

# Reliability analysis and design of backfill in a cut-and-fill mining method

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## Abstract

*In underground mining, hydraulic backfill materials, such as waste tailings, river sand, and cement, are often used to fill underground mined stopes. In cut-and-fill mining methods with blasthole stoping and delayed backfill, after extraction of adjacent pillars that contain economic minerals, the backfill is often subject to exposure of free standing on at least one side. A key concern for mining engineers is the stability of this immediately-bordered backfill body, because the backfill stability has a significant effect on the dilution/loss rate and the safety of mining operations. It is found that backfill stability is one of the mining subjects most dominated by uncertainty. Rock and backfill properties, environmental conditions, and analytical models are such factors contributing to uncertainty. Conventional methods simplified the problem by considering the uncertain parameters to be deterministic, and accounted for the uncertainties through the use of empirical factors of safety.*

*This paper aims to conduct stability analysis of backfill in underground mining using a probabilistic reliability method, which is an extension of conventional deterministic methods. The parameters of backfill properties are modelled as random variables. In order to determine the failure probability of the backfill in a cut-and-fill mining method of an underground mine in China, a three-dimensional wedge model is set up for the backfill and a corresponding limit state function is established to characterize the backfill stability for the purpose of reliability analysis. The influences of the mean values, coefficients of variation, probability distribution types, and correlation between random variables are carefully investigated through sensitivity analysis. The results obtained give insights into the mechanism of backfill stability and could provide some useful clues on how to choose the right backfill materials.*

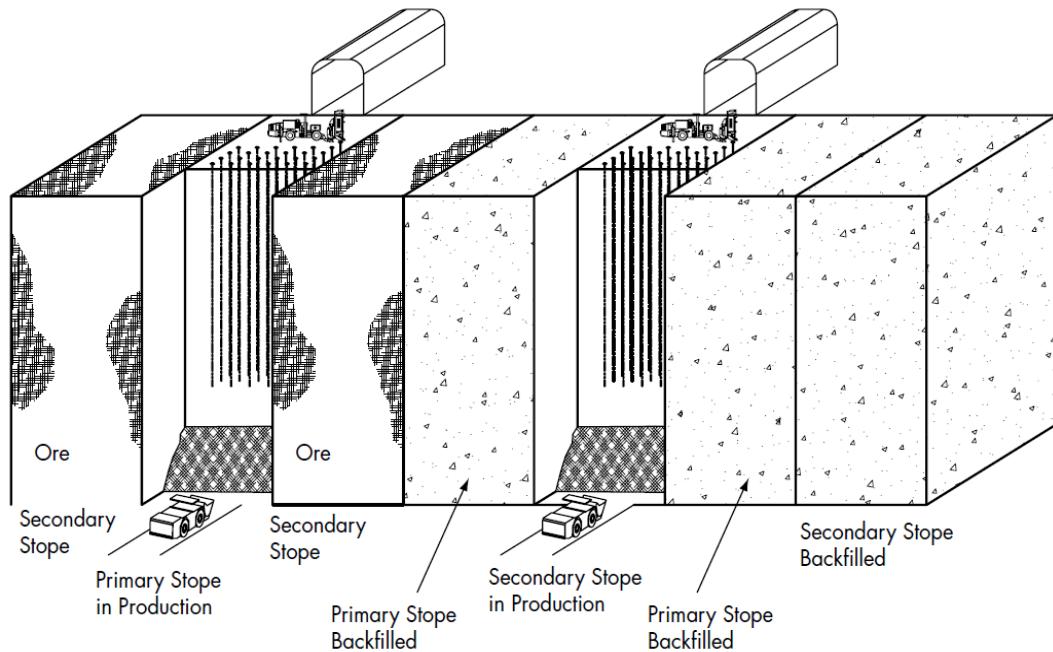
## 1 Introduction

Backfilling has been used increasingly in underground mines as a support medium in/around mining areas. The mining method known as cut-and-fill, which incorporates backfilling in its operational cycle, is ranked as a major method of underground mining. Even the room-and-pillar mining and sublevel stoping methods also employ backfills such that the remnant pillars can be recovered after the primary mining operations have been completed. As a general rule, backfill performs the following unique functions:

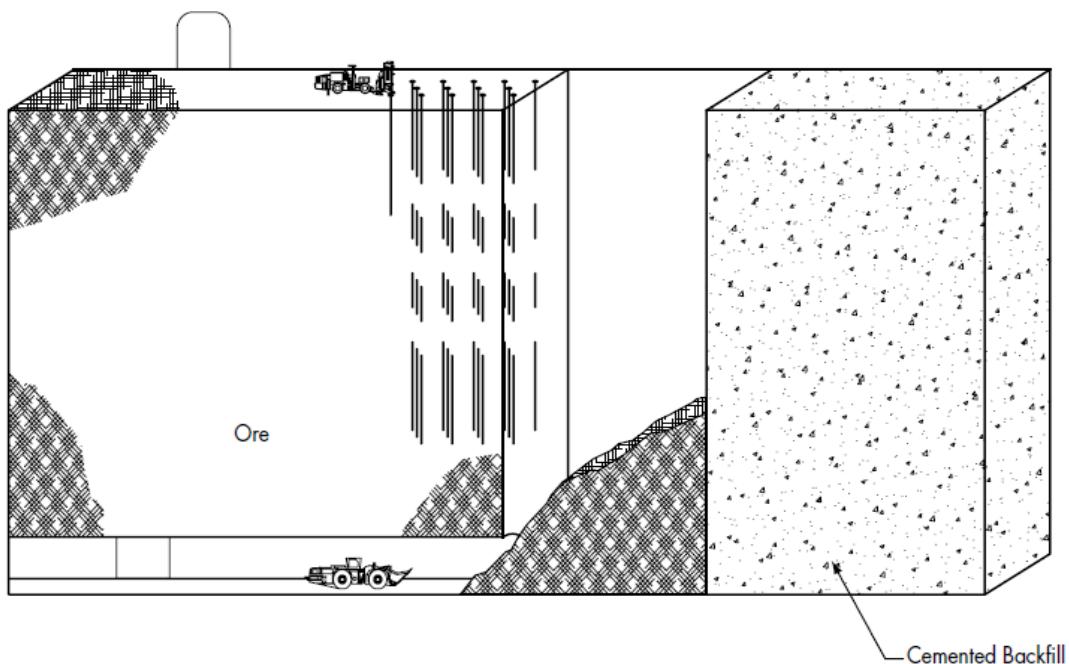
- Serves as a ground control measure: wall support and stope stabilization.
- Provides a working floor: artificial roof in underground cut-and-fill stoping.
- Fills voids.
- Disposes of waste rock and/or tailings.
- Serves as subsidence and fire control, rockburst control, etc.

In cut-and-fill methods, blasthole stoping with delayed backfill is a method that can be applied to ore bodies that are generally tabular and vertical or near vertical. The rock quality of both the ore and the waste needs to be fair to good. Cemented fill for the primary stopes is required when a primary/secondary plan is pursued.

Cemented fill allows secondary stopes to be mined subsequently. The secondary stopes may be backfilled partially or completely with uncemented fill. Where the ore zone has sufficient width, the long-axis orientation of the stopes may be perpendicular to the strike (Darling, 2011). These are transverse stopes (see Figure 1). Where the ore zone is narrow and the long axes of the stopes are parallel to the strike, the stope orientation is called longitudinal (Darling, 2011). These conditions require sequences of longitudinal stopes followed by backfilling to prevent stability problems and dilution from the collapse of the waste material between ore lenses (see Figure 2).



