

Required strength evaluation model and its application for vertically exposed backfill: inspired by Mitchell's physical model tests

GS Liu BGRIMM Technology Group, China

XC Yang BGRIMM Technology Group, China

A Pan Lassonde Institute of Mining, University of Toronto, Canada

LJ Guo BGRIMM Technology Group, China

L Li École Polytechnique de Montréal, Canada

Abstract

Cemented paste backfill is an increasingly popular technique to improve ground stability in underground mines. This technique is used in several mining methods that require strength evaluation for the vertically exposed cemented backfill following the excavation of an adjacent stope on one side. The critical strength is generally evaluated with an analytical solution proposed by Mitchell et al. (1982). Despite its wide acceptance in academia and application in the mining industry, the Mitchell solution has received only a few updates in the literature, including some new developments given by the authors and colleagues.

The original Mitchell solution and its variants were mainly validated against the physical model test results obtained by Mitchell et al. (1982). Even though the Mitchell model debatably assumed a zero backfill friction angle, the required strengths predicted by the Mitchell solution corresponded quite well to those obtained by physical model tests.

This study reanalysed the Mitchell solution and its physical model. The testing conditions and procedures for measuring the shear strength parameters are investigated. The stability of the cemented backfill upon removal of a confining wall is analysed with FLAC3D. The comparisons between the numerical modellings, experimental results and analytical solutions are presented, and the applicable range of the classical Mitchell solution is discussed.

A new analytical solution is proposed to evaluate the minimum required strength of the cemented backfill confined by two sidewalls exposed on one side and subject to pressure from uncemented backfill on the opposite side. The proposed analytical solution is validated by numerical simulations with FLAC3D.

The proposed analytical solution is used to determine the theoretical strength requirement of cemented backfill in primary stopes of an iron mine that employs stage stoping with subsequent backfill mining. The floating Factor of Safety (FS) characterising the current backfilling quality control level of this mine was statistically evaluated with a large amount of uniaxial compressive strength (UCS) data after testing vast drilled samples from field stopes. The engineered strength requirement of the cemented backfill in primary stopes had been finalised by combining the analytical results and floating FS of the mine.

Keywords: cemented backfill, required strength, Mitchell, physical model tests, FLAC3D

1 Introduction

Open stoping with delayed backfill is an increasingly popular technique in underground mines. The primary stopes are first mined out and filled with cemented backfill to form self-supporting structures, which must remain stable during the exploitation of the adjacent secondary stope(s). The cementation is mainly achieved through the addition of Portland cement and/or other binders. Backfill strength increases with

binder content, which results in increased cost. Therefore, it is critical to have an accurate understanding of the required strength of the cemented backfill (Darling 2011).

The simplicity and effectiveness of analytical models has promoted their use to estimate backfill strength requirements. Prior to the 1980s, the required strength for backfill in primary stopes were preliminarily determined through two methods: (1) the overburden solution ($\sigma_c = \gamma h$; where σ_c is uniaxial compressive strength and γ is bulk unit weight of backfill, h is depth from top surface of backfill); and (2) the plane strain slope solution ($\sigma_c = \gamma H/2$; where H is the overall height of the exposed backfill) (Duncan & Wright 2005). These two approaches neglected the three-dimensional characteristics and confining effects of side rock walls, which often resulted in uneconomical and overly conservative strength designs (Mitchell et al. 1982).

The confining effects of two side rock walls were addressed by Mitchell et al. (1982). The proposed analytical solution had been validated through physical model tests that evaluate the stability conditions of cemented backfill exposed on one side in the laboratory. The Mitchell method ($\sigma_c = 2c = \gamma/(1/H+1/L)$; where c is the cohesion and L is length of the exposed backfill) has been largely accepted in academia and in the industry (Arioglu 1984; Chen & Jiao 1991; Dirige et al. 2009; Zou & Nadarajah 2006). The application of this solution has led to significant economic benefits for the mining industry.

Li and colleagues revisited the Mitchell model and provided several updates that accounted for stope geometry, internal friction angle of the backfill, shear strengths along three confining walls, and failure mechanism of exposed backfill (Li 2014a, 2014b; Li & Aubertin 2012, 2014). These analytical solutions contributed to greater understanding and better evaluation of the backfill strength requirements.

However, in the Mitchell et al. (1982) solution, which was originally employed in the industry to estimate the strength requirement of cemented backfill cured for about 28 days, the internal friction angle of the backfill was debatably assumed to be zero. Under this assumption, the Mitchell analytical solution corresponded well to physical model test results. Besides, the backfill friction angle obtained by comparing the experimental results of Mitchell with the solutions of Li and Aubertin (2012, 2014) and Li (2014a) based on the curve fitting technique, is in the range of 12–17°, which is still far from the typical values of backfill's effective friction angle that is usually close to 30–35°. This led to the following questions: what are the governing principle and mechanics of backfill, and what kind of effect do they make on the application of the Mitchell method?

The Mitchell model and subsequent updated solutions were developed for cemented backfill with one open face associated with the excavation of one adjacent secondary stope. In practice, the first secondary stope is often filled with an uncemented backfill and the cemented backfill must be exposed on the opposite side due to the excavation of the adjacent second secondary stope. This case cannot be analysed by the original and modified Mitchell et al. (1982) solutions as they do not consider the pressure exerted by the uncemented backfill in the first secondary stope.

In this study, physical model tests performed by Mitchell et al. (1982) will be revisited and numerically simulated with FLAC3D to try to see if and under what conditions the physical model tests of Mitchell can be reproduced. The applicable conditions of the Mitchell solution are discussed. The stability of the cemented backfill in primary stopes exposed on one side and subjected to a lateral pressure on the opposite side by the uncemented backfill in a secondary stope is analysed. A new analytical solution is formulated and validated by numerical simulations. A case study illustrating the use of the new proposed solution is introduced to calculate the strength requirement of cemented backfill in primary stopes in an underground iron mine.

