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Three-dimensional limit equilibrium slope stability analyses, method, and design acceptance criteria uncertainties

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Abstract

A review of the information available in the public domain has shown that there are geological and mathematical uncertainties in the method used and the design acceptance criteria for three-dimensional limit equilibrium stability analyses offered to prepare slope designs in asymmetrical, closely jointed and interbedded and folded rocks. The geological and mathematical criteria assumed are concluded to be impracticable and not well enough understood with respect to the slope configurations that they are being applied. It is recommended that a geotechnical research project be undertaken to resolve these issues and uncertainties.

Keywords: *three-dimensional, limit equilibrium, uncertainty, design acceptance criteria*

1 Introduction

In a keynote address to the International Society for Rock Mechanics and Rock Engineering 14th International Congress on Rock Mechanics held in Brazil in 2019 (Read 2019), the author noted that the open pit mine geotechnical engineer is responsible for creating slope designs that are expected by the owners, management, the workforce, the regulators, and the public domain to be stable for the life of the mine. Economic requirements are always implicit in the designs but in today's world, producing slope angles that ensure the workers in the pit are protected against death or injury have become additional moral and legal requirements. These factors require that the geotechnical engineer has a skill set that at a minimum encompasses regional and site-specific knowledge of the geological and geohydrogeological models, understands the spatial and temporal distribution of the structural defects that are likely to affect the stability of the pit slopes, together with the properties of the soil and rock materials in which the slope will be excavated. They also must have access to the stability analyses necessary to transform this knowledge into robust slope designs, thus reducing uncertainty, reducing risk, identifying the preferred development option, and identifying opportunities.

Historically, the main types of analyses used in slope design studies include the following:

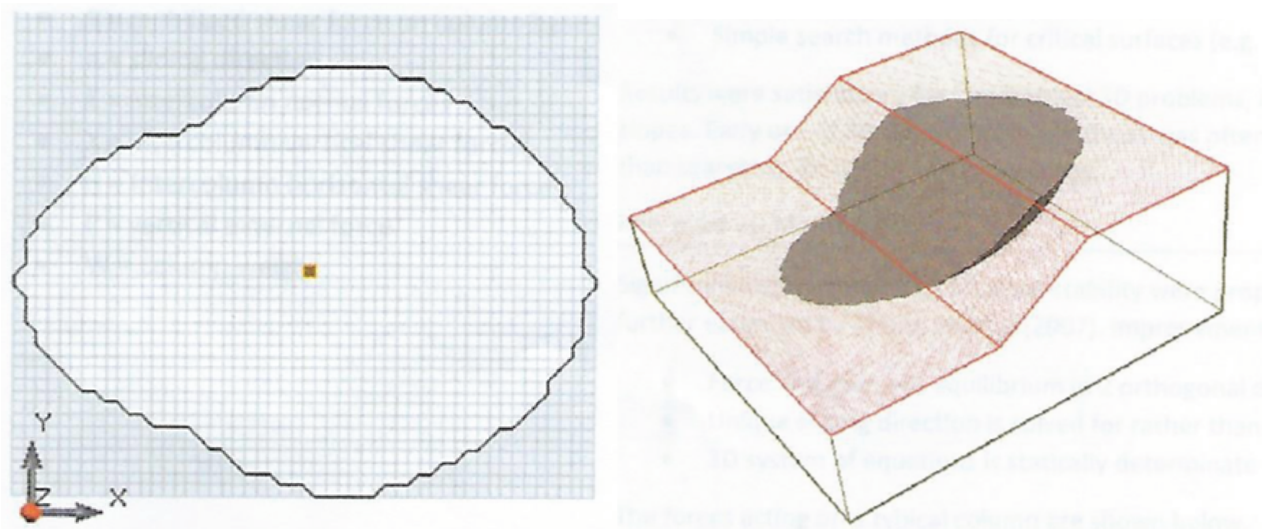
- Kinematic planar and wedge analyses based on the spatial orientation of the joint fabric at bench scale.
- Kinematic and limit equilibrium analyses applied to potential structurally controlled bench and inter-ramp planar and wedge slope failures.
- Limit equilibrium analyses applied to inter-ramp and overall slopes where stability is controlled by rock mass strength, with or without structural anisotropy.
- Numerical analyses applied to the design of inter-ramp and overall slopes.

