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Development of a computer program for underground mine stope optimisation using a heuristic algorithm

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Abstract

Stope layout optimisation improves the economic potential of any underground mining operation and maximises profitability over mine life. A limited number of algorithms are available for underground stope layout optimisation. However, the available algorithms do not guarantee an optimal solution in three-dimensional space. In this paper, existing algorithms for underground stope layout optimisation were reviewed and a computer program called Stope Layout Optimizer 3D (SLO3D) was developed as a C# user interface to implement a heuristic algorithm for optimisation of underground stope boundaries. SLO3D provides an interactive environment to define and edit important parameters related to the stope layout optimisation, including block model parameters, stope geometry, and economic factors. Finally, an example is presented to demonstrate the implementation of algorithm with different stope limits and selection type strategies.

Keywords: *underground mining, stope layout optimisation, heuristic algorithms, SLO3D*

1 Introduction

Hard rock mining is divided into two major categories: surface mining and underground mining. Open pit mining as a form of surface mining is the most popular and economically efficient method when the ore is located close to the surface. Underground mining is suitable for deep deposits located in sensitive areas with high reclamation costs associated with open pit operation. Underground mining methods are higher risk and more complex than surface mining, and have higher mining costs (O'Sullivan & Newman 2015). Table 1 shows a comparison of these two mining methods.

Table 1 Comparison of open pit mining and underground mining (O'Sullivan & Newman 2015)

Attribute	Open pit mining	Underground mining
Complexity	Less complex	More complex
Waste mining	Very high	Low
Stockpiling ore	Yes	No
Environmental disruption	Large footprint	Small footprint
Safety	Relatively safe	High risk
Extraction costs	Low	High
Reclamation costs	High	Relatively low

A variety of methods have been proposed in order to find the optimum limit of open pit mines (Ovanic & Young 1995). Underground mine optimisation has attracted more attention in the last 15 years. Less effort has been done in the field of underground mining and only a few algorithms are available for economic optimisation of underground stope boundaries (Topal & Sens 2010). The major reasons for the lack of research in stope layout optimisation include (Mirzaeian & Ataee-Pour 2011):

- Generality: The existing underground mining methods are numerous; development of a general and common optimisation algorithm for all mining methods is difficult.
- Complexity: Due to various restrictions and conditions of underground mining optimisation; geological, geomechanical and economical modelling of these parameters are more complex.
- Acceptability: Computer programs have contributed to the automatic design of mining areas. However, traditional methods and practices have a greater adherence to general designs by software, especially in Iran.

Due to computational complexity and size of the problem, most of the proposed algorithms in this area work in two-dimensional (2D) space. Few computer packages have been developed to determine and optimise underground stope layout in three-dimensional (3D) space. In this paper, a computer program will be presented, named Stope Layout Optimizer 3D (SLO3D) based on a heuristic algorithm. It is written in the C# programming language. A step-by-step description of this program is outlined in this paper. This computer program has successfully been applied to determine the location of all stopes into an actual copper deposit.

2 Literature review

The existing algorithms for underground stope layout optimisation can be classified in two groups: level-oriented and field-oriented. The level-oriented algorithms such as dynamic programming (Riddle 1977), branch and bound (Ovanic & Young 1999), optimum limit integrated probable stope (OLIPS) algorithm (Jalali & Ataee-Pour 2004; Jalali et al. 2007a); mixed integer programming (Grieco & Dimitrakopoulos 2007), Greedy (Jalali & Hosseini 2009), and global optimisation for underground mining area (GOUMA) algorithm (Jalali et al. 2016) are applied on a level or panel. In contrast, in the field-oriented algorithms, the economic value of each block is considered as a constant value and the determination of underground mining limits takes place on the entire mining area before dividing the mining area into levels or panels. Floating stope method (Alford 1995), maximum value neighbourhood (MVN) algorithm (Ataee-Pour 2000, 2005), multiple pass floating stope process (Carwse 2001), heuristic approaches (Topal & Sens 2010; Sandanayake et al. 2015a, b), network flow method (Bai et al. 2013, 2014) and octree-division algorithm (Cheimanoff et al. 1989) were developed in this manner. Most of the previously proposed algorithms relied on simplification of a large and complex problem. For instance, a 3D problem was converted to a 2D problem. In the following sections, available computer programs for underground stope layout optimisation in 2D and 3D space are discussed. Table 2 shows the summary of the proposed computer programs in underground stope layout optimisation.