**Figure 1** Typical transverse blasthole stoping with delayed backfill (Darling, 2011)



**Figure 2** Typical longitudinal blasthole stoping with delayed backfill (Darling, 2011)

After extraction of adjacent pillars that contain economic minerals, the backfill is often subject to exposure of free standing on at least one side. The stability of the primary stope backfill is a key issue to apply these

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blasthole stoping with delayed backfill methods. If the primary stope backfill in Figure 1 and the cemented backfill in Figure 2 lose their stability, the backfill materials would collapse and then mix up with the excavated ore. This collapse movement of the backfill materials has an negative effect on the separation of the ore and backfill region in the mucking pile, causing either ore loss (the ore is wrongly categorized as backfill waste and sent to the waste dump) and/or ore dilution (backfill material is wrongly categorized as ore and sent to the processing plant). The dilution or loss of mineral are two important factors in grade control of a mine. The backfill would also act as a resistance to the ground stress and a support for subsidence of overhead stratum. Therefore, the backfill stability has a significant effect on the dilution/loss rate and the safety of mining operations. The correct design and selection of backfill are necessary conditions for these backfill benefits to be obtained effectively. The backfill stability problem becomes extremely significant when the presence of uncertainties has been recognized in the analysis and design of mining process. Rock and backfill properties, environmental conditions, and analytical models are such factors contributing to uncertainty. Conventional methods simplified the problem by considering the uncertain parameters to be deterministic, and accounted for the uncertainties through the use of empirical factors of safety.

This paper aims to do stability analysis of backfill in underground mining using a probabilistic reliability method. In Section 2, after introducing the mining geology of an underground mine in China, a three-dimensional wedge model was set up to simulate the backfill performance. A corresponding limit state function was established to characterize the backfill stability for the purpose of reliability analysis. In Section 3, the uncertainty in backfill was modelled as random variables and the first-order-reliability-method was used to determine the failure probability of backfills. Sensitivity analysis was performed to investigate the influence of various parameters on backfill stability. Section 4 concludes the paper and measures were suggested to maintain the backfill stability in this mine.

## 2 Model

### 2.1 Mining geology

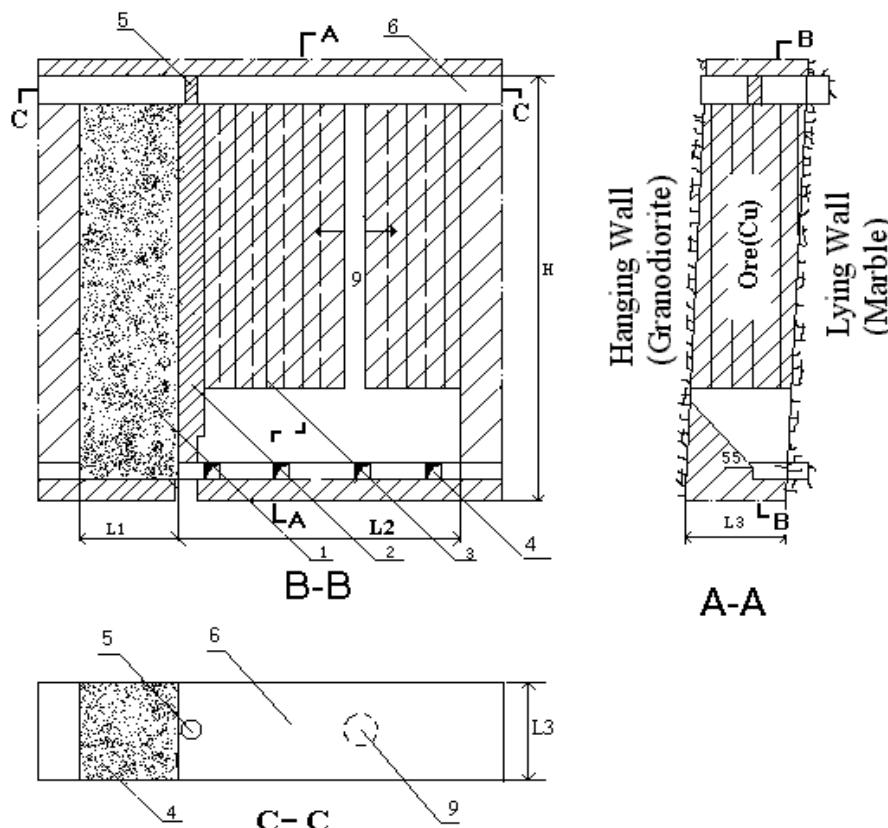
The underground copper mine is located in Anhui province, southeastern China. It is one of the major non-ferrous mines in China. The ore body geometry looks like a thin plate near vertical inserting in the ground. Its length and width on the horizontal ground are about 600 and 20 m, respectively. Its depth is deeper than 400 m. The two plate surfaces dip at an angle between 75 and 85 to the horizontal. The plate-shaped ore rock forms the narrow interface material between granodiorite and marble. The granodiorite is located above the ore rock and forms the longitudinal hanging walls for the ore excavation. The marble is situated below the ore rock and becomes the longitudinal lying wall for the ore excavation. The ore rock is made of Cu<sub>2</sub>O-ferroferrite and Cu<sub>2</sub>O-garnet. The Protodyakonov rock strength index is between 16 and 26. The average ore grade is 0.93%. Ore grade is the percentage of the useful part weight in the whole ore-body weight in general. Table 1 gives the mechanical and strength properties of the ore rock, the granodiorite, the marble and the backfilled materials.

