information gives management the ability, as uncertainty is resolved, to take advantage of opportunities and mitigate risks.

While these techniques offer more information and some guidance with respect to how different strategies will react to the resolution of different uncertainties, they are an analysis tool, requiring the planner to undertake an iterative approach of evaluating many discrete alternatives. They do not offer the decision-maker guidance as to how they should create those discrete alternatives. Maybee (2012) showed that tools from the management of fixed income portfolios can be used as a means of controlling a mining company's exposure to risky cash flows by identifying how the planned cash flows will offset the risk associated with the different sources of funds that are used to finance the operations, providing additional information for the decision-maker.

Portfolio theory is used to manage investment risk in the financial sector by combining assets so that either the expected return is increased, the risk is decreased, or both. Harry Markowitz is considered the father of modern portfolio theory (MPT). Markowitz (1952) first proposed the use of expected return and variance of return in the portfolio selection process. His theory was built upon two objectives; having high returns, and having stable returns not subject to uncertainty. To aid an investor in the selection of an optimal portfolio considering his or her risk–return preferences, he also distinguished efficient and inefficient portfolios, later known as the efficient frontier. The implementation of MPT requires the estimation of the correlation coefficients of returns between all assets considered for the portfolio in addition to the expected future mean returns and the variance of returns for each asset. Although the last two inputs can be estimated using historical information, the estimation of the common movement of assets represents a complex issue (Elton & Gruber 1973).

To handle the difficulty involved in estimating correlations, different techniques have been developed on the groundings of three types of models, namely, historical models, mean models, and index models (Elton & Gruber 1973; Elton et al. 1978). The first method uses past correlation coefficients as estimates of future correlation. The second approach assumes the average of previous periods' correlation coefficients as the correlation coefficient between all assets. Finally, index models assume that asset returns are dependent on one or more common indices.

There have been different attempts to estimate the appropriate inputs for mean-variance portfolio theory; however, the most straightforward method was developed by Sharpe (1963) following the work of Markowitz (1959). This SIM assumes that returns on assets systematically move together because of their common relationship with a single market index that is believed to be the most important influence on asset returns. The linear regression equation for the SIM of an asset can be written as:

$$R_{i(t)} = \alpha_i + \beta_i R_{M(t)} + e_{i(t)} \tag{1}$$

where:

$R_{i(t)}$	=	the t^{th} random return of asset *i*.
α_i	=	the non-random expected return of asset *i*.
β_i	=	measures the sensitivity of asset *i* to the market index.
$R_{M(t)}$	=	the t^{th} random return of the market index.
$e_{i(t)}$	=	the t^{th} random risky return of security *i*.

The SIM breaks the return of an asset into a firm-specific component $\alpha_i + e_{i(t)}$ and a market-related component $\beta_i R_{M(t)}$. The values for the coefficients can be calculated through a regression of the asset returns on the return of the market index over a historical period of time. Because of its convenience and performance, the use of the SIM as an input estimator for mean-variance portfolio theory was later extended to its use in the composition of optimum portfolios, where it is the most widely used model. This model comprises estimating the beta parameter through a least square regression technique, and then uses that beta to estimate future correlation coefficients. Elton et al. (2014) argue that although this method implies possible estimation errors, it is the most convenient methodology to use.

Elton et al. (1976) defined the cut-off rate for inclusion or rejection of an asset in a portfolio. They formulated a method for determining the assets to include in an optimal portfolio, including the amount that should be invested in each of them based on a ranking criterion and the cut-off rate. This method is based on two important assumptions: first, the co-movement of assets results solely from a common response to a market index and, second, there exists a riskless asset with return (Rf). Elton et al. (2014) argue that an investor is then able to rank assets based on their additional return per unit of unsystematic risk; that is, their excess return (as measured by the expected return on the asset minus the riskless rate of return) to beta ratio:

$$\text{Excess return to beta} = \frac{\bar{\mathrm{R}}_{\mathrm{i}} - \mathrm{R}_{\mathrm{f}}}{\beta_i} \tag{2}$$

where:

R_i = expected return for asset *i*.

R_f = risk-free rate of return.

β_i = measures the sensitivity of asset *i* to the market index.