Table 2 Summary of the proposed computer programs in underground stope layout optimisation

Computer program	Year	Algorithm	Dimension	Mining method	Optimality
Fortran (Riddle 1977)	1977	Riddle	2D	Block caving	No
Datamine (Alford 1995)	1995	Floating stope	3D	All	No
LINGO–CPLEX (Ovanic & Young 1999)	1999	Branch and bound	1D	All	Yes
SLO (Ataee-Pour 2000)	2000	MVN	3D	All	No
SBO (Jalali et al. 2007a)	2007	OLIPS	2D	All	Yes
MATLAB (Topal & Sens 2010)	2010	Heuristic algorithm	3D	All	No
GOUMA-CP (Jalali et al. 2016)	2016	GOUMA	2D	All	Yes

2.1 Dynamic programming

Riddle (1977) proposed an algorithm based on dynamic programming that was an extension of 3D dynamic programming method for ultimate open pit optimisation that had been developed by Johnson and Sharp in 1971 (Johnson & Sharp 1971). The algorithm was written in Fortran and implemented on hypothetical economical block models. This algorithm defines the stope boundaries for block caving operations. The major drawback of this algorithm is that it is only able to define stope boundaries in 2D space and optimality is not guaranteed in 3D space.

2.2 Floating stope

Alford (1995) developed a floating stope method to determine stope boundaries for ore blocks within a resource block model. During the process of this algorithm a stope is floated with the entire block model with a specified origin and an increase in three-dimensional space. Based on this algorithm, Datamine software was presented by a company with the same name (Datamine 1995). The major problem of this method is producing overlapping stopes where some stopes share high-grade blocks (Alford et al. 2007).

2.3 Branch and bound

A mathematical programming technique was developed by Ovanic & Young (1999). Mixed integer programming (MIP), in combination with piecewise linear function, optimise the origin and last location for mining for an entire row or mining panel. The MIP approach is applied with a special kind of variable named 'Special Ordered Sets Types 2' (SOS2) to determine an optimum mining limit. There are many programs to solve mathematical problems, especially branch and bound techniques including LINGO, MPS and GAMS/CPLEX packages.

2.4 Maximum value neighbourhood

Ataee-Pour (2000) introduced MVN as a heuristic approach based on stope/block and the order of neighbourhood that are restricted by the mine geometry constraints. Different sets of stopes are provided with different starting points for the algorithm which does not generate a mineable stope shape. Ataee-Pour (2000) developed a computer program named Stope Limit Optimizer (SLO) for stope limit optimisation in 3D space.

2.5 OLIPS

Jalali et al. (2007b) developed the OLIPS algorithm based on the dynamic programming method. This algorithm complies with all technical and geometric constraints and provides mathematical proof. OLIPS has two major steps. At the first step, a conventional economic model of mining panel is constructed. Then, in the second step, the probable stope economic model and integrated probable stope economic model are derived from conventional model. Based on OLIPS algorithm, a computer program named Stope Boundary Optimizer (Jalali et al. 2007b) was developed and validated by 2D hypothetical models.

2.6 Topal and Sens heuristic approach

Topal and Sens (2010) introduced a heuristic algorithm to optimise underground stope layout with different stope sizes and strategies in 3D space. During the optimisation, all stopes with specified height, width and length are generated based on their economic values in the MATLAB software. The major drawback of this approach is selecting stopes in descending order while removing overlapping stopes.

2.7 GOUMA

Jalali et al. (2016) presented a new comprehensive algorithm called GOUMA. The value of underground mining area blocks vary with geometry and location of the panel or level. So the algorithm runs on a special model named 'Variable Value Economic Model' (VVEM). For easy use of this algorithm in large-scale problems, a computer program was written in C++ programming language named GOUMA-CP (Jalali et al. 2016).

