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Evaluation of a non-Newtonian two-layer model for high concentration suspensions

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Abstract

High concentration thickened tailings slurries that appear to be homogeneous mixtures often contain coarse particles that settle in the pipeline under laminar flow conditions. During pipeline transport, these coarse particles may eventually settle to the pipe invert. Frequently, these high concentration suspensions are misclassified as homogeneous slurries, leading to the use of incorrect models for predicting the pressure gradient and flow behaviour.

This paper discusses the use of a non-Newtonian two-layer model to predict the pressure gradient of a high concentration suspension with a sliding bed in laminar flow conditions. The success of the model is measured by comparing the results obtained by applying the model to experimental results for a typical iron ore tailings slurry. It was found that the model predicted the laminar flow pressure gradient with less than 10% error for slurries with carrier fluid yield stresses above 10 Pa.

Keywords: *high concentration suspensions, laminar, pressure gradient, two-layer, pipe flow*

1 Introduction

Tailings slurries frequently comprise relatively wide particle size distributions. The concept of the 'fine' particle fraction combining with the fluid (water) to form a homogeneous carrier fluid in which the 'coarse' particles can be considered to be conveyed has been well described by Sellgren and Wilson (2007).

In the case of thickened and paste tailings, pipeline systems conveying these slurries may operate in the laminar flow regime due to the high laminar–turbulent transition velocity associated with viscous thickened/paste tailings properties. Many authors have found that although these slurries appear to be homogeneous and non-segregating under static or unsheared conditions, segregation of coarse particles and stratified flow occurs under laminar flow conditions (Pullum & Graham 2000; Cooke 2002; Thomas et al. 2004).

When these slurries are sheared (as when flowing in a pipeline), the structures and bonds associated with the yield stress are overcome or broken and the support that the yield stress provides to coarse particles under static conditions is lost. It is seen in concentration profiles for laminar flow conditions that coarse particles quickly migrate to the bottom of the pipe forming a coarse particle lower (sliding bed) layer (Pullum & Graham 2000; Graham et al. 2002). In the case of laminar flow conditions (as opposed to turbulent flow), there is no mechanism to re-suspend coarse particles that have settled to the bed layer. This sets up a two-layer, laminar flow geometry in the pipe, with a lower layer of coarse solids and a particle-lean upper layer.

This paper presents an evaluation of a laminar flow two-layer model against pipe loop pressure gradient data for an iron ore tailings slurry comprising a non-Newtonian yield stress carrier and a coarse particle fraction. In particular, the analysis assumes a 'gelled bed' condition as described by Talmon and Mastbergen (2004). It is found that there is good agreement between the model and the pipe loop data.

2 Two-layer laminar flow model theory

The two-layer laminar flow model as described by Pullum et al. (2004) and Talmon et al. (2014) applies the two-layer concept as originally applied by Wilson (1976) for turbulent two-layer flow of coarse solids in a Newtonian fluid (typically sands or gravels in water). For application to laminar flow of slurries with a non-Newtonian, yield stress carrier fluid and containing a coarse (settling) particle component, the two-layer model assumes all solids that are not part of the carrier fluid are transported in a lower sliding bed layer with an upper, particle-lean layer flowing above. The carrier fluid is assumed to be formed by the mixture of the liquid phase (water) plus fine particles. In this analysis, the division between fine and coarse particles is made at 45 µm.

The two-layer model geometry, and the forces acting at the interfaces of the layers, are shown in Figure 1.