The underground copper mine is vertically divided into levels, with each level being 60 m high for accommodation of the drilling machine. Each level is horizontally divided into stopes and each stope is considered as an extraction unit. In each unit, the cut-and-fill, blasthole stoping with delayed backfill method is used for the mining operation. Each of the vertical blasting holes is of 165 mm in diameter and 50 m in length. After being blasted in the stope, the ore blocks are drawn down into the transportation cars on the drift located at the depth 360 m below the ground. Then the ore is transported along the shaft onto the ground. Figure 3 shows the layout of the mining method in an extraction unit of Level -300 m. Each stope is divided into substope A and substope B. The substope A is 15 m long along the ore longitudinal direction.

The substope B is 35 m. A temporary ore wall is needed between the substope A and B to support a pillar in the drilling room. The ore rock in the substope A is extracted at first. The ore blocks in the substope A are directly drawn down to the drift. Once the ore in the substope A is completely extracted, the tailings mixed with Porland cement will be used to backfill the open space of the substope A. Until the substope A is completely backfilled, mining activities will not commence in the substope B. The blasted ore blocks in the substope B are drawn via a vibrating machine. This vibration drawing machine requires the substope B to be 35 m long. The substope B will be backfilled once its ore is completely extracted. The mining operations will commence and repeated for the next stope.

**Table 1 Properties of the surrounding rock, ore and backfill**

Property	Granodiorite (Hanging wall)	Marble (Lying wall)	Copper (Ore body)	Cemented tailings (Backfill)
Uniaxial Compressive Strength (MPa)	38.0	28.5	60.0	3.5
Tensile Strength (MPa)	1.2	2.3	5.6	0.5
Young's Modulus (MPa)	8000	21000	65000	1000
Poisson's Ratio	0.16	0.26	0.31	0.30
Cohesion (MPa)	0.60	2.16	4.30	0.65
Internal Friction angle ( $^{\circ}$ )	53	50	51	35
Density (kN/m <sup>3</sup> )	28.0	28.0	40.4	22.1



1 = Substope A (backfilling); 2 = temporary ore wall; 3 = Substope B; 4 = Stope basement structure; 5 = the pillar in the drilling room; 6 = drilling room; 9 = blasting cut hole; L1 = length of substope A; L2 = length of substope B; L3 = width of ore-body; H = height of level

**Figure 3 Schematic diagram of cut-and-fill mining method**

The stability of the backfill in substope A plays a critical role in controlling ore dilution rate and loss rate. It can be seen that during the extraction and mucking in substope B, the backfill's exposure surface is so high

and so large that there is a significant tendency to lose stability. Hence, both the dilution rate and the loss rate (or extraction rate) depend heavily on the stability of the backfill in substope A. The correct selection of backfill will determine whether the backfill is stable or not. This is the key technique in the blasthole stoping with delayed backfill method. A three-dimensional wedge model is used to study the problem of backfill stability.