2 Analyses of Mitchell's physical model tests

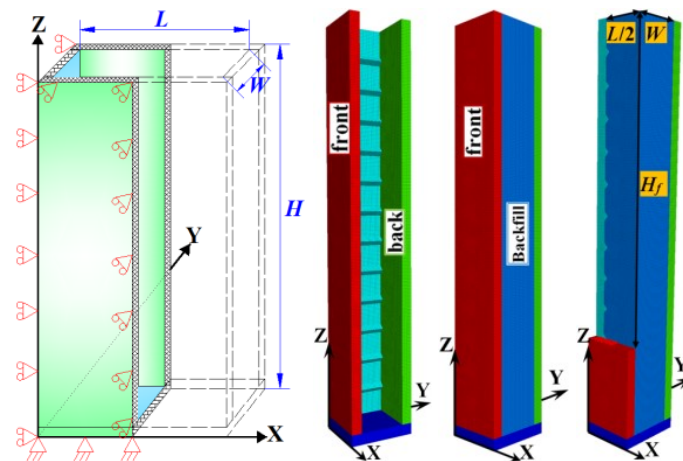
The Mitchell's limit equilibrium wedge failure model is validated through a series of physical box stability tests. These experiments evaluated the maximum stable exposed height, H_f , of cemented backfill for different block dimensions and strengths. Various tested data was obtained to illustrate the relationship

between the critical strength of vertically exposed backfill and the corresponding backfill dimension (length, width, height) with the analytical results.

Figure 1(a) shows backfill that has collapsed following exposure of a vertical face in one of the physical model tests in Mitchell et al. (1982). The width of the cemented backfill in the 'primary stope' varied from 0.2 m to 0.6 m, and the block length varied from 0.4 m to 0.8 m. The box was sealed with waterproof calking, rubber straps and plastic sheeting. The front wall of the box was constructed with 100 mm square section timbers. These sections are removed from the top towards the bottom to simulate the excavation of an adjacent secondary stope. Mitchell et al. (1982) used two types of cemented backfill, with tailings and clean, washed sand. Both had a cement content of 2.2% and the initial slurry density was 70%. The tailings were characterised by D_{10} around 20 μm and D_{60} around 80 μm , having a hydraulic conductivity around 10^{-7} m/s. The sand, which has a higher hydraulic conductivity than tailings, was characterised by D_{10} around 70 μm and D_{60} around 0.2 mm. For both cases, the addition of cement decreased the hydraulic conductivity of the cemented backfill.



(a)



(b)

Figure 1 (a) Typical views of collapsed backfill at critical exposure height H_c ; (b) schematic views of the physical model and the confining structures before and after backfilling and close to H_c

Once a physical model was constructed, the filling process started and was completed within about 30 minutes. Following the drainage of excess water on the top of the backfill slurry in the physical box, the valve of the drainage at the bottom of the model was closed to keep the fill in a saturated state. A thin layer of slimes was placed on the pour surface to eliminate seepage erosion. A plastic sheet was applied over the front wall to avoid the entrance of air to exposed backfill. It should be note that these measures decreased the drainage of the cemented backfill at the bottom of the model.

The physical study was used to find out the critical self-standing height (H_f) of different backfills based on laws of simulation for a reduced-scale physical model that can be built in a laboratory to about 2 m in height. This was achieved through lowering the strength of the cemented backfill and limiting the curing time to about 2.5 to 5 hours. Extra control samples were taken during each pour in the physical model to obtain the uniaxial compressive strength and shear strength of the cemented backfill by unconfined compression or direct shear tests at predetermined times. Pore water pressures were not measured during the unconfined compression and direct shear tests, which were realised very quickly (failure in about one minute for each control sample).

The backfill in the physical model tests of Mitchell et al. (1982) is shown to exhibit undrained behaviour. The internal friction angle (ϕ) is near to the undrained friction angle (ϕ_u) of 0, and the cohesion is probably the total cohesion (c) close to the undrained cohesion (c_u) – usually called undrained shear strength in soil mechanics), that is, $c \approx c_u$ and $\phi \approx \phi_u = 0$. These observations are consistent with Mitchell et al. (1982): “Two pieces of testing apparatus were developed to evaluate the short-term strength of control samples and of samples taken after the fill was exposed.”

The physical assumptions were numerically verified with FLAC3D. The numerical model analysed the short-term stability of exposed cemented backfill under an undrained condition with undrained shear strength parameters ($c = c_u$ and $\phi = \phi_u = 0$). Figure 1(b) shows schematically the physical model, and the confining structures before backfilling, after backfilling and close to critical exposure height H_f .

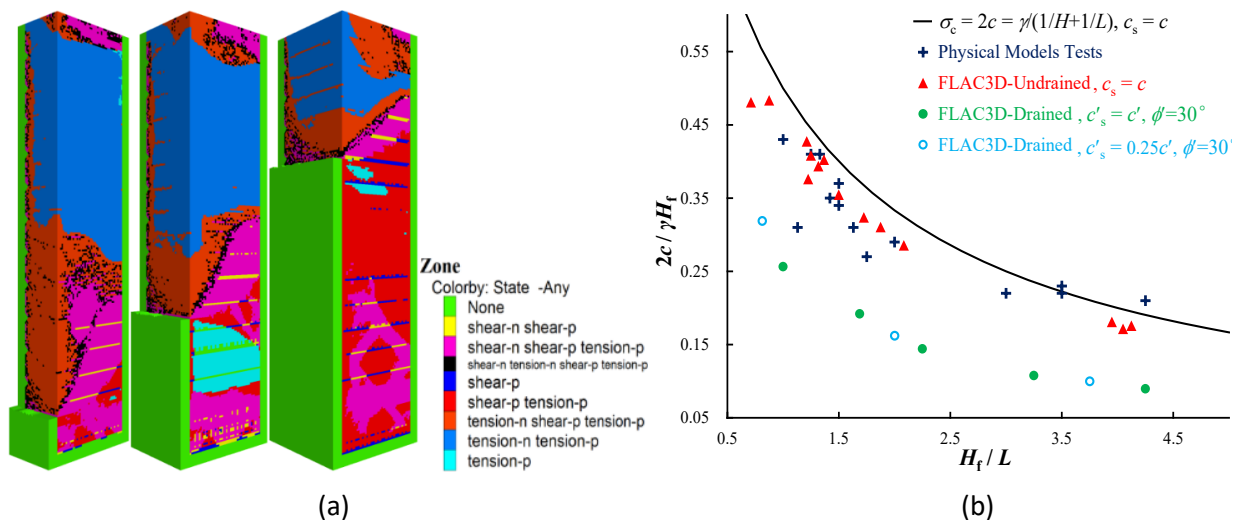


Figure 2 (a) Parts of numerical results of yield states of cemented backfill upon exposure to a critical height in different physical model tests: T25 at $H_f = 1.62$ m, S20 at $H_f = 1.25$ m, S7 at $H_f = 0.57$ m; (b) Variation of minimum required strength with the exposed height of backfill

For a physical model test, trial and error numerical modellings have to be done to obtain the critical height H_f of exposure at which the cemented backfill starts to collapse, beginning from the cohesions in tests given in Table 1. If the cemented backfill remained stable upon an exposure of the full height (1.8 m), these tests were simulated by using reduced cohesions and searching the critical exposed height. For a given case, a number of numerical simulations have to be made to find the critical strength. Figure 2(a) shows the yield states of the cemented backfill exposed to a critical height H_f when the backfill block dimensions change from 0.4 m x 0.2 m to 0.8 m x 0.4 m ($L \times W$). More details are given in Table 1.