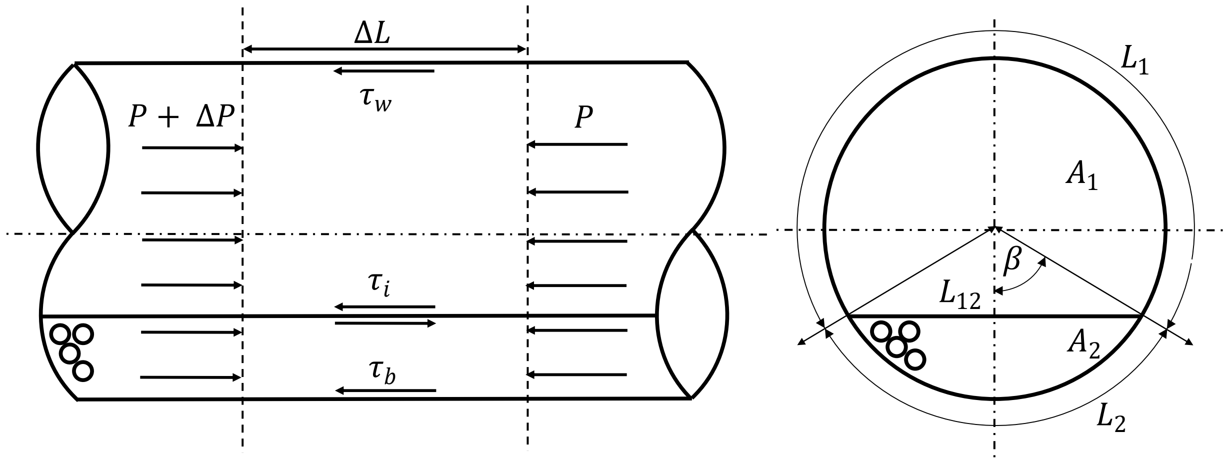

Figure 1 Two-layer geometry and shear stresses acting on the layers

In Figure 1, A1 is the cross-sectional area of the upper layer and A2 is the cross-sectional area of the lower sliding bed layer. β is the half-angle defining the interface between the upper and lower layers. The force balances applied to the upper and lower layers respectively are set out in Equations 1 and 2.

$$\Delta P A_1 = \tau_w D(\pi - \beta)\Delta L + \tau_i D \sin(\beta)\Delta L \quad (1)$$

$$\Delta P A_2 + \tau_i D \sin(\beta)\Delta L = \tau_b D \beta \Delta L \quad (2)$$

where:

ΔP = differential pressure over length ΔL.

τ_w = shear stress acting at the interface between the upper layer and the pipe wall.

τ_i = shear stress acting at the interface between the two layers.

τ_b = shear stress acting at the interface between the lower layer and the pipe wall.

It should be noted that in this analysis, the shear stress at the lower layer pipe wall interface is treated as generally described by Talmon et al. (2014). That is, the bed layer exhibits gelled bed behaviour and solids sliding friction at the bed-pipe interface is negligible. The behaviour and properties of the two layers and in particular the gelled bed lower layer is discussed further in this paper, under separate headings.

2.1 Upper layer analysis

The wall shear stress for the flow in the upper layer is determined considering the measured viscous properties of the carrier fluid and applying the Buckingham equation using an equivalent pipe diameter for the upper layer.

The interfacial shear stress between the upper and lower layers is determined in the same way, but with the flow velocity treated as the velocity difference between the two layers.

2.2 Lower layer analysis – gelled bed behaviour

Sliding friction between the bed layer and the pipe wall is a key component of the original (Wilson 1976) two-layer model, and also in the application of the two-layer model for laminar flow in a Newtonian carrier fluid (Gillies et al. 1999).

For the non-Newtonian yield stress fluid two-layer laminar flow analysis presented here, we assume gelled bed behaviour for the bed layer. In this condition, the packing concentration of the bed is lower than that of a closed-packed, granular bed, with separation of the particles maintained by the yield stress in the bed layer. Two possible conditions for the settled bed are described by Talmon et al. (2014):

1. A bed with particles supported by the yield stress of the interstitial fluid is referred to as a 'gelled bed'.
2. The bed with particles in direct interparticle contact is referred to as a 'granular bed'.

In the gelled bed condition, the weight of the coarse particles is not transferred through intergranular contact to the pipe wall. The significance of this for the two-layer analysis is that this eliminates solids sliding friction between the bed and the pipe wall. The shear stress between the bed layer and pipe wall is evaluated considering only the viscous properties of the bed layer.

The shear stress between the bed layer and the pipe wall is calculated considering the viscous properties of the bed (yield stress and plastic viscosity modified by the coarse particles as described below). Specifically, the wall shear stress is approximated by the value calculated for the full pipe cross-section occupied with a homogeneous mixture with rheology matching the bed layer rheology.