An optimal portfolio is formed through the inclusion of all assets whose excess return to beta ratios are higher than the cut-off rate (C*). The cut-off rate comprises the characteristics of all assets that are being considered for inclusion in the optimum portfolio and provides a tool to easily understand the impact of introducing a new asset into the portfolio. In order to calculate C*, the variable C_i is defined as a candidate and determined as:

$$\mathrm{C}_i = \frac{\sigma_m^2 \sum_{j=1}^{i} \frac{(\bar{\mathrm{R}}_{\mathrm{j}} - \mathrm{R}_{\mathrm{f}})\beta_j}{\sigma_{e_j}^2}}{1 + \sigma_m^2 \sum_{j=1}^{i} \frac{\beta_j^2}{\sigma_{e_j}^2}} \tag{3}$$

where:

σ^2_m = the variance of returns on the market index.

$R_{i(t)}$ = the t^{th} random return of asset *i*.

α_i = the non-random expected return of asset *i*.

β_i = measures the sensitivity of asset *i* to the market index.

$R_{M(t)}$ = the t^{th} random return of the market index.

$e_{i(t)}$ = the t^{th} random risky return of security *i*.

All the assets considered for inclusion in a portfolio must first be ranked according to their excess return to beta ratio. C_i is then calculated for all assets, as per Equation 3. The cut-off rate C* is the C_i with highest value, and thus, the optimal portfolio consists of all assets used in the calculation of C_i.

Considering that the composition of the optimum portfolio is already established, the percentage of investment in each asset needs to be calculated. For this, Z_i is defined as the relative investment in each asset, calculated as:

$$\mathrm{Z}_i = \frac{\beta_i}{\sigma_{e_i}^2}\left(\frac{\bar{\mathrm{R}}_{\mathrm{i}} - \mathrm{R}_{\mathrm{f}}}{\beta_i} - C^*\right) \tag{4}$$

where:

$R_{i(t)}$ = the t^{th} random return of asset *i*.

α_i = the non-random expected return of asset *i*.

β_i = measures the sensitivity of asset *i* to the market index.

$R_{M(t)}$ = the t^{th} random return of the market index.

$e_{i(t)}$ = the t^{th} random risky return of security *i*.

X_i is then calculated as the percentage to be invested in each asset, determined as:

$$X_i = \frac{Z_i}{\sum_{included} Z_j} \tag{5}$$

where:

$R_{i(t)}$ = the t^{th} random return of asset *i*.

α_i = the non-random expected return of asset *i*.

β_i = measures the sensitivity of asset *i* to the market index.

$R_{M(t)}$ = the t^{th} random return of the market index.

$e_{i(t)}$ = the t^{th} random risky return of security *i*.

This process represents a solution for the portfolio problem faced by any investor. Although the process may seem tedious, it is more straightforward than other methodologies that required more advanced mathematical modelling.

3 Application to the mining sector

Scheduling is an important component of the planning process in any mining operation. As such, it is developed for different time horizons (life-of-mine, medium-term and short-term), trying to identify the optimal allocation of limited resources (e.g. manpower, equipment, money, etc.) over different mine areas. Hall (2012) argues that one of the most important objectives for mine scheduling is to maximise the net present value (NPV) of the project by feeding the mill with the highest ore grades possible. He claims that this is achieved through the elaboration of a strategic plan that involves estimating a cut-off grade, equipment requirements and operating and capital costs, among others. Albanese and McGagh (2011) argue that the methodologies that are used for mine scheduling are expected to evolve. For instance, they note that short-term scheduling could be developed in a way that allows it to respond to short-term market fluctuations.

This study proposes the use of the SIM as part of the mine scheduling process, providing a tool to assist decision-makers in selecting an optimum set of mine areas so that feasible mining sequences can be established with the most effective allocation of investment resources. This process is similar to that of an investor considering a number of assets for investment, and using the SIM to identify the assets, as well as the respective proportions, for inclusion in their portfolio.

An approach based on the SIM is applied to an ongoing underground mining operation in this section to recommend an optimal mining sequence at the level of discrete mine areas. In the underground mining context, there is interdependency between the various extraction units (i.e. primary/secondary stoping patterns, constraints between various extraction levels, etc.). This interdependency may limit the extraction sequences that can be scheduled. In this paper, the SIM provides strategic guidance working at the level of discrete mine areas. The assumption being made is that each of the mine areas is independent, and can be mined at any time. As such, within each mine area there are many smaller extraction units (i.e. stopes), which have been scheduled in a manner that accounts for the various constraints and dependencies that exist.