Table 1 Variation of the critical height of backfill exposure H_f with different cohesion c

Test no.	Unit weight γ (kN/m ³)	Length L (m)	Width B (m)	Physical tests		Numerical results					
				c (kPa)	H_f (m)	Undrained		Drained			
						c (kPa)	H_f (m)	$c'_s = c'$		$c'_s = 0.25c'$	
								c' (kPa)	H_f (m)	c' (kPa)	H_f (m)
S1A	20.0	0.4	0.2	3.2	1.4	2.9	1.65	1.4	1.3	1.5	1.5
S4	19.7	0.4	0.2	2.2	0.6	2.2	0.69	1.5	1.7	–	–
S16	19.3	0.6	0.2	3.0	0.8	3.0	0.79	–	–	–	–
S17	19.6	0.6	0.2	3.0	0.9	3.0	0.75	–	–	–	–
S18	19.4	0.6	0.2	3.2	0.8	3.2	0.82	–	–	–	–
S13	19.5	0.8	0.2	3.3	0.8	3.3	0.70	2.0	0.8	–	–
S14	19.3	0.8	0.2	4.0	1.0	4.0	0.97	2.5	1.35	2.0	0.65
T25	18.0	0.4	0.4	2.9	1.4	2.5	1.62	–	–	–	–
T26	18.2	0.4	0.4	3.2	1.7	2.6	1.58	–	–	–	–
S20	18.5	0.6	0.4	3.6	1.8	3.3	1.25	1.8	1.35	1.8	1.2
S7	19.7	0.8	0.4	2.7	0.9	2.7	0.57	–	–	–	–
S8	18.8	0.8	0.4	4.0	1.3	4.0	1.20	–	–	–	–
T9	19.0	0.8	0.4	3.5	1.4	3.5	0.98	–	–	–	–
T11	18.9	0.8	0.4	4.4	1.6	4.4	1.50	–	–	–	–

The numerical results of the Mitchell's physical model tests are summarised in Table 1. The comparison of the required strengths between the numerical modelling and the physical model tests is shown in Figure 2(b), on which the required strength predicted by the Mitchell et al. (1982) analytical solution is also plotted. The numerical and analytical analysis showed a good fit with the experimental results, when the parameters of the backfill were set up to a short-term undrained shear strength, i.e. $c \approx c_u$ and $\phi \approx \phi_u = 0$. This suggests that physical models were in an undrained state.

The physical model tests conducted by Mitchell et al. (1982) were used to validate the analytical solution for the strength requirement of cemented backfill under fully drained conditions. The initial purpose was to reproduce the test results by numerical modelling with a consideration of the long-term stability of the exposed backfill under drained condition. Without success, the short-term stability of the exposed backfill under undrained condition was investigated, which demonstrated a good fit between the numerical modelling and experimental results.

Additional numerical modellings of the long-term stability of the exposed cemented backfill are shown in Table 1 and Figure 2(b). The friction angle (ϕ') of tailings or sand material was taken as 30° . The effective cohesion (c') of the backfill was based on measured results by Mitchell et al. (1982). The interface friction angle δ'_s along the side walls was assumed to be two-thirds of the friction angle of the backfill, that is, $\delta'_s = 2\phi'/3$. The interface cohesion along the two side walls were taken as $c'_s = c'$, and $c'_s = 0.25c'$ respectively, which considered the roughness of the side walls in the physical box (Figure 1(a)).

The required strengths of the cemented backfill in fully drained conditions obtained by numerical modelling are much smaller than those obtained by the physical model tests and predicted by the analytical solution of

Mitchell et al. (1982), shown in Table 1 and Figure 2(b). These results indicate that the exposed cemented backfill of the physical model tests by Mitchell et al. (1982) should not be considered as fully drained.

The strength parameters obtained and used in Mitchell et al. (1982) are mostly undrained shear strengths. The stability of the cemented backfill exposed in the physical model tests of Mitchell et al. (1982) was mostly controlled by the short-term strength under undrained condition, not the long-term strength under fully drained condition. Therefore, the Mitchell et al. (1982) analytical solution ($\sigma_c = 2c = \gamma/(1/H+1/L)$) is valid for estimating the required strength to ensure the short-term stability of a vertically exposed cemented backfill under an undrained condition. It is not valid for evaluating the long-term stability of exposed cemented backfill under a fully drained condition.

3 New analytical solution to evaluate backfill strength requirement

The investigation has demonstrated that the Mitchell analytical solution ($\sigma_c = 2c = \gamma/(1/H+1/L)$) is mostly valid for estimating the short-term strength requirement under undrained conditions. In practice, the cemented backfill in primary stopes is commonly designed to be exposed with the excavation of adjacent secondary stopes after 28 days of curing time at least.

As shown in Figure 3(a), the cemented backfill in the primary stope (1) is subjected to the pressure exerted on the back wall by the uncemented backfill (or fill slurry) placed in the first secondary stope (2). The cemented backfill must remain stable following the exposure of the opposite side with the excavation of the second secondary stope (3). This scenario cannot be analysed by the Mitchell et al. (1982) analytical solution and its modifications (Arioglu 1984; Chen & Jiao 1991; Dirige et al. 2009; Li 2014a, 2014b; Li & Aubertin 2012, 2014; Zou & Nadarajah 2006). These models do not account for the pressure exerted by the uncemented backfill in the first secondary stope (2). A new analytical solution is proposed to evaluate the backfill strength requirement in this case. The isotropic drawing of the primary stope (1) is shown in Figure 3(a) and the forces and pressures acting on the cemented backfill body are shown in Figure 3(b) with a static overburden pressure on the back wall of the vertically exposed cemented backfill.