Thomas (1999) has presented correlations for estimating how the presence of coarse particles modify the yield stress and plastic viscosity of the carrier fluid, shown in Equations 3 and 4. These correlations are used to estimate the bed layer yield stress and plastic viscosity. A value of 62% is applied for the maximum bed packing concentration of the coarse solids (this is a reasonable and generally accepted value for relatively narrowly graded partially angular particles). A value of 35% was assumed for the packing concentration of coarse solids in the gelled bed. Good agreement between model and pipe loop data (see under 'Results') appears to support this assumption.

$$\frac{\tau_{y(coarse+fine)}}{\tau_{y(fines)}} = \left(1 - \frac{C_{v\,bed}}{k\,C_{v\,max}}\right)^{-2.5} \tag{3}$$

$$\frac{K_{(coarse+fine)}}{K_{(fines)}} = \left(1 - \frac{C_{v\,bed}}{k\,C_{v\,max}}\right)^{-2.5} \tag{4}$$

where:

$\tau_{y\,(coarse\,+\,fines)}$	=	predicted bed layer yield stress.
$\tau_{y\,(fines)}$	=	measured carrier fluid yield stress.
$K_{(coarse\,+\,fines)}$	=	predicted bed layer plastic viscosity.
$K_{(fines)}$	=	measured carrier fluid plastic viscosity.
$C_{v\,bed}$	=	coarse solids packing concentration (by volume) in the bed layer.
$C_{v\,max}$	=	coarse solids maximum packing concentration (by volume).
k	=	correlating parameter, with value nominally 1.5.

2.3 Two-layer model solution

The pipeline friction pressure gradient (ΔP/ΔL in Equations 1 and 2) is determined by solving for β to satisfy the force balances in Equations 1 and 2 and volume flow continuity of the carrier fluid (liquid plus fines) and coarse solids.

3 Materials and methods

Tests were conducted in a DN 100 pipe loop in the Paterson & Cooke Slurry Test Facility in Cape Town. Data on the laminar flow pipeline friction pressure gradient was collected to compare with the model prediction. Figure 2 and Table 1 show details of the pipe loop. The system consists of:

- Centrifugal slurry pump controlled using a variable speed drive.
- Differential pressure transducer and static pressure tappings used to measure the differential pressure over a 4 m length.
- Viewing section of clear PVC pipe with inner diameter matched to the pressure measurement pipe.
- Magnetic flow meter to measure the slurry flow rate. The flow meter is installed in vertical piping to minimise the possible influence of asymmetrical velocity and concentration profiles.
- Sampling point just downstream of the pump from where samples are drawn for particle size distribution (PSD) analysis and slurry density measurement.
- Temperature probe located in the sump to record the slurry temperature.

The slurry density is measured by manual sampling from a sampling point just downstream of the pump discharge. All instrument signals are logged on a data logging unit.