For this application, an underground gold mine with six operating levels is divided into 11 distinct mine areas (Table 1). Each of these mine areas is divided into independent east and west sides, with the exception of the deepest level (3933) that is considered one mine area. The smaller extraction units within each mine area are then scheduled individually as a standalone asset to account for the various dependencies that exist. Under this assumption, the development costs for each level are fully borne by each side in the level (i.e. east and west) independently of whether or not the other side in the same level is scheduled for production. This is required so that each area can be considered as a standalone asset.

Table 1 Mine area characteristics

Mine area	Stoping (t)	Backfill (t)	Development (m)	Gold (g/t)	Life (months)
3933	0	0	1,126	3.8	13
3914E	14,891	16,380	427	4.1	7
3914W	18,661	20,527	834	3.7	11
3895E	13,667	15,034	459	4.1	8
3895W	20,911	23,002	886	4.2	12
3876E	12,543	13,797	459	4.3	8
3876W	29,172	32,089	895	4.6	12
3857E	32,601	35,861	609	4.5	8
3857W	8,785	9,663	765	2.8	10
3838E	17,184	18,902	698	5.4	9
3838W	11,159	12,275	719	3.3	11

Based on the SIM, it is suggested that the returns for the mine areas are correlated due to their common response to a single economic force, namely the commodity price. In this study, the change in each mine area's internal rate of return (IRR) is related to the change in gold price in the same period. The SIM equation to represent changes in IRR related to changes in gold price is outlined as:

$$\Delta\, IRR_i = \alpha_i + \beta_i \times \Delta\, Price + e_i \tag{6}$$

where:

$R_{i(t)}$ = the t^{th} random return of asset i.

α_i = the non-random expected return of asset i.

β_i = measures the sensitivity of asset i to the market index.

$R_{M(t)}$ = the t^{th} random return of the market index.

$e_{i(t)}$ = the t^{th} random risky return of security i.

Estimates of the coefficients and their standard deviations are computed by running a time series regression. However, before proceeding with this analysis, Studenmund (2011) suggests that the input data for use in the regression should be properly prepared, reflecting the characteristics of the underground operation under consideration. Although mine plans are developed using the most accurate information available at the time of estimation, the fact is that divergences between the mine conditions are expected and actually encountered. These differences lie in the uncertainty associated with operating and geological factors. In this analysis, time series data is generated to account for the uncertainty associated with the gold price, development and production costs, grades, and tonnages. This uncertainty is not homogeneous across all the mine operation and thus varies between mine areas.

The uncertainty derived from grades and tonnages increases as mining operations extend at depth. This is reflected in the model input data generated through different variability limits for each factor in different levels. All data points have base case values (Table 2) with the factors fitting triangular distributions (see distribution limits in Tables 3 and 4). From these bases, one thousand data points were generated.

Table 2 Price and cost base case calculations

Gold price (USD/oz)	Base case 1,200
Development cost (USD/m)	5,000
Stoping cost (USD/t)	100
Backfill cost (USD/t)	50

Table 3 Price and cost certainty levels

	Lower limit	Upper limit
Gold price (USD/oz)	1,100	1,300
Development cost	95%	110%
Stoping cost	95%	110%
Backfill cost	95%	110%

Table 4 Grade and tonnage certainty levels

Level	Lower limit	Upper limit
3838	97%	103%
3857	95%	105%
3876	90%	110%
3895	85%	115%
3914	80%	120%
3933	90%	110%

From this information, the regression analysis (Table 5) suggests that the sensitivity of the change in IRR to the change in gold price is the highest for mine area 3838W, as interpreted by the high beta coefficient. Also, mine area 3933 has a negative beta, which, as described by Elton et al. (2014), may result from an error during the period of the regression.