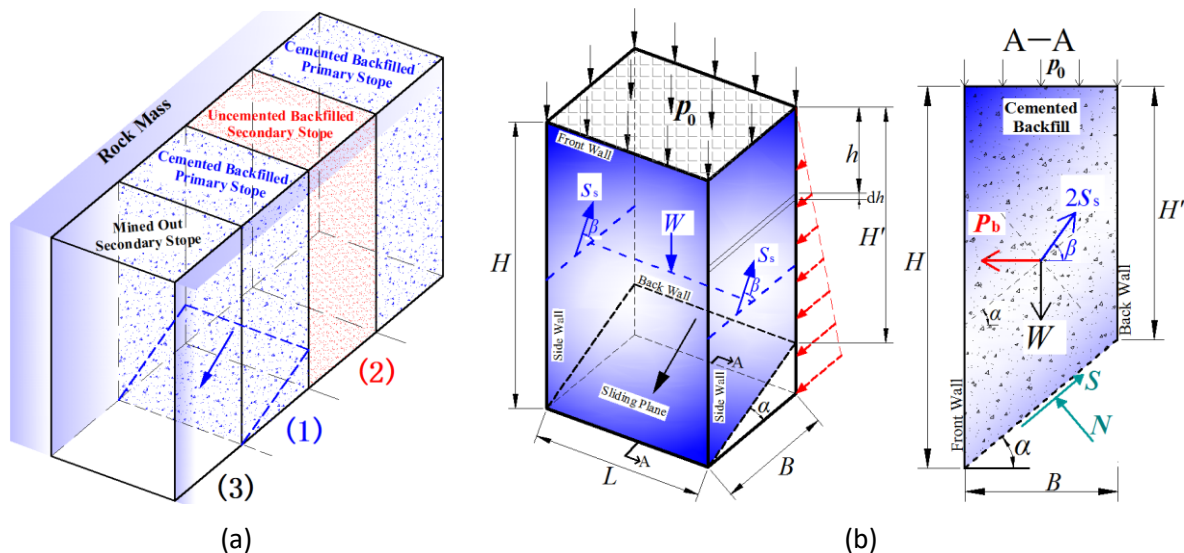


Figure 3 (a) A cemented backfill in the primary stope (1) exposed on one side due to the excavation of the second secondary stope (3) and pressured on the opposite side by the uncemented backfill placed in the first secondary stope (2); (b) An isolated presentation of the cemented backfill of the primary stope (1) with the various acting forces and pressures

The primary stope (1) has a high height-to-width aspect ratio and the sliding plane is shown to intercept the back wall ($H \geq B \tan \alpha$). The uncemented backfill slurry placed in the secondary stope (2) is considered in a slurry state with no shear strength. These conditions have been observed in several operations following the immediate placement of the backfill in stopes, and these uncemented backfills usually show a very slow

consolidation process due to the presence of a large quantity of fine grains (El Mkadmi et al. 2014; Thompson et al. 2012). The pressure exerted by the uncemented backfill slurry on the back wall of the cemented backfill is then equal to the isotropic pressure ($\gamma_u h$; γ_u (kN/m³), is the bulk unit weight of the uncemented backfill; h (m) is the depth). The resulting force P_b (kN) on the back wall of the sliding wedge can be expressed as follows:

$$P_b = \int_0^{H'} \gamma_u h L dh = \frac{1}{2} \gamma_u L (H - B \tan \alpha)^2 \quad (1)$$

where:

$H' (= H - B \tan \alpha; \text{m})$ = the height of the sliding wedge on the back wall.

The effective weight of the sliding wedge W' (kN) is obtained by considering the weight (W) of the sliding wedge and the load on the top surface of the backfill as follows:

$$W' = W + p_0 LB = (\gamma H^* + p_0) LB \quad (2)$$

where:

γ (kN/m³) = the bulk unit weight of the cemented backfill in the primary stope.

$H^* (= H - (B \tan \alpha)/2; \text{m})$ = the equivalent height of the sliding cemented backfill wedge.

The shear resistant force S_s (kN) along the side rock walls can be expressed as:

$$S_s = \int_0^{H'} \tau_s B dh + \int_{H'}^H \tau_s \frac{H-h}{\tan \alpha} dh \quad (3)$$

where:

h (m) = the depth from the top surface of the backfill.

τ_s = the shear strength along the interfaces between cemented backfill and side rock walls in primary stope and is calculated using

$$\tau_s = c_s + \sigma_h \tan \delta_s = r_s c + \sigma_h \tan (r_i \phi); \text{kPa.}$$

c (kPa) and ϕ (°) = the cohesion and friction angle of the cemented backfill, respectively.

r_s and r_i = the adherence and friction angle ratios of the fill-wall interfaces, respectively.

σ_h (kPa) = the horizontal stress normal to the interfaces between the cemented backfill and side rock walls at a depth h .

Horizontal stress can be obtained using a typical two-dimensional arching solution as:

$$\sigma_h = \frac{\gamma L}{2 \tan \delta_s} \left(1 - e^{\frac{-2Kh \tan \delta_s}{L}} \right) + K p_0 e^{\frac{-2Kh \tan \delta_s}{L}} \quad (4)$$

where:

K = earth pressure coefficient. For most cases within backfilled stopes, it takes a value close to the Rankine active earth pressure coefficient ($K = K_a = \tan^2 (45^\circ - \varphi/2)$).

Introducing Equation 4 to Equation 3 leads to the following equation for the shear force S_s :

$$S_s = BH^* r_s c + X \quad (5)$$

where:

$$X = \frac{LB}{2} \left[\gamma H^* - \left(\frac{\gamma L}{2K \tan \delta_s} - p_0 \right) \right] + \frac{L^2}{4K \tan \alpha \tan \delta_s} \left(\frac{\gamma L}{2K \tan \delta_s} - p_0 \right) \left(e^{\frac{-2K \tan \delta_s}{L} H^*} - e^{\frac{-2K \tan \delta_s}{L} H} \right) \quad (6)$$

Considering the equilibrium of the sliding wedge leads to the following expressions for the normal ($N = Y - 2S_s \sin(\beta - \alpha)$) and shear ($S = Z - 2S_s \cos(\beta - \alpha)$) forces on the sliding plane, the Factor of Safety (FS) of the sliding wedge can then be expressed as follows:

$$\begin{aligned} FS &= \frac{\text{Resisting forces}}{\text{Driving forces}} = \frac{c \times (\text{area of the sliding plane}) + N \tan \phi}{S} \\ &= \frac{c \times L(B / \cos \alpha) + [Y - 2S_s \sin(\beta - \alpha)] \tan \phi}{Z - 2S_s \cos(\beta - \alpha)} \end{aligned} \quad (7)$$

where:

$$Y = W' \cos \alpha - P_b \sin \alpha$$

$$Z = W' \sin \alpha + P_b \cos \alpha$$

This results in a general solution describing the stability of the cemented backfill in the primary stope (1). The FS depends on the angle values α and β .