3 SLO3D computer program

In order to facilitate the implementation of a heuristic algorithm previously developed by Sandanayake et al. (2015a) with some strategies that have been added to the process of this algorithm, a user-friendly interface (UI) computer program was developed in C# and SQL-Server programming language named SLO3D. The main difference between this computer program and other programs and software tools is the kind of algorithm used. As shown in Figure 1, this computer program has three main steps: block economic value (BEV) producer, stope generator and stope layout optimiser. After defining the block economic model using economic parameters in the first step, the algorithm generates all possible stopes in an economic block model considering the stope dimension in 3D space. After that, the algorithm generates a family of non-overlapping stopes over all possible stopes and selects the highest economic value as an optimal solution. For large and complex problems, to save solution time, three new probabilistic strategies have been added to this algorithm with stope layout optimisation performed in the third step.

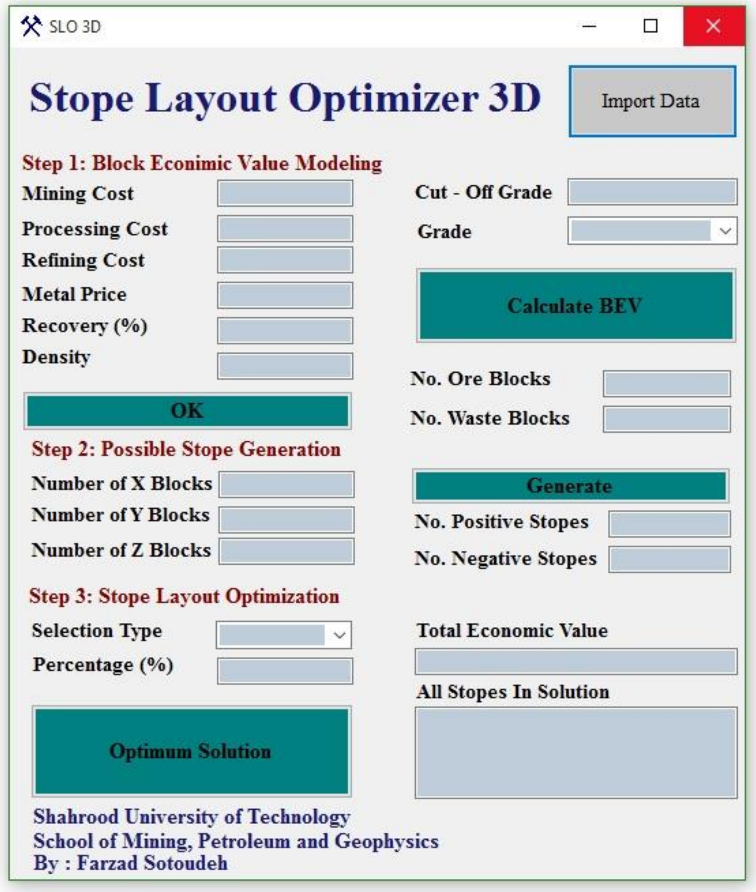

Figure 1 Stope Layout Optimizer 3D interface

3.1 Block economic value

A grade block model with regularised dimension as the input file must be created in Microsoft Excel format before running the program. The input file structure consists of seven major columns, similar to a spreadsheet shown in Figure 2; including the coordinate of a regularised block (XC, YC and ZC), block dimension (XINC, YINC and ZINC), grade of a block (percent or gram per tonne), value of a block (BEV (USD)), density (tonne/m^3), total weight and metal weight of every block. After importing the input file by clicking on the specified button named 'Import Data', the user should import economic factors such as mining cost (C_m), processing cost (C_p), refining cost (R), metal price (P), grade (g), weight of a block (T) and recovery (Y). Then, with the cut-off grade defined, blocks below the cut-off grade are termed as waste blocks. After clicking on the 'Calculate BEV' button, the grade block model is converted to an economic model based on Equation 1 and the number of ore and waste blocks with consideration given to the cut-off grade all reported.

$$\mathrm{BEV} = \{(\mathrm{P}-\mathrm{R}) \times \mathrm{g} \times \mathrm{Y} - (\mathrm{C_m} + \mathrm{C_p})\} \times \mathrm{T} \tag{1}$$