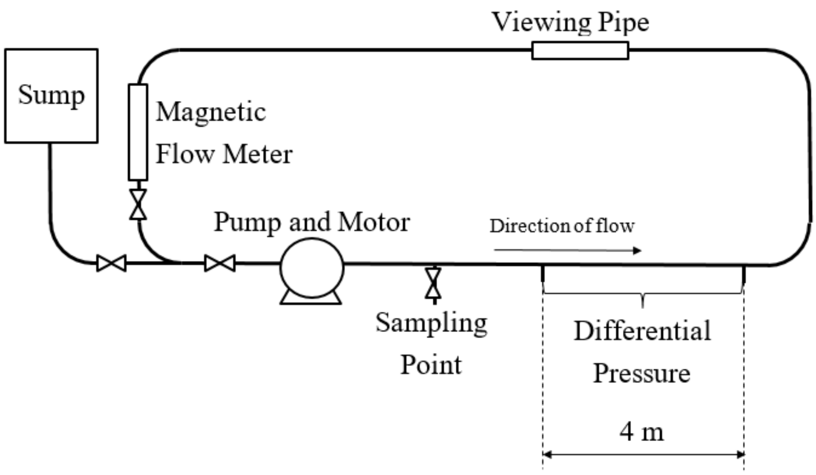

Figure 2 Schematic diagram of the pipe loop

Table 1 Pipe loop constants

Property	Value
Internal diameter	106 mm
Hydraulic roughness	4 µm

Five slurry blends were constituted by mixing a coarse fraction (+45 µm) and a fines fraction (-45 µm) in various proportions. The rheological properties of the carrier fluid were determined and are summarised in Table 2. An Anton Paar RheolabQC rotational viscometer with a temperature control bath was used and the carrier fluid was tested and analysed using the ISO 3219 method (International Organization for Standardization 1993). The properties of each of the five slurries are summarised in Table 3.

The PSDs of the five slurries are plotted in Figure 3.

Table 2 Rheological correlations for the carrier fluid

Bingham plastic model	
Applicable concentration range: 17.6%v < C_{vf} < 27.6%v	
Plastic viscosity	Bingham yield stress
$K = \mu_w + 10.1\ C_{vf}^{3.89}$	$\tau_y = 34.2\ \times 10^3\ C_{vf}^{4.98}$

Cvf = volume concentration of solids (fines) in carrier fluid

Table 3 Summary of the properties of the iron ore tailings slurries tested

Property	**Test 1**	**Test 2**	**Test 3**	**Test 4**	**Test 5**
Solids density (kg/m³)	3785	3820	3875	3800	3835
Slurry density (kg/m³)	1705	1810	1920	1895	1985
Slurry volume concentration – C_v (%)	25	29	32	32	35
Carrier fluid volume concentration – C_{vf} (%)	19	20	19	23	23
Carrier fluid Bingham yield stress – τ_y (Pa)	8.4	10.2	7.6	24.8	23.7
Carrier fluid plastic viscosity – K (Pa.s)	0.0162	0.0187	0.0151	0.0365	0.0353
Bed packing concentration* (%)	35	35	35	35	35
Percentage fines (%)	68.2	60.1	47.9	64.8	56.5

*Packing concentration of coarse solids in bed layer (assumed value)