In previous solutions (Arioglu 1984; Chen & Jiao 1991; Dirige et al. 2009; Li 2014a, 2014b; Li & Aubertin 2012, 2014; Mitchell et al. 1982; Zou & Nadarajah 2006), the angle α was determined as $\alpha = 45^\circ + \phi/2$ based on an active state. This has been partly verified by physical model tests (Mitchell et al. 1982). The angle β of the shear force, S_s , has been postulated to be $\beta = 90^\circ$ in the Mitchell et al. (1982) and extension models. Li and Aubertin (2014) performed preliminary numerical analyses on the movement of exposed backfill. They found that the sliding wedge could be divided into two zones: the upper zone with $\beta = 90^\circ$ and the lower zone with $\beta \approx 45^\circ + \phi/2$.

These analyses were based on the exposed cemented backfill confined by three rock walls. The two angles α and β can be expected to change more or less compared to the Mitchell et al. (1982) because the proposed extension models with a lateral pressure have been applied on the back wall of the cemented backfill. Therefore, four possible cases are considered.

3.1 Model 1, $\alpha = 45^\circ + \phi/2$ and $\beta = 90^\circ$

Introducing $\alpha = 45^\circ + \phi/2$ and $\beta = 90^\circ$ into Equation 7 leads to the following expression for the required backfill cohesion c :

$$c = \frac{FS \cdot Z - Y \tan \phi - 2X(FS \cdot \sin \alpha - \cos \alpha \tan \phi)}{LB / \cos \alpha + 2BH^* r_s (FS \cdot \sin \alpha - \cos \alpha \tan \phi)} \quad (8)$$

3.2 Model 2, $\alpha = \beta = 45^\circ + \phi/2$

Introducing $\alpha = \beta = 45^\circ + \phi/2$ into Equation 7 leads to the following expression for the required backfill cohesion c :

$$c = \frac{FS \cdot (Z - 2X) - Y \tan \phi}{LB / \cos \alpha + 2 \cdot FS \cdot BH^* r_s} \quad (9)$$

3.3 Model 3, $\alpha = \beta$

This model considers that the sliding plane and direction of the sliding wedge are parallel in any direction ($90^\circ \geq \alpha = \beta \geq 0^\circ$). The required cohesion, c , has the same expression with Equation 9. The critical angle $\alpha (= \beta)$ can be obtained from the partial derivative to the function FS (Equation 7) with respect to the variable α .

By imposing $\partial FS / \partial \alpha = 0$, the critical angle $\alpha (= \beta)$ can readily be calculated with Microsoft Excel[®]. The required cohesion can then be calculated with Equation 9.

$$\frac{\partial FS}{\partial \alpha} = \frac{\left[c + \gamma_u H' \tan \phi - \gamma B \tan \phi / (2 \tan \alpha) \right] LB \sin \alpha / \cos^2 \alpha - Z \tan \phi}{Z - 2S_s} - \frac{cLB / \cos \alpha + Y \tan \phi}{(Z - 2S_s)^2} \times \left\{ \frac{B \left[2r_s cB - 2\gamma_u LH' \cos \alpha + \gamma LB(1 - \sin \alpha) \right]}{2 \cos^2 \alpha} + Y \right. \\ \left. + \frac{L^2}{2K \tan \delta_s} \left(\frac{\gamma L}{2K \tan \delta_s} - p_0 \right) \left[\left(\frac{1}{\sin^2 \alpha} - \frac{4KB \tan \delta_s}{L \sin 2\alpha} \right) e^{\frac{-2K \tan \delta_s}{L} H'} - \frac{1}{\sin^2 \alpha} e^{\frac{-2K \tan \delta_s}{L} H} \right] \right\} \quad (10)$$

3.4 Model 4 (proposed solution), $\alpha = 45^\circ + \phi/2$, $90^\circ \geq \beta \geq 0^\circ$

This model considers that the sliding plane is not affected by the lateral pressure ($\alpha = 45^\circ + \phi/2$), but the sliding tendency can be in any direction ($90^\circ \geq \beta \geq 0^\circ$). The FS is expressed by Equation 7. Performing partial derivative to the function FS with respect to the variable β and solving the equation $\partial FS / \partial \beta = 0$ lead to the following equation for the critical angle β :

$$\beta = \alpha + \arcsin \frac{2S_s U - Z \sqrt{U^2 - 4S_s^2 + Z^2}}{Z^2 + U^2} \quad (11)$$

where:

$$U = Y + cLB / (\cos \alpha \tan \phi).$$

The minimum required backfill cohesion is determined through an iterative process involving the critical angle β given by Equation 11 and involves cohesion c , Equation 7. The critical angle, β , is taken as $\beta = 45^\circ - \phi/2$, i.e. $\alpha - \beta = \phi$ for typical slope geometry and material properties.

Then, Equation 7 leads to the following expression for the stability of the cemented backfill in the primary slope (1):

$$FS = \frac{cLB / \cos \alpha + (Y + 2S_s \sin \phi) \tan \phi}{Z - 2S_s \cos \phi} \quad (12)$$

Considering $FS = 1$ in Equation 12 leads to the minimum required cohesion c as follows:

$$c = \frac{Z \cos \phi - Y \sin \phi - 2X}{LB \frac{\cos \phi}{\cos \alpha} + 2BH^* r_s} \quad (13)$$

Equations 12 and 13 constitute the proposed solution to evaluate the stability and required cohesion of the exposed backfill based on the numerical modelling presented herein.

Numerical simulations with different slope geometries and backfill properties were carried out to determine the most reasonable model for calculation of backfill strength requirement from the aforementioned four models. Figure 4(a) illustrates half of a physical model of the cemented backfill

(Stope 1) with its front wall exposed (Stope 3) and back wall in contact with an uncemented backfill slurry (Stope 2), and the corresponding numerical model constructed with FLAC3D.