Block	XC	YC	ZC	XINC (m)	XINC (m)	XINC (m)	Grade (%)	density (ton/m3)	BEV ($)	Block Weight	Metal Weight
1	3348413	223248	1525.5	5	5	5	0.0192559	2.54			
2	3348413	223248	1530.5	5	5	5	0.0409784	2.54			
3	3348413	223248	1535.5	5	5	5	0.021165	2.54			
4	3348413	223248	1540.5	5	5	5	0.0099024	2.54			
5	3348413	223248	1545.5	5	5	5	0.021478	2.54			

Figure 2 Input file structure before running program (Excel format)

3.2 Stope generation

In this step, all possible stopes with specified dimensions are generated based on the underground mining method and geotechnical stability parameters. By clicking on the 'Generate' button, the algorithm specifies the origin of the economic block model (i' j' k'). Then, assuming that the block increases in size increments in X, Y, and Z directions as stope dimension parameters, the last stope block (i'' j'' k'') is determined. The constructed stope is floated through the economic model and all possible stopes are generated based on two conditions that are shown in Equations 2 and 3.

$$(i', j', k') \leq (i, j, k) \leq (i'', j'', k'') \tag{2}$$

$$(i' \leq i_{max}\ ,\ j' \leq j_{max}\ ,\ k' \leq k_{max}) \tag{3}$$

The (i_{max} j_{max} k_{max}) as shown in Equation 3 is the maximum stope block (i j k) within the economic model. The average grade of each stope is calculated by summing the grade of each block within the stope and dividing this by the total number of blocks within the stope. Also, the economic value of each stope is calculated by summing the values of all the blocks. The output file structure consists of seven major columns including (Figure 3): Stope ID, Stope Dimension in X, Y and Z direction (XINC, YINC and ZINC), Stope Grade (percent), Stope Economic Value (SEV), Total Weight, Metal Weight and identifier of origin and last block of each possible stope. Finally, based on the SEV column, the number of positive and negative stopes are reported.

Stope ID	XINC(m)	YINC(m)	ZINC(m)	Grade (%)	SEV ($)	TotalWeight (ton)	MetalWeight (gram)	X-Origin	Y-Origin	Z-Origin	X-Last	Y-Last	Z-Last
1	15	15	15	0.45056	-2320.9109	8572.5	38624240.76	1	1	1	3	3	3
2	15	15	15	0.425464	-23862.393	8572.5	36472888.57	1	1	2	3	3	4
3	15	15	15	0.363886	-61263.955	8572.5	31194117.89	1	1	3	3	3	5
4	15	15	15	0.521916	36243.4358	8572.5	44741242.88	1	1	4	3	3	6
5	15	15	15	0.451764	-7337.854	8572.5	38727436.04	1	1	5	3	3	7

Figure 3 Output file structure after generating all possible stopes

3.3 Stope layout optimisation

In order to determine the optimum location of underground stopes, all possible sets of non-overlapping stopes are generated according to the algorithm shown in Figure 4. For this purpose, all negative stopes generated in the previous step are removed and positive stopes imported to the algorithm. Two major null family of sets, S_T and S_O, are created. S_T is all possible sets of non-overlapping stopes that are generated during the algorithm and S_O is a unique derived set of S_T. During the processing of the algorithm, each stope is compared with all stopes within any set of non-overlapping stopes (S_P). If the imported stope does not overlap with other stopes, all stopes are combined and a new set of non-overlapping stopes are created (S_{Pnew}). This process iterates until all positive stopes have participated in the algorithm. Finally, the high value of non-overlapping stope sets is selected as the optimum solution.