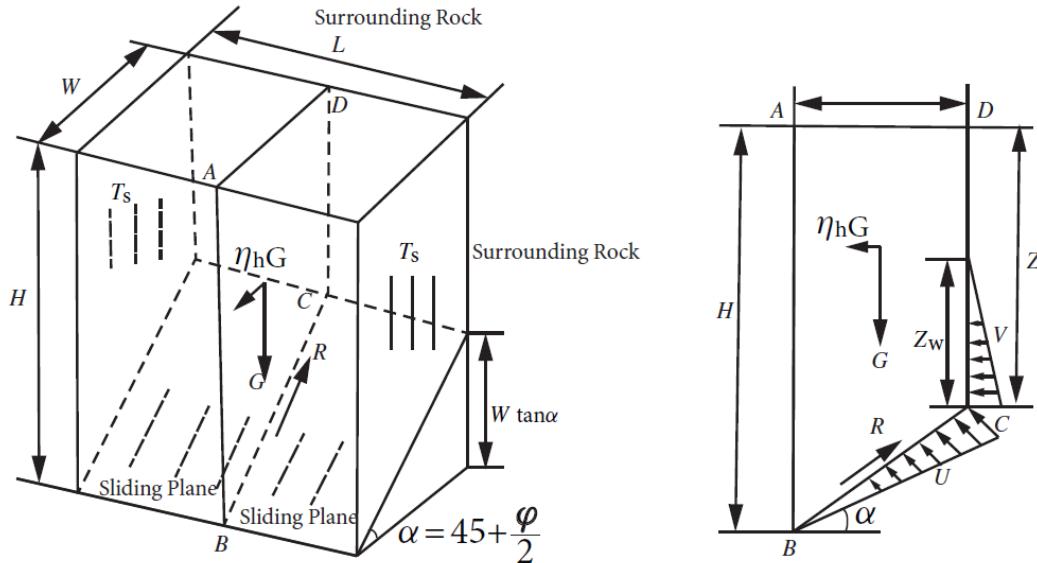


Figure 4 Three-dimensional wedge model for backfill of substope A.

## 2.2 Three-dimensional wedge model for backfill of substope A

In order to assess the stability of the backfill of substope A, a 3-D wedge model is set up in Figure 4. The loads on the backfill wedge are determined as follows,

$$R = CA_b + N \tan \varphi \quad (1)$$

$$S = G(\sin \alpha - \eta_h \cos \alpha) + V \cos \alpha - 2T_s \quad (2)$$

where  $R$  and  $S$  are the resistance force of sliding and the driving force of sliding, respectively, and

$$G = \frac{1}{2} \gamma_f (2H - W \tan \alpha) WL = \gamma_f H_a WL \quad (3)$$

$$N = G(\cos \alpha - \eta \sin \alpha) - U - V \sin \alpha \quad (4)$$

$$H_a = H - \frac{1}{2} W \tan \alpha \quad L_h = \frac{W}{\cos \alpha}, \quad U = \frac{1}{2} \gamma_w Z_w L_h L \quad V = \frac{1}{2} \gamma_w Z_w^2 \quad (5)$$

$$A_b = \frac{L_w}{\cos \alpha} \quad A_s = H_a W \quad T_s = C_j A_s + N_s \tan \varphi_j \quad (6)$$

$$N_s = \frac{WH_a C(1 - \sin \varphi_j)}{\sin \varphi_j} \quad \eta_h = K_h C_Z a_i \quad C_Z = 0.25, \quad a_i = 1.0 \sim 2.5 \quad (7)$$