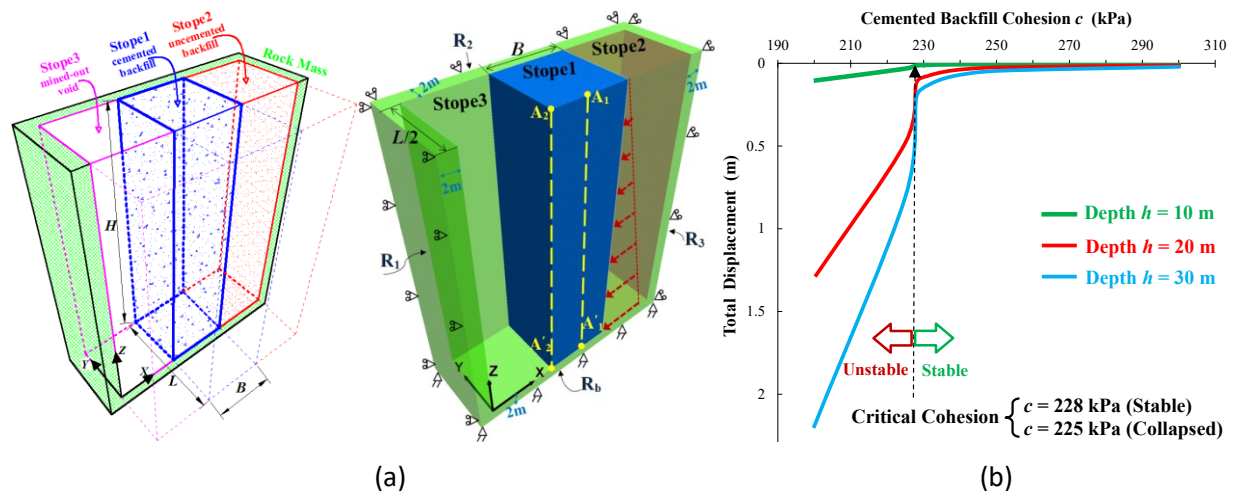


Figure 4 (a) A physical model of exposed cemented backfill in a primary stope and the corresponding numerical model; (b) Total displacement variations at three depths ($h = 10, 20, 30$ m) along the monitoring line $A_2A'_2$ with different backfill cohesion

Figure 4(b) illustrates the variation in the total displacements at three depths $h = 10, 20, 30$ m along the monitoring line $A_2A'_2$ with different backfill cohesions after the excavation of Stope 3 (Figure 4(a)). The simulations were made with length, L , of 10 m, width, B , of 5 m, height, H , of 40 m, angle of frictional resistance, ϕ , set equal to interface shear coefficient, δ_s , of 33° , and $r_s (= c_s/c) = 0.5$. The displacements are everywhere very small with $c = 300$ kPa, which indicates exposed backfill is in a stable state. When the backfill cohesion decreases from 300 to 230 kPa, the displacements at the three monitoring points show very small and progressive increments. When the backfill cohesion further decreases from 230 to 200 kPa, a jump of the displacements takes place at cohesion around 228 kPa. The large displacements at cohesion less than 228 kPa indicate the collapse of the exposed backfill. Therefore, the critical cohesion of the exposed cemented backfill is determined as $c = 228$ kPa. This procedure based on displacement monitoring for determining the critical strength of exposed backfill will be repeated for all the other simulations.

Figure 5 shows the variation in the critical cohesions obtained by numerical modelling and predicted by four analytical solutions with different stope width B (Figure 5(a), $L = 10$ m, $H = 40$ m), length L (Figure 5(b), $B = 10$ m, $H = 40$ m), height H (Figure 5(c), $L = 10$ m, $B = 8$ m), and backfill friction angle ϕ (Figure 5(d), $L = 20$ m, $B = 10$ m, $H = 40$ m). Other parameters are $\gamma = \gamma_u = 21$ kN/m³, $\phi = 33^\circ$, $r_s = 0.5$, $r_i = 1$ and $p_0 = 0$ kPa. In all cases, it is seen that the minimum required strengths (that is, $FS = 1$) predicted by the solution of Model 4 are the closest to the critical cohesions obtained by the numerical modelling. The equations given in Model 4 (Equation 13) are recommended as the proposed analytical solution for evaluating the minimum required cohesion of the vertically exposed cemented backfill pressured on the back wall.