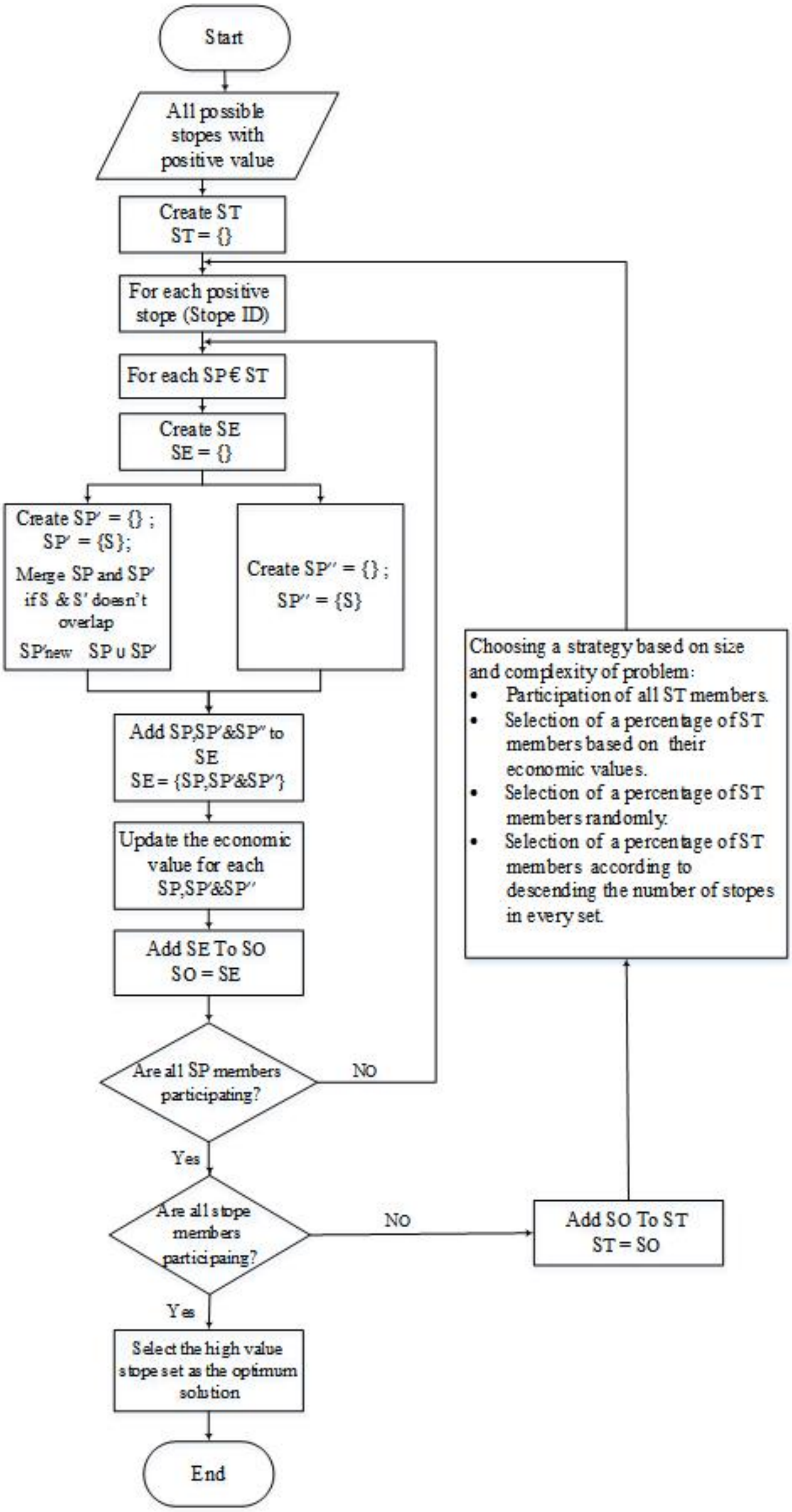

Figure 4 Revised stope layout optimisation algorithm after Sandanayake et al. (2015a)

In large and complex problems, the sets of non-overlapping stopes (S_P) are infinite. So the solution time increases exponentially. To overcome this problem, three strategies have been added to this algorithm. The first strategy is sorting of all sets (S_T members) according to their economic value to the lowest value and selecting a percentage of the sorted collection. The major drawback of the first strategy is removing some stope sets with low economic value. In this strategy, the possibility of the combination of removed sets and other stopes to derive a set with a higher total value is discarded. Owing to this disadvantage, a second strategy with a probabilistic background was proposed.

- Selecting a percentage of S_T members randomly and frequently.
- Selecting a percentage of S_T members according to the descending number of stopes in each set.

As shown in Figure 1, the user should define the percentage of selection in the case of using a specified strategy amongst three strategies. By clicking on 'Optimum Solution', the total economic value of the underground layout and stopes' IDs in the mining area is reported.