Table 5 Regression output for each mine area

Mine area	α_i	β_i	σ_{ei}
3933	2.775	(3.725)	14,806.862
3914E	0.032	1.531	0.068
3914W	0.090	2.492	0.247
3895E	0.014	1.437	0.027
3895W	0.019	1.700	0.038
3876E	0.009	1.569	0.016
3876W	0.004	1.089	0.008
3857E	0.001	1.018	0.002
3857W	0.001	0.890	0.002
3838E	0.001	1.166	0.003
3838W	0.010	2.510	0.012

The construction of the optimal portfolio using the SIM requires calculating the expected return for each mine area, identifying a risk-free rate of return, and obtaining the variance in the market index (gold price) in addition to the betas already calculated. The expected (mean) returns are obtained from Monte Carlo simulations with 10,000 iterations. A monthly risk-free rate of 0.5% (6% annually) was deemed appropriate for this analysis. Finally, the monthly volatility of gold price was estimated as 3.75% using data from 2012 and 2013 (Figure 1).

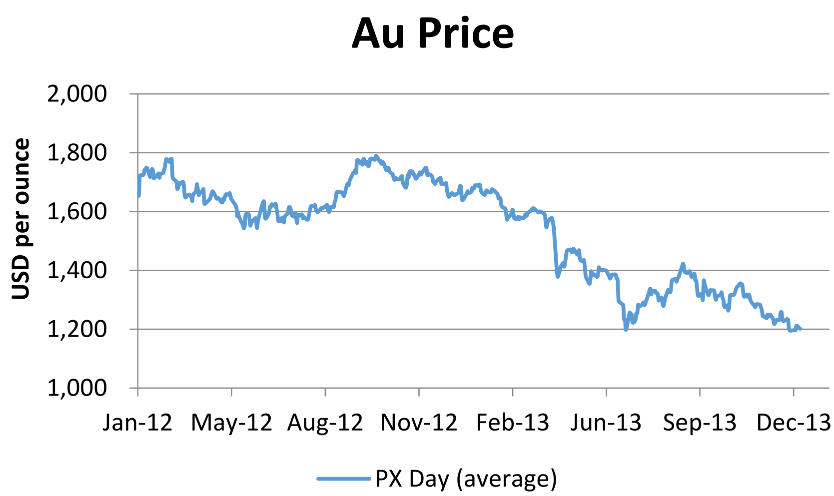

Figure 1 Market price of gold (2012–2013)

Table 6 draws together the input information for each mine area considered for inclusion in the optimum portfolio, that is, a portfolio of mine areas that maximises value for the mining firm. A sequence can then be established based on the relative attractiveness and physical location of each mine area (asset) using optimisation techniques.

Table 6 Input data for the SIM

Mine area	Mean expected return	Standard error	Beta
3933	2.25%	12,168.35%	(3.725)
3914E	35.19%	26.13%	1.531
3914W	33.89%	49.72%	2.492
3895E	44.18%	16.46%	1.437
3895W	46.75%	19.53%	1.700
3876E	42.01%	12.77%	1.569
3876W	70.91%	9.05%	1.089
3857E	67.05%	4.97%	1.018
3857W	65.65%	4.85%	0.890
3838E	64.29%	5.15%	1.166
3838W	19.08%	11.02%	2.510

Following the method developed by Elton et al. (1976), mine areas are ranked in Table 7 according to their excess return to beta ratio where the asset with the highest ratio is the most desirable area. The mine areas are then selected for inclusion in the optimum portfolio according to their candidate score, calculated using Equation 3 and a resulting cut-off rate of 0.421.

Table 7 Asset (mine area) ranking and C_i candidates

Asset number i	Mine area	$(R_i-R_f)/\beta_i$	C_i	X_i
1	3857W	0.73	0.235	39%
2	3857E	0.65	0.355	32%
3	3876W	0.65	0.381	10%
4	3838E	0.55	0.421	19%
5	3895E	0.30	0.417	0%
6	3895W	0.27	0.412	0%
7	3876E	0.26	0.403	0%
8	3914E	0.23	0.401	0%
9	3914W	0.13	0.398	0%
10	3838W	0.07	0.342	0%
11	3933	0.00	0.342	0%

Based on the results in Table 7, the optimum mine schedule (portfolio) for this underground mine currently consists of four mine areas; 3857W, 3857E, 3876W and 3838E. As these areas are exhausted by the mining operations this analysis would be re-run to identify the next areas to be included.