where  $T_s$  is the shear force between rock and backfill,  $T_b$  is the resistance force of the sliding face,  $N_s$  is the total lateral force on the backfill from the surrounding rock (Lu, 1994),  $N_b$  is the support force of the backfill below the sliding surface to the slide body,  $G$  is the weight of backfill,  $\gamma_f$  is the density of the backfill,  $\gamma_w$  is the water density,  $A_b$  and  $A_s$  are the area of the sliding plane and the area of lateral faces, respectively.  $H$ ,  $L$ , and  $W$  are the height, width, and length of the backfill, respectively.  $\alpha$  is the sliding angle,  $\alpha = 45^\circ + \varphi/2$ .  $C$  and  $\varphi$  are the cohesion and internal friction angle of the backfill, respectively.  $C_j$  and  $\varphi_j$  are the cohesion and internal friction angle between the surrounding rock and the backfill,

respectively.  $Z_w$  is the water depth between the rock and the backfill,  $\eta_h$  is the horizontal coefficient for blasting earthquake. By using this 3D wedge model, a corresponding limit state function will be set up for the purpose of reliability analysis in the following subsection.

### 2.3 Limit state function for the backfill wedge

By using the resistance force  $R$  and the driving force  $S$ , the factor of safety of the backfill wedge is set up as follows

$$F_s = R/S \quad (8)$$

where  $R$  and  $S$  can be found from Equations 1-7 as follows

$$R = \frac{CLW}{\cos \alpha} - \frac{1}{2} \gamma_f (2H - W \tan \alpha) WL (\cos \alpha - \eta_h \sin \alpha) - \frac{\gamma_w Z_w LW}{\cos \alpha} - \gamma_w Z_w^2 \sin \alpha \tan \varphi \quad (9)$$

$$S = \frac{1}{2} \gamma_f (2H - W \tan \alpha) WL (\sin \alpha - \eta_h \cos \alpha) + \frac{1}{2} \gamma_w Z_w^2 \cos \alpha - 2C_j \left( H - \frac{1}{2} W \tan \alpha \right) W + \frac{2WH_a C(I - \sin \varphi_j)}{\cos \varphi_j} \quad (10)$$

By substituting all the variable values into Equation 8, the factor of safety is found to be 2.17. This result can be confirmed by the results obtained from energy method and numerical method (Deng et al., 1999), which shows that the RS model in Equation 8 is correct. When  $F_s = 1$ , the backfill wedge is in a state of limit. Following the concept of reliability analysis (Deng et al., 2005), the limit state function can be given by

$$Z = R - S \quad (11)$$

If there is no water between the backfill and the surrounding rock, the limit state function is reduced to

$$R = \frac{CLW}{\cos \alpha} - \frac{1}{2} \gamma_f (2H - W \tan \alpha) WL (\cos \alpha - \eta_h \sin \alpha) \tan \varphi \quad (12)$$

$$S = \frac{1}{2} \gamma_f (2H - W \tan \alpha) WL (\sin \alpha - \eta_h \cos \alpha) - 2C_j \left( H - \frac{1}{2} W \tan \alpha \right) W + \frac{2WH_a C(I - \sin \varphi_j)}{\cos \varphi_j}. \quad (13)$$

## 3 Reliability analysis and design

Structural reliability methods deal with the statistical nature of basic variables in structural safety analysis and design (Deng et al., 2005). If a system is described by  $n$  state variables,  $X_1, X_2, \dots, X_n$ , and the limit state function is described as

$$Z = g(X_1, X_2, \dots, X_n) \quad (14)$$

Failure occurs when  $Z < 0$ . Therefore, the probability of failure,  $p_f$ , is given by the integral

$$p_f = \int \cdots \int f_X(X_1, X_2, \dots, X_n) dX_1 dX_2 \cdots dX_n \quad (15)$$

where  $f_X(X_1, X_2, \dots, X_n)$  is the joint probability density function for  $X_1, X_2, \dots, X_n$  and the integration is performed over the region in which  $g(X_1, X_2, \dots, X_n) < 0$ . The relationship between the probability of failure and the reliability index is  $p_f = \Phi(-\beta)$ , where  $\beta$  is the reliability index and  $\Phi$  is the cumulative distribution function for a standard normal variable.