4 Application of SLO3D

4.1 Orebody modelling

To introduce the capability of the SLO3D program, it was applied on an actual copper deposit located in southeast Iran. The study deposit is a copper vein with a thickness of 20 m, a 400-m length in a longitudinal direction and reaching 100 m in the vertical direction (Table 3). Datamine software was used to generate an orebody model and produced 6,400 blocks of which 2,259 are oreblocks according to the cut-off grade. All blocks are 5 × 5 × 5 m in size. All blocks were estimated according to exploration and information data. The output files retrieved from the Datamine software contain the block centre coordinate, density and average grade of each block and prepared as an input file for SLO3D. A horizontal section of the grade distribution for the entire block model is shown in Figure 5. Economic parameter assumptions for converting the geological model to an economic model (BEV) are provided in Table 4. The values of all blocks calculated were based on these assumptions. Consequently, the range of economic value variation was from USD -994,234 to 2,763,576.

Table 3 The coordinates of study area

Direction	Minimum	Maximum
X	3348410	3348810
Y	223245	223265
Z	1523	1623

Table 4 Economic parameters for optimisation

Parameter	Value
Mining (USD/tonne)	20
Processing cost (USD/tonne)	10
Refining cost (USD/tonne)	90
Copper price(USD/tonne)	6,500
Recovery (%)	90
Cut-off grade (%)	0.52

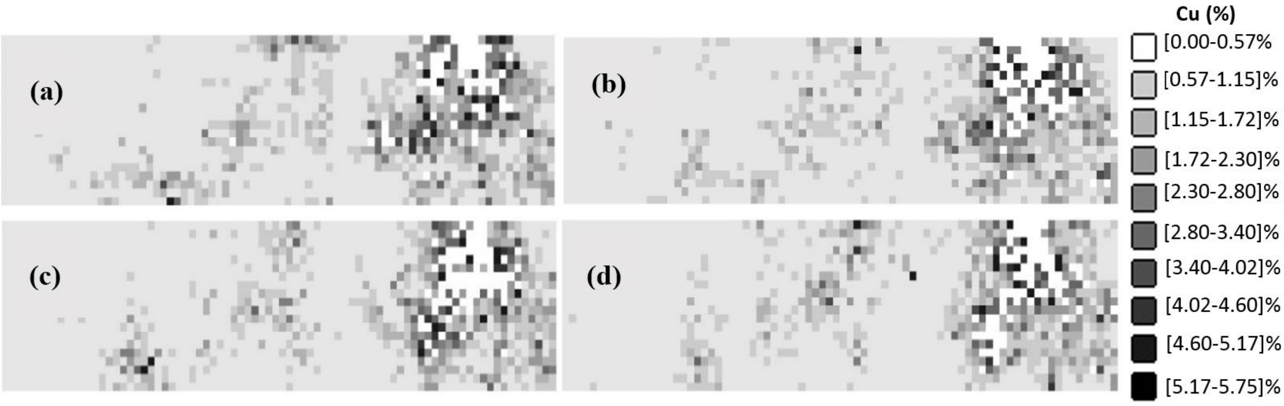

Figure 5 Horizontal section of grade block model, in N23°W direction at (a) Y = 223245; (b) Y = 223250; (c) Y = 223255; and, (d): Y = 223260

4.2 Stope generation

The mining method in this orebody is longitudinal stoping – a similar method to the method of sublevel stoping. In longitudinal stoping the direction of mining is the same as sublevel stoping (along the strike of the orebody in a longitudinal direction). This method is designed for orebodies with a thickness in the range of circa 5–20 m (Tatiya 2005). Determination of stope dimensions in these methods can, in most cases, be achieved by designing stopes with high vertical and short horizontal dimensions or stopes having short vertical and long horizontal dimensions (Villaescusa 2000).

For this study, stope dimensions are considered as 50 × 20 × 25 m. As calculated by the computer program (SLO3D), 1,136 possible stopes were generated, of which 974 stopes had a positive value and 162 stopes had a negative value (Table 5).