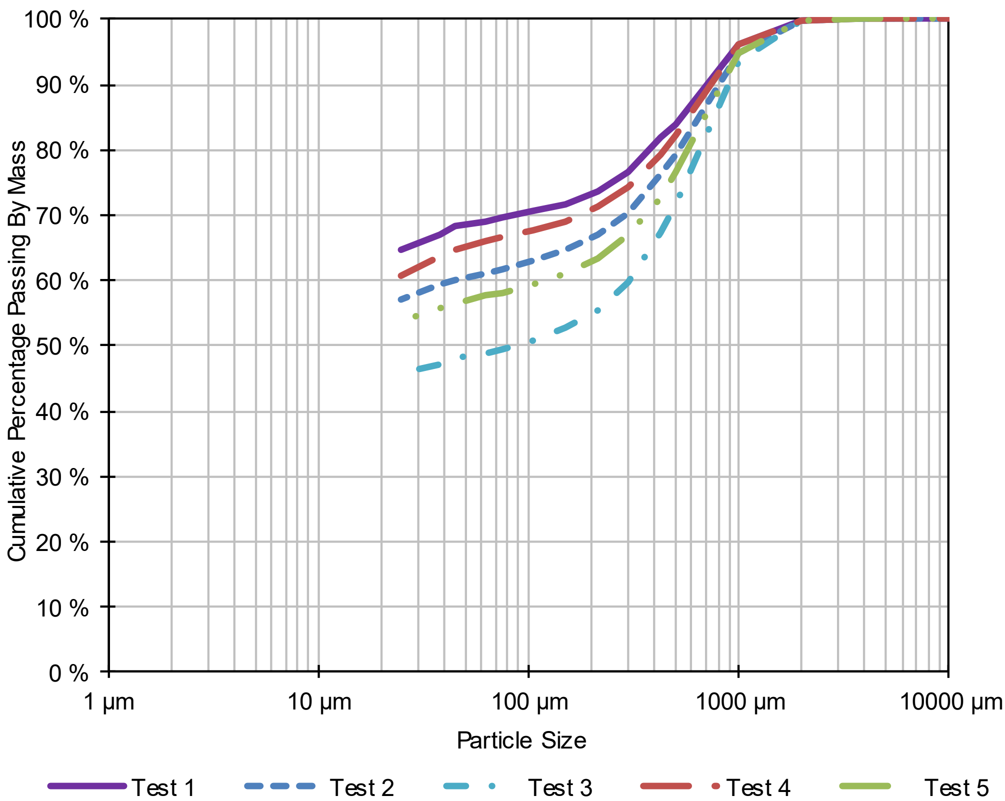

Figure 3 Particle size distribution of slurries tested

4 Results

Figures 4 to 8 show the comparison of the model prediction and pipe loop data for Tests 1 to 5 respectively. The model shows good agreement in the laminar flow regime for Tests 2, 4 and 5 (Figures 5 to 8) with the largest error between the pipe loop data and the model being 6.9%. The laminar–turbulent transition velocity is estimated from the change in slope of the plotted pipe loop data.

For Tests 1 and 3 (Figures 4 and 6), the model slightly underpredicts the experimental results. It is suspected that this is due to the low carrier fluid yield stresses (τ_y < 10 Pa) for these cases and could be exacerbated by the large percentage coarse particles in the case of Test 3. The error between the measured values and the model for Tests 1 and 3 are in the range of 10 and 25%, respectively. The model therefore appears to be applicable to high concentration suspension slurries with a carrier fluid yield stress >10 Pa, with the upper limit still to be determined.

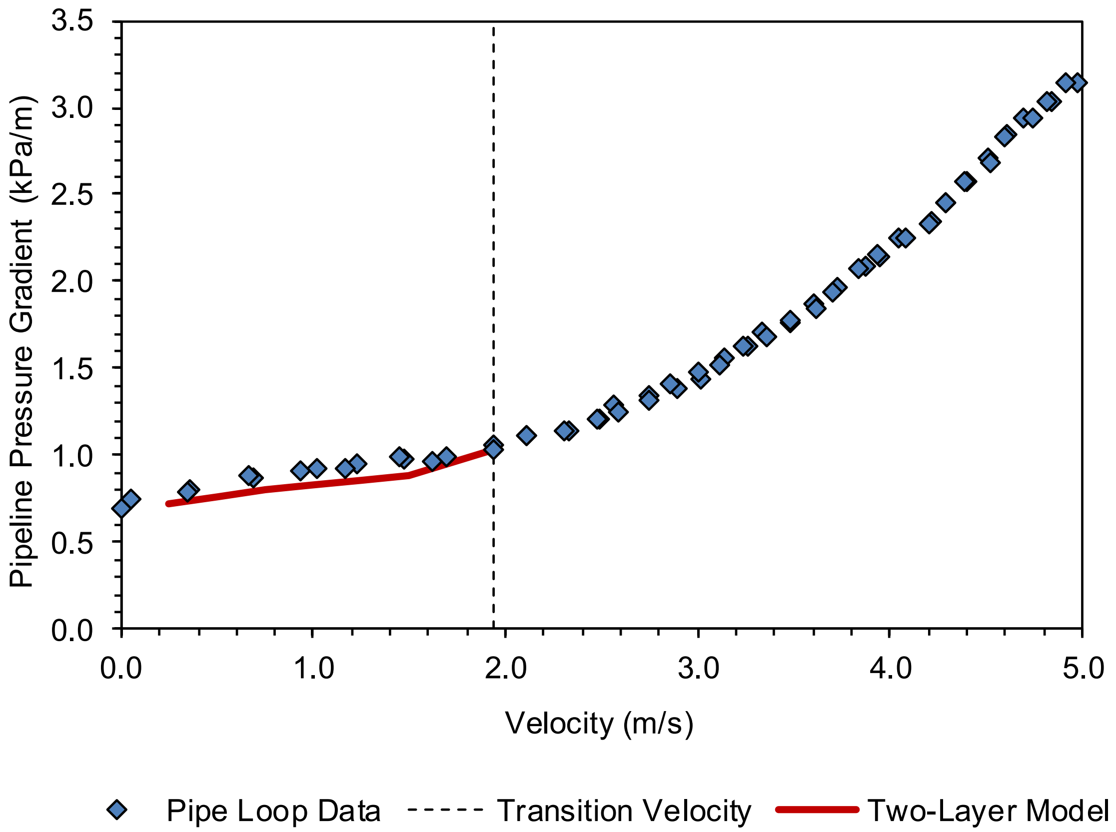

Figure 4 Test 1 comparison of two-layer model and pipe loop pressure gradient data