The computation of  $p_f$  by Equation 15 is called the full distributional approach and can be considered to be the fundamental equation of structural reliability analysis. In general, the joint probability density function of random variables is practically impossible to obtain. Even if this function is available, the evaluation of the multiple integral is extremely complicated. Therefore, one possible approach is to use approximations of this integral that are simpler to compute. The Monte-Carlo simulation (MCS), the first-order-reliability-method

(FORM) and point estimate method (PEM) are three methods that have been widely used to estimate the failure probability. FORM is used in this paper because it can take full advantage of random variables' probability distributions as compared to PEM, and the calculation effort is small as compared to MCS. The details of FORM can be found in (Haldar and Mahadevan, 2000). The influences of the mean values, coefficients of variation, probability distribution types, and correlation between random variables were carefully investigated through sensitivity analysis.

### 3.1 Effect of variation

The effects of the variance of backfill cohesion ( $C$ ), backfill internal friction angle  $\varphi$ , and the horizontal seismic coefficient ( $\eta_h$ ) on the backfill wedge are considered, when the mean values are  $\mu_C = 0.6 \text{ MPa}$ ,  $\mu_{\tan \varphi} = 0.78$ , and  $\mu_{\eta_h} = 0.1$ . The variance of variables is represented by the coefficient of variation, of which the typical values are between 0.1 and 0.5. Figure 5 shows that the effect of variance of backfill cohesion is the largest. To obtain a higher reliability index of the backfill wedge, uniform backfill cohesion is suggested.

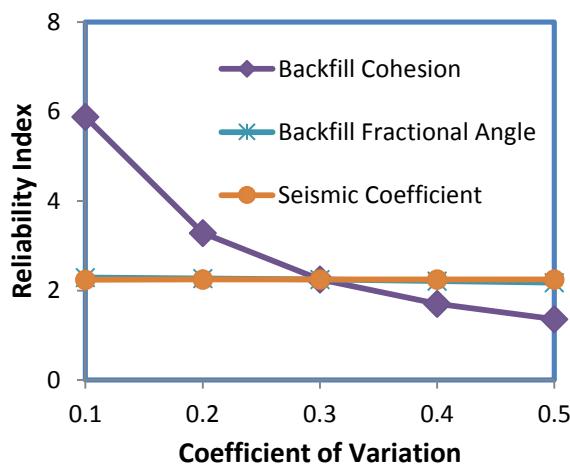


Figure 5 Effect of the variance of random variables

### 3.2 Effect of probability distribution of random variables

The influence of probability distribution of random variables is investigated: backfill cohesion ( $C$ ), backfill internal friction angle  $\varphi$ , and the horizontal seismic coefficient ( $\eta_h$ ). The mean values are  $\mu_C=0.6 \text{ MPa}$ ,  $\mu_\varphi=38^\circ$ ,  $\mu_{\eta_h}=0.25$ , and the coefficients of variation are taken as  $V_C=0.2$ ,  $V_{\tan \varphi}=0.2$ ,  $V_{\eta_h}=0.25$ . Table 2 shows

that the smallest reliability index comes from the case when all random variables are normal distribution. If all random variables are assumed normal distribution, the result appears to be conservative.