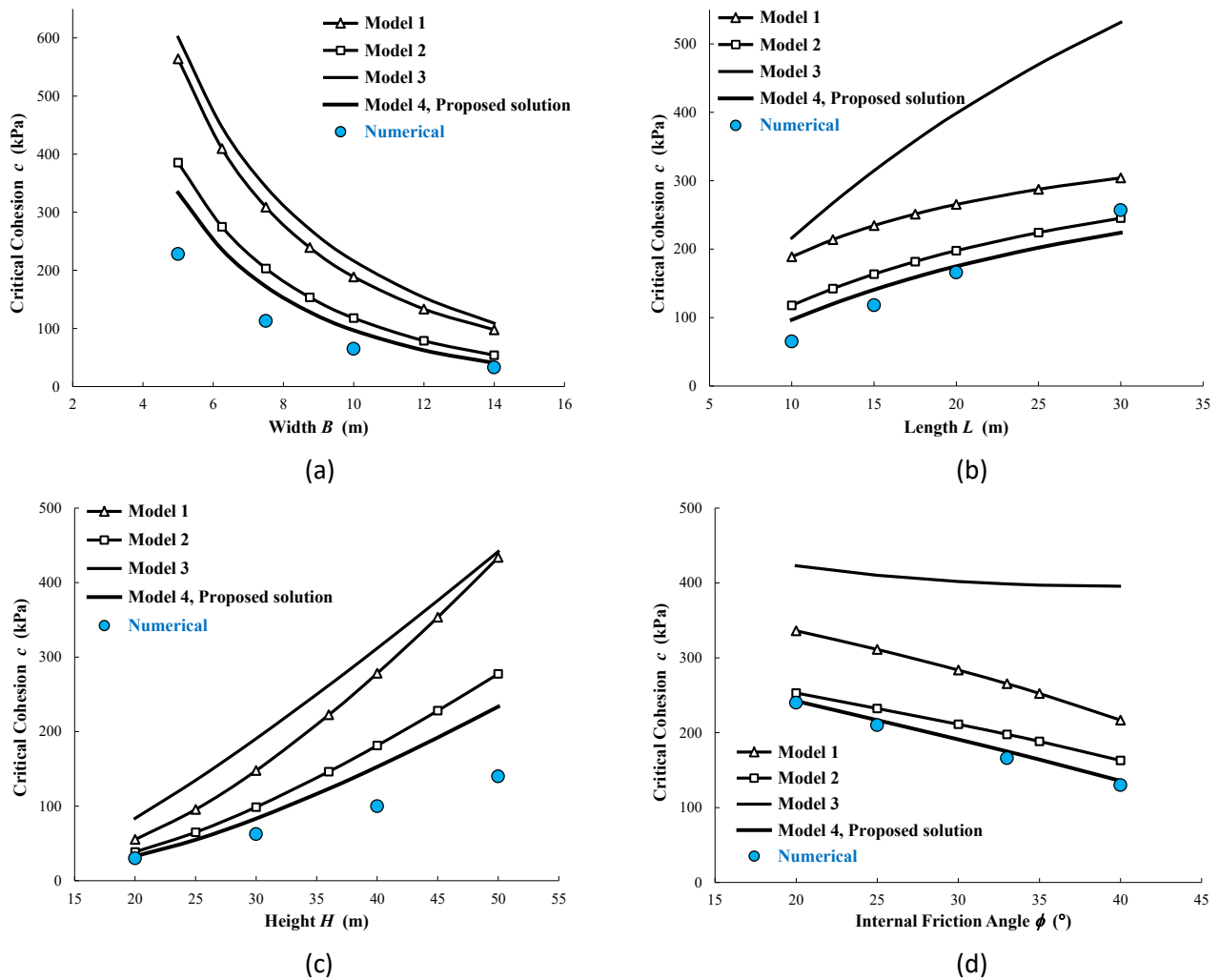


Figure 5 Variation of the critical cohesions obtained by numerical modelling and of the minimum required cohesion predicted by the four analytical solutions with different slope: (a) Width B ; (b) Length L ; (c) Height H ; (d) Friction angle of the cemented backfill (ϕ)

4 Application for backfill strength design in an iron mine

The proposed method has been applied in a mine in Anhui Province, China. Staged open stoping with subsequent backfill mining is the main method used to excavate the thick iron orebody. Cemented backfill made with unclassified tailings and cement is used to fill the primary stopes, while the secondary stopes are usually filled with unclassified tailings without binders (or very few). The previous cemented backfill strength requirement in primary stopes was determined by empirical analogy of the adjacent mines in Anhui Province, without considering the practical stope geometries and backfill properties of this mine, and neglecting the differences of backfill quality control levels in different mines. The over-conservative design for the strength requirement of the cemented backfill in primary stopes leads to a waste of binder, which increases the unnecessary cementing costs.

The designed stope geometries of the primary and secondary stopes in this mine are as follows: stope width B is kept as 18 m, the length L of the stope is the thickness of the orebody, which varies from 30 to 70 m, and the common height H of the exposed cemented backfill is about 60 m. The unit weight of the cemented backfill γ in primary stopes is around 21 kN/m^3 , the unit weight of the uncemented backfill (or slurry) γ_u in secondary stopes is around 20 kN/m^3 , and the internal friction angle of the cemented backfill is around $\phi = 33^\circ$. The parameters are set to the values of $r_s = r_i = 1$ for the adherence and friction angle ratios of the very rough fill-wall interfaces and no surcharge on the backfill top surface $p_0 = 0 \text{ kPa}$.

The theoretical strength requirement of cemented backfill is obtained through introducing these values to Equation 13. The required cohesion, c_{\min} , for minimum slope length, L_{\min} , of 30 m is 0.122 MPa and the required cohesion, c_{\max} , for maximum slope length L_{\max} of 70 m is 0.228 MPa under FS of 1.0. A further assumption that the cemented backfill follows the linear Mohr–Coulomb criterion leads to a conversion formula between uniaxial compressive strength, σ_c , with the cohesion, c , and internal friction angle, ϕ , that is, $\sigma_c = 2c \times \cos\phi/(1-\sin\phi)$. The required strength of the cemented backfill, $\sigma_{c \min}$, for the minimum slope length, L_{\min} , of 30 m would be 0.450 MPa and the required strength of the cement backfill, $\sigma_{c \max}$, for the maximum slope length L_{\max} of 70 m would be 0.839 MPa under FS of 1.0.

The theoretical strength requirement of cemented backfill in primary stopes under *factor safety* of 1.0 has been calculated in the iron mine from 0.450 MPa to 0.839 MPa. However, in the practical mine backfill, the strength values of the backfill body in different parts of the field stopes are not uniform, which is caused by the settlement, consolidation, segregation and stratification of a complex process of evolution from slurry to backfill body. The quality control of the field strength in practical stopes usually reflects the degree of the mine filling technology. In this case, a large number of backfill samples had been drilled from the field stopes and the uniaxial compressive tests had been carried out to form a statistical sample including 1,684 pieces of uniaxial compressive strength (UCS) values. These samples basically obey the normal distributions as shown in Figure 6.