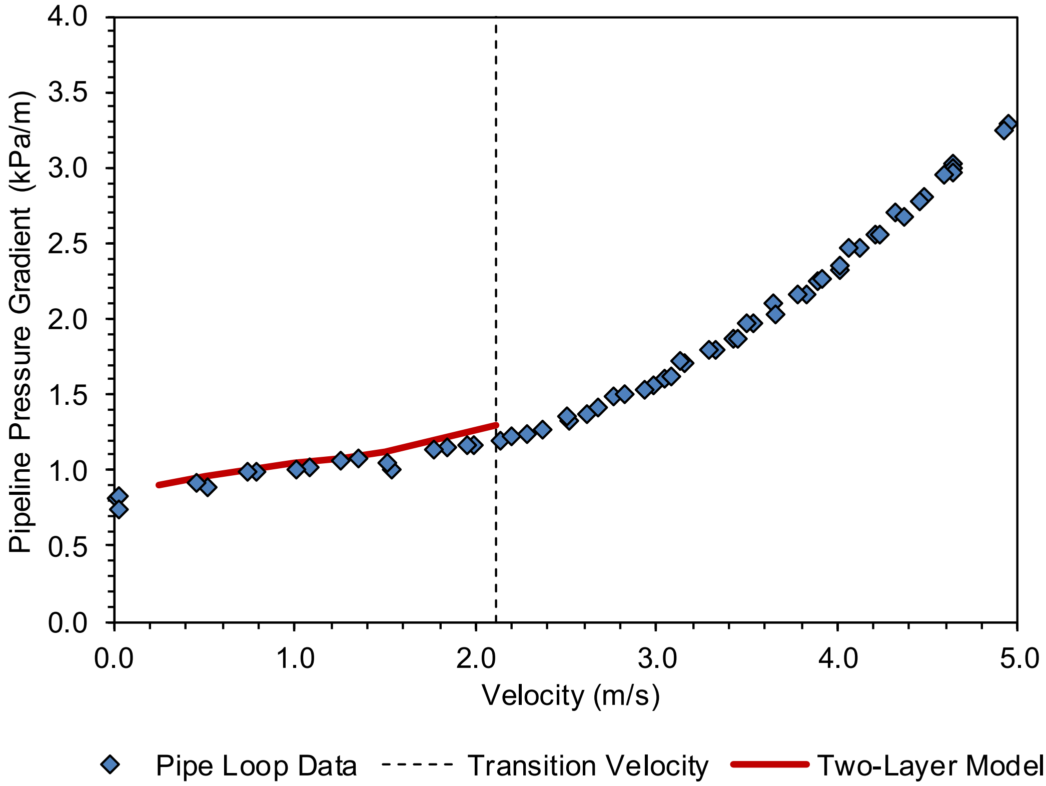

Figure 5 Test 2 comparison of two-layer model and pipe loop pressure gradient data