Similarly, if the exogenous influences on this mine change, the analysis could also be re-run to find the new set of mine areas to be included within the mining portfolio. The planner would then assess the costs to switch from the current portfolio to this new set to determine if it is appropriate to do so. An analysis of this nature is shown in Figure 2, where the optimum set (portfolio) of mine areas for different monthly volatilities of gold price are displayed. As can be seen, at low levels of gold price volatility, up to nine of

11 mine areas are scheduled for production. Levels 3838W and 3933 are not selected due to their low excess return to beta ratios. At more likely levels of monthly price volatility (i.e. around 3.75%) only four areas are deemed appropriate to schedule with this number of areas decreasing as volatility increases.

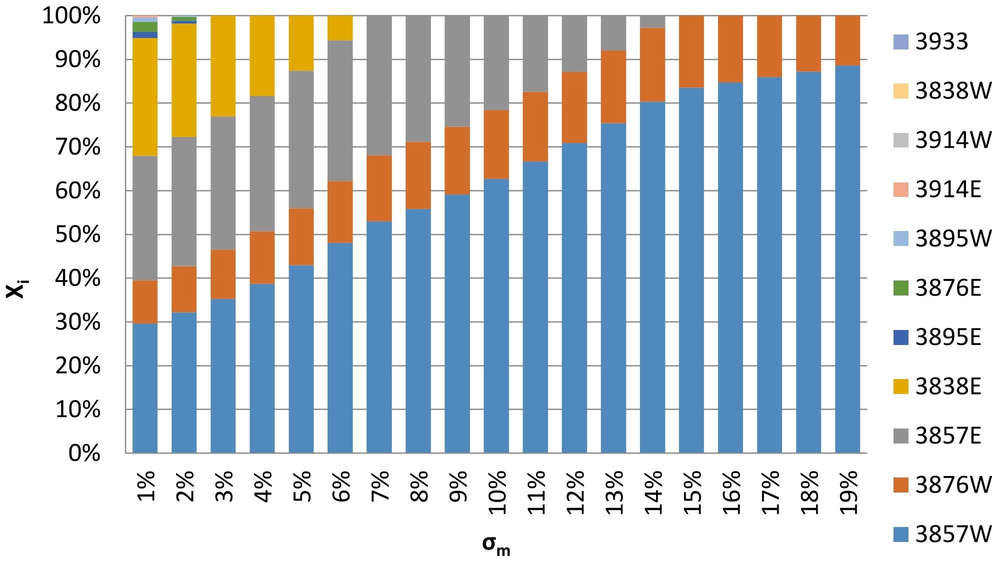

Figure 2 Investment allocation based on the SIM

This shows that when only the one source of volatility is analysed, there is more security in limiting the operations to those areas of highest value. It is expected that as more sources of uncertainty are incorporated into the optimisation (multi-index model) a different solution may be provided to limit the impact of the combined uncertainties.

4 Conclusions

The mining industry operates with many unknowns that can have severe impacts upon what was considered to be a potentially profitable project. As a result, planners must make decisions and create extraction plans within an environment of heightened uncertainty. Thus good planning practices dictate that they should use all of the information available to them at the time. While numerous advances have been made in the evaluation tools used in the mining industry to provide more information early in the decision-making process, they are, for the most part, based upon a predefined plan, and fail to offer guidance to the planner when creating their strategic plan.

There are limited theoretical tools to assist mine planning professionals in selecting the mine areas to which their limited resources should be applied. As a result, this study proposes the use of a tool from the financial portfolio management world (the single index model) early in the mine scheduling process to assist decision-makers in selecting an optimum mining portfolio that utilises an effective allocation of resources.

This paper has identified a tool that can be used in the mine planning process to assist with the identification of the mine areas to be included in the short-term schedule, as well as the amount of limited resources that should be allocated to these areas. However, as the mining industry is a business, it is driven by the maximisation of NPV. The challenge is to marry this use of finance tools with the objective of maximising NPV so as to provide planners with a tool that not only hedges their risky cash flow position, but does so in a manner that keeps in line with the corporate objective of maximising NPV, and thus shareholder value.

Future studies will extend upon the use of the SIM by identifying how the proportions suggested for investment under the situation of limited resources are transferred to implementable mine schedules at the tactical level. This reconciliation of the proposed risk-mitigating portfolio, with the objective function of maximising NPV, will provide a more robust tool for mine planners, which could be compared to results obtained using standard DCF valuation and ranking techniques.

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