Table 5 Summary of possible stopes generated

Parameter		Value
Number of possible stopes	Positive value	974.00
	Negative value	162.00
Grade (%)	Min	0.20
	Max	1.76
	Average	0.83
SEV (USD)	Min	-867,419.14
	Max	4,658,403.18
	Average	1,2693,07.13
Metal weight (tonne)	Min	129.60
	Max	1,120.00
	Average	526.60

4.3 Stope layout optimisation

After generation of all possible stopes, stopes with negative economic value were removed and all positive stopes imported as the input file in order to produce all combinations of the combined non-overlapping stopes through the algorithm (Section 3.3). A unique set of non-overlapping stopes containing 29 stopes was selected as the optimum solution, with a value of USD 37.05 M. Table 6 shows the summary of the stope layout optimisation step. Stope ID, Stope Dimension in X, Y and Z direction (XINC, YINC and ZINC), Stope Grade (percent), Stope Economic Value (SEV), Total Weight, Metal Weight and identifier of origin and last block of each stope within the optimum limit are shown in Table 7.

Table 6 Summary of stopes generated within optimum layout

Parameter		Value
Number of stopes		29
Total economic value (USD)		37.05 M
Grade (%)	Min	0.46
	Max	1.28
	Average	0.83
SEV (USD)	Min	49,579.30
	Max	2,905,380.96
	Average	1,277,587.73
Metal weight (gram)	Min	296.50
	Max	815.20
	Average	527.60

Table 7 Values of stopes in optimum solution

No.	Stope ID	Grade (%)	SEV (USD)	Metal (tonne)	Origin block			Last block		
					X	Y	Z	X	Y	Z
1	1	0.81	1,273,906.816	514.84	1	1	1	10	4	5
2	6	0.79	1,244,680.488	503.34	1	1	6	10	4	10
3	11	1.01	2,070,867.724	646.82	1	1	11	10	4	15
4	80	1.04	1,999,699.462	664.16	5	1	16	14	4	20
5	161	0.94	1,561,925.662	596.91	11	1	1	20	4	5
6	166	0.65	549,999.2011	415.59	11	1	6	20	4	10
7	171	0.87	1,303,862.937	555.62	11	1	11	20	4	15
8	322	0.46	49,579.29928	296.72	21	1	2	30	4	6
9	330	0.69	900,844.0652	441.21	21	1	10	30	4	14
10	368	0.50	138,315.9739	321.26	23	1	16	32	4	20
11	481	1.13	2,283,708.359	720.39	31	1	1	40	4	5
12	486	1.10	2,188,453.484	698.57	31	1	6	40	4	10
13	491	1.23	2,699,146.127	783.57	31	1	11	40	4	15
14	560	1.14	2,318,046.64	726.77	35	1	16	44	4	20
15	641	0.71	781,486.3839	451.44	41	1	1	50	4	5
16	646	0.56	319,377.4155	361.84	41	1	6	50	4	10
17	651	0.77	1,079,351.313	491.00	41	1	11	50	4	15
18	752	0.63	549,862.6853	401.61	47	1	16	56	4	20
19	801	0.83	1,333,063.971	527.70	51	1	1	60	4	5
20	806	0.73	985,764.3348	463.80	51	1	6	60	4	10
21	811	1.06	2,220,607.588	678.12	51	1	11	60	4	15
22	944	0.57	375,454.2742	362.59	59	1	16	68	4	20
23	961	0.98	1,746,695.373	623.21	61	1	1	70	4	5
24	966	0.83	1,303,448.075	531.93	61	1	6	70	4	10
25	971	1.28	2,905,380.967	815.62	61	1	11	70	4	15
26	1104	0.90	1,475,802.093	574.68	69	1	16	78	4	20
27	1121	0.60	464,234.7939	383.00	71	1	1	80	4	5
28	1126	0.57	367,185.6118	363.71	71	1	6	80	4	10
29	1131	0.60	559,293.018	383.96	71	1	11	80	4	15

5 Conclusion

The geometry of the mining area in an underground operation is one of the most important issues and maximises the economic value of a project. The complexity of underground mining methods has discouraged the development of computer programs and most of these computer programs do not guarantee the optimal layout, especially in 3D space. In this paper, a new computer program (SLO3D) developed to optimise stope layout according to economic factors, cut-off grade and specified stope dimension, has been presented. Also, for large and complex problems, three strategies that have been added to a heuristic algorithm in order to save solution time, were presented. The implementation of SLO3D was applied to an actual copper mine located in southeast Iran that resulted in an optimised layout with 29 stopes within the orebody with a value of USD 37.05 M.

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