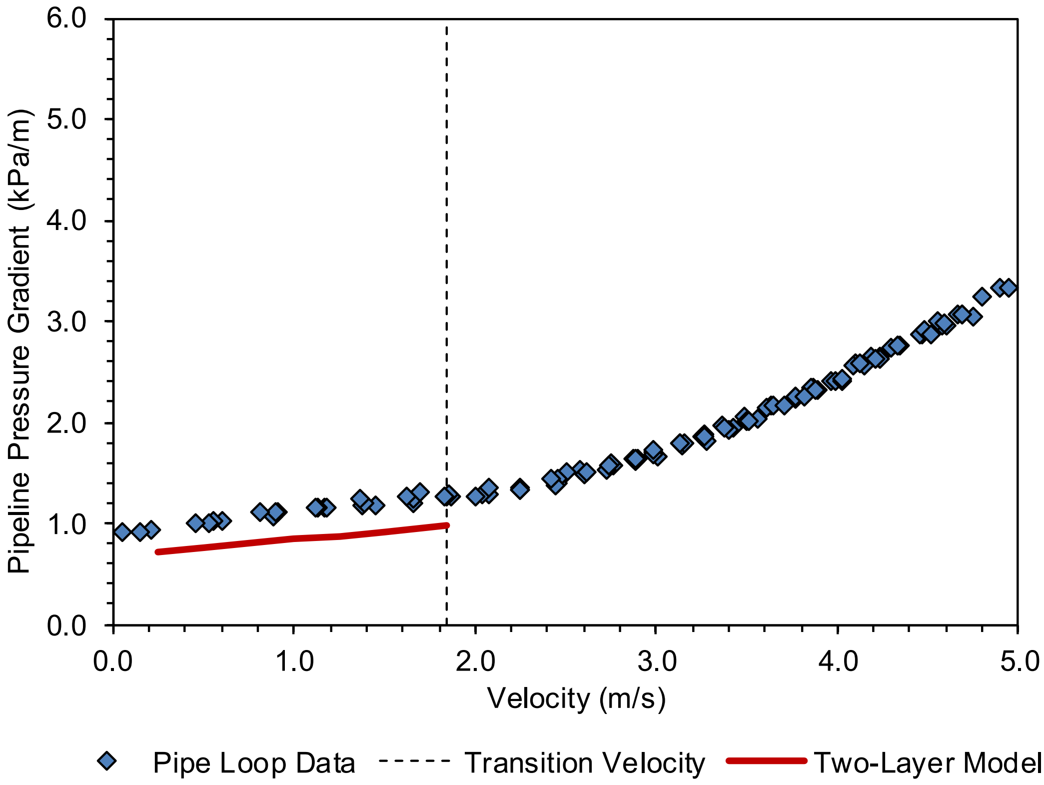

Figure 6 Test 3 comparison of two-layer model and pipe loop pressure gradient data