Table 2 Effect of probability distribution

Types of probability distribution	Reliability index	Failure probability
$C$ , $\varphi$ , and $\eta_h$ are Normal distribution	3.3713	0.3758E-3
$C$ , $\varphi$ , and $\eta_h$ are Lognormal distribution	3.7446	0.8842E-4
$C$ and $\varphi$ are Normal and $\eta_h$ is Lognormal distribution	3.3730	0.3735E-3

### 3.3 Effect of mean values of random variables

The effect of mean values of backfill cohesion ( $C$ ), backfill internal friction angle  $\varphi$ , and the horizontal seismic coefficient ( $\eta_h$ ) on the backfill wedge is considered, when the coefficient of variation are constant  $V_C=0.3$ ,  $V_{\tan \varphi}=0.3$ ,  $V_{\eta_h}=0.25$ . The change of mean values are (0.60~0.35) MPa for cohesion, (40~20) for friction angle, and (0.25~0) for seismic coefficient. Six cases are considered, which is uniformly distributed in their range,

respectively. Figure 6 illustrates that the effect of mean values of backfill cohesion is the largest. The next random variable is backfill internal friction angle, and the smallest effect comes from seismicity. If possible, cemented backfill will greatly increase the stability because the use of cement would increase the backfill cohesion.

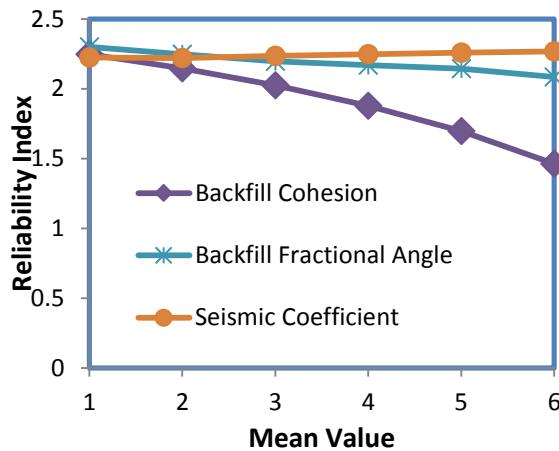


Figure 6 Effect of mean values of random variables

### 3.4 Effect of correlation among random variables

The effect of correlation between backfill cohesion ( $C$ ) and backfill internal friction angle  $\varphi$  is studied, when the coefficient of variation and mean values are constant:  $V_C = 0.2$ ,  $V_{\tan \varphi} = 0.2$ ,  $V_{\eta_h} = 0.25$  and  $\mu_C = 0.6 \text{ MPa}$ ,  $\mu_\varphi = 38^\circ$ ,  $\mu_{\eta_h} = 0.1$ . Both variables are assumed normal distributions. The results are shown in Table 3. When the correlation coefficient decreases from positive to negative values, the reliability increases. However, the increase of reliability index is relative small. As a consequence, the analysis results may be acceptable for practical engineering purpose even without consideration of correlation among random variables.

Table 3 Effect of correlation

Correlation coefficient $\mu_{C,\varphi}$	0.5	0.3	0.1	0	-0.1	-0.3	-0.5
Reliability index $\beta$	2.6911	2.8812	3.1253	3.3713	3.4879	3.7325	4.1248

## 4 Conclusion remarks

This paper introduces a blasthole stoping cut-and-fill mining method with delayed backfill. The details of an example in Southeastern China is presented. A three-dimensional wedge model was set up for the backfill and a corresponding limit state function was established to characterize the backfill stability. The parameters of backfill property were modelled as random variables, and the first-order-reliability-method was used to determine the failure probability and/or the reliability index of backfills. The influences of the mean values, coefficients of variation, probability distribution types, and correlation between random variables were carefully investigated through sensitivity analysis.

- The effect of variance of backfill cohesion is the largest. To obtain a higher reliability index of the backfill wedge, uniform backfill cohesion is suggested.
- If all random variables are assumed normal distribution, the result appears to be conservative.
- The effect of mean values of backfill cohesion is the largest. Cemented backfill will greatly increase the stability.
- The correlation among random variables may be neglected without introducing too large errors in reliability index.

These results give insights into the mechanism of backfill stability and could provide some clues on how to choose the right backfill materials.

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