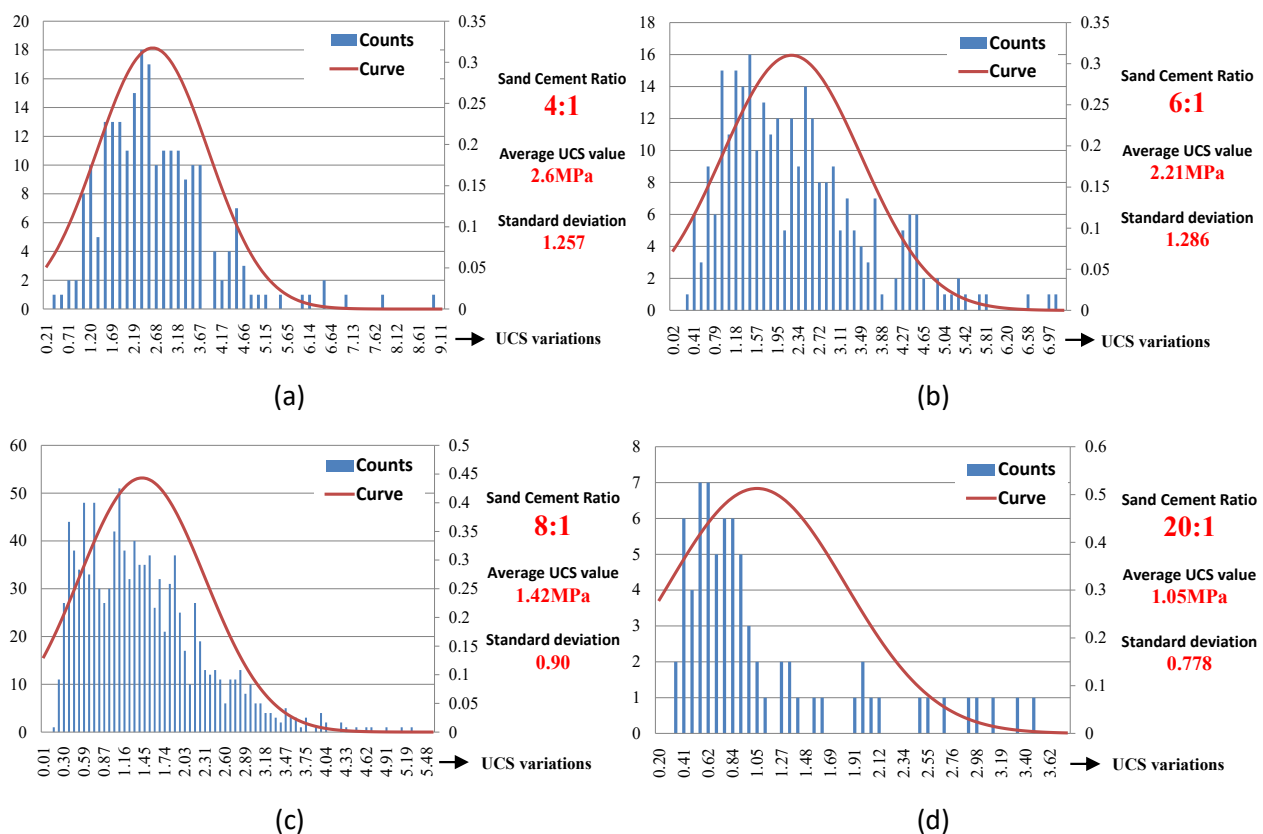


Figure 6 Normal distributions for UCS values of drilled backfill samples with different sand–cement ratios

Figure 6 shows the normal distributions for UCS values of drilled backfill samples with different sand–cement ratios (4:1, 6:1, 8:1 and 20:1) in this iron mine. In every normal distribution of the backfill UCS data for each sand–cement ratio, a floating FS is calculated by comparing the differences between the average UCS value and two times the standard deviation, which would cover 95.5% of the statistical data. As is known in the engineering field, a piece of data in the big sample will be called an ‘outlier’ if the deviation of this data from the average is more than two times the standard deviation. After calculating the FS of each normal distribution of each sample corresponding to different sand–cement ratios, a finalised floating FS characterising the current backfilling quality control level of this mine was further calculated with the

method of weighted mean. The floating FS of this iron mine was determined to $FS = 2.21$ after the statistical calculation. Then the strength requirement of the cemented backfill in primary stopes of the iron mine was obtained ($\sigma_c = 0.995$ MPa for the minimum stope length $L_{\min} = 30$ m and $\sigma_c = 1.854$ MPa for the maximum stope length $L_{\max} = 70$ m, with constant values for stope width $B = 18$ m and height $H = 60$ m).

5 Conclusion

The strength parameters obtained and used in the Mitchell et al. (1982) solution, including the backfill in their physical model tests, are mostly undrained shear conditions. The classical Mitchell et al. (1982) analytical solution ($\sigma_c = 2c = \gamma/(1/H+1/L)$) is valid for estimating the required strength to ensure short-term stability of a vertically exposed cemented backfill under an undrained condition.

The new proposed analytical model with the sliding plane making an angle $\alpha = 45^\circ + \phi/2$ and sliding direction making an angle $\beta = 45^\circ - \phi/2$ is the most appropriate solution for cemented backfill design to estimate the minimum required strength of vertically exposed cemented backfill next to a secondary stope filled with an uncemented (or low cemented) paste fill.

The new proposed analytical solution has been successfully applied to calculate the theoretical strength requirement of cemented backfill in primary stopes under $FS = 1.0$ in an iron mine ($\sigma_c = 0.450$ – 0.839 MPa). Combined with the backfill samples and drilling and testing from field stopes, a reasonable floating FS characterising the current backfilling quality control level of this mine was statistically evaluated ($FS = 2.21$). The engineered strength requirement of cemented backfill in primary stopes of this mine had been finalised ($\sigma_c = 0.995$ – 1.854 MPa for varied orebody thickness 30–70 m) by combining the analytical results and the floating FS, which led to a safe and economical backfill strength design.

Acknowledgement

Financial support from the National Natural Science Foundation of China (grant no. 51774040, 51804031) and the Youth Science and Technology Innovation Fund of BGRIMM Technology Group (grant no. QCJ201804) is gratefully acknowledged. The authors gratefully thank Professor Robert Mitchell for his excellent pioneering works and permission to use part of his published materials.

References

- Arioglu, E 1984, 'Design aspects of cemented aggregate fill mixes for tungsten stoping operations', *Mining Science and Technology*, vol. 1, no. 3, pp. 209–214.
- Chen, LJ & Jiao, DY 1991, 'A design procedure for cemented fill for open stoping operations', *Mining Science and Technology*, vol. 12, no. 3, pp. 333–343.
- Darling, P 2011, *SME Mining Engineering Handbook*, 3rd edn, Society for Mining Metallurgy & Exploration Inc., Littleton.
- Dirige, APE, McNearny, RL & Thompson, DS 2009, 'The effect of stope inclination and wall rock roughness on back-fill free face stability', in M Diederichs & G Grasselli (eds), *Rock Engineering in Difficult Conditions, Proceedings of the 3rd Canada-US Rock Mechanics Symposium*, Toronto, paper 4152.
- Duncan, JM & Wright, SG 2005, *Soil Strength and Slope Stability*, John Wiley & Sons, Hoboken.
- El Mkadmi, N, Aubertin, M & Li, L 2014, 'Effect of drainage and sequential filling on the behaviour of backfill in mine stopes', *Canadian Geotechnical Journal*, vol. 51, no. 1, pp. 1–15.
- Li, L 2014a, 'Generalized solution for mining backfill design', *International Journal of Geomechanics*, vol. 14, no. 3.
- Li, L 2014b, 'Analytical solution for determining the required strength of a side-exposed mine backfill containing a plug', *Canadian Geotechnical Journal*, vol. 51, no. 5, pp. 508–519.
- Li, L & Aubertin, M 2012, 'A modified solution to assess the required strength of exposed backfill in mine stopes', *Canadian Geotechnical Journal*, vol. 49, no. 8, pp. 994–1002.
- Li, L & Aubertin, M 2014, 'An improved method to assess the required strength of cemented backfill in underground stopes with an open face', *International Journal of Mining Science and Technology*, vol. 24, no. 4, pp. 549–558.
- Mitchell, RJ, Olsen, RS & Smith, JD 1982, 'Model studies on cemented tailings used in mine backfill', *Canadian Geotechnical Journal*, vol. 19, no. 1, pp. 14–28.
- Thompson, BD, Bawden, WF & Grabinsky, MW 2012, 'In situ measurements of cemented paste backfill at the Çayeli Mine', *Canadian Geotechnical Journal*, vol. 49, no. 7, pp. 755–772.
- Zou, S & Nadarajah, N 2006, 'Optimizing backfill design for ground support and cost saving', *Golden Rocks 2006, Proceedings of the 41st US Symposium on Rock Mechanics (USRMS)*, American Rock Mechanics Association, Alexandria, pp. 7–21.