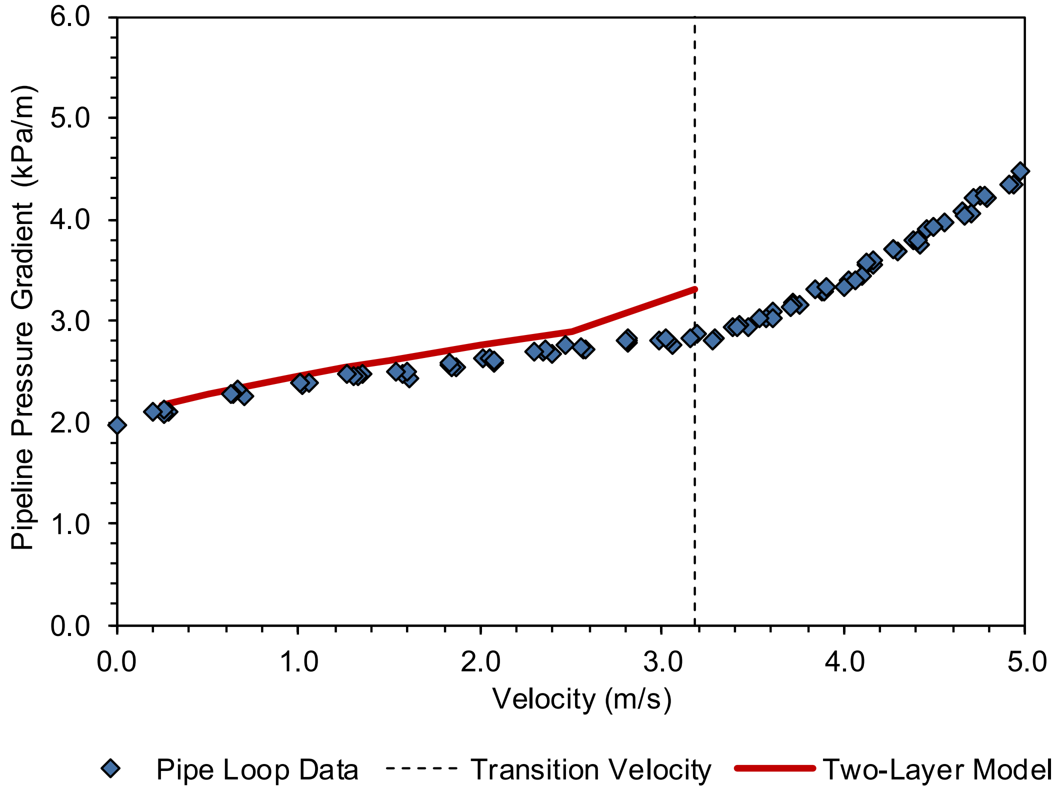

Figure 7 **Test 4 comparison of two-layer model and pipe loop pressure gradient data**

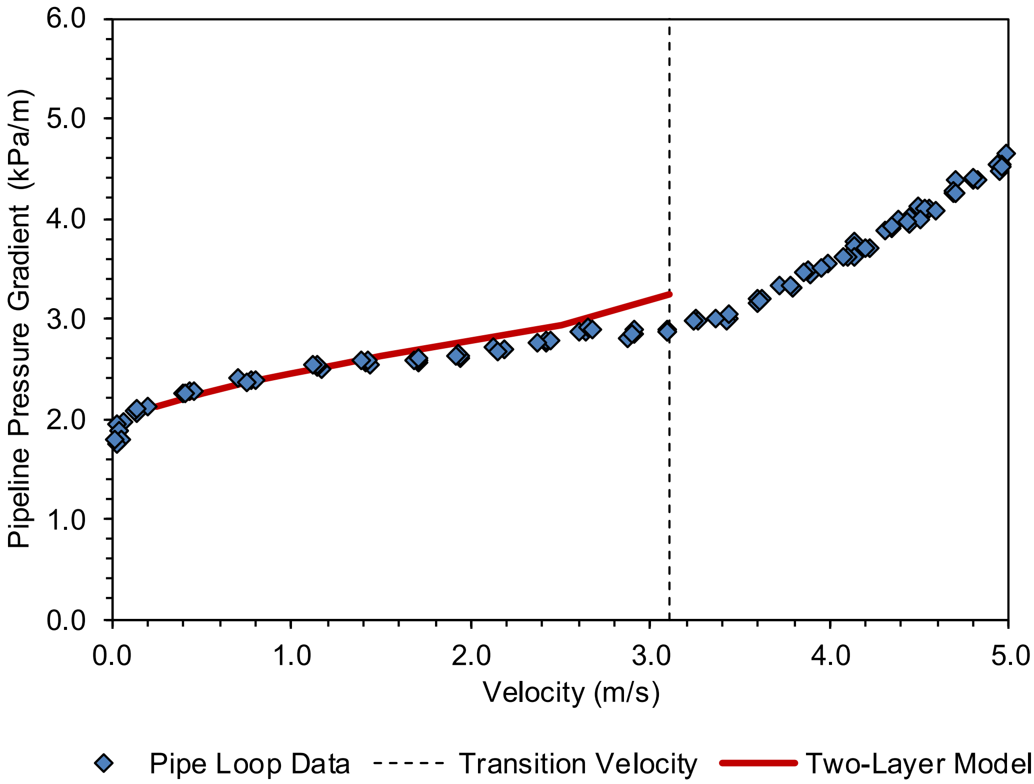

Figure 8 **Test 5 comparison of two-layer model and pipe loop pressure gradient data**

5 Conclusion

The model is in good agreement with the results obtained from the pipe loop tests. It appears that the gelled bed assumption is valid for slurries with yield stress >10 Pa in this case.

The model predicts the laminar pressure gradient accurately for the slurries tested with an average error of less than 7% in the specified range (τ_y > 10 Pa).

To better establish the range of applicability of the model, further testing and investigation are required.

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