When applying these analyses, a frequently made caution is not to plunge headlong into performing endless numerical stability analyses at the outset of the project studies (Read 2014; Read & Stacey 2016). Numerical analyses can and have been used to model many of the complex conditions found in rock slopes, including non-linear stress–strain behaviour, anisotropy, and changes in geometry. However, they should not be the initiating activity but rather the penultimate activity in the design process. They should not be performed until the failure mechanisms perceived for each mine design sector have been thoroughly evaluated using common sense and limit equilibrium methods, and a need for such numerical analyses has been demonstrated. Which brings us to the main topic of this paper: how and where do three-dimensional (3D) limit equilibrium (LE) analyses belong in this process?

2 Origins of three-dimensional limit equilibrium analyses

The first 3D LE analyses appeared in the literature in 1969 (Anagnosti 1969) and 1975 (Baligh & Azzouz 1975). Anagnosti extended the Morgenstern and Price (1965) method to estimate the Factor of Safety (FoS) of general slip surface, establishing a series of LE equations on thin vertical slices of a sliding body where the slip surface was not restricted to any specific shape. Baligh and Azzouz's method, expanded in two subsequent publications (Azzouz & Baligh 1978, 1983) was based on circular arc theory where the shape of the slip surface was assumed to be a combination of a cylindrical centre part with conical or ellipsoidal ends.

Since 1975, almost all of the analyses presented in the literature convert the two-dimensional (2D) method of slices into 3D by dividing the candidate sliding body into a series of square-shaped vertical columns with their bases lying on a spherical surface with a unique axis of rotation, as illustrated in Figure 1.

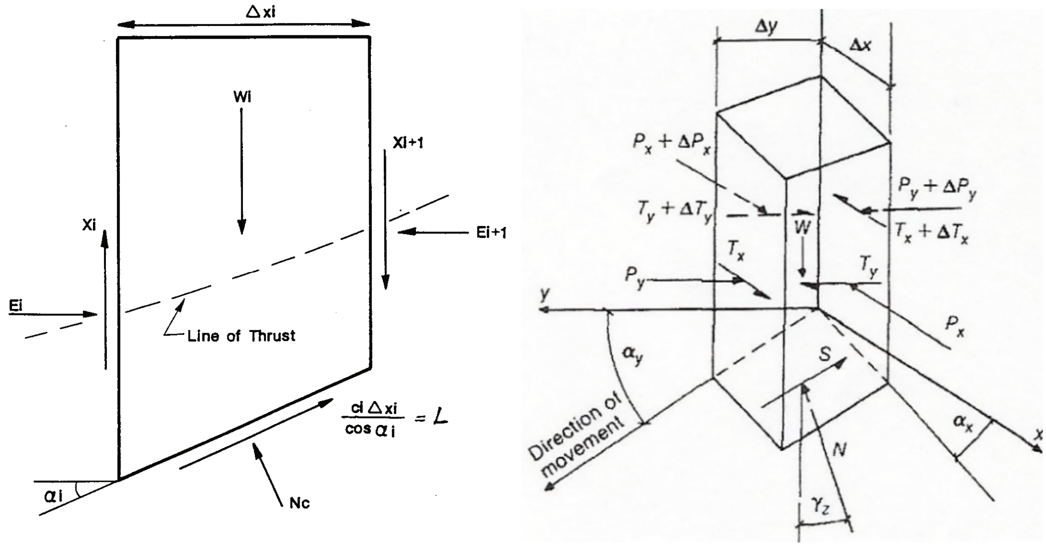


(a) (b)

Figure 1 (a) Plan view of sliding body divided into a series of vertical columns arranged in rows of uniform width; (b) Image of three-dimensional spherical failure surface (after Rocscience 2017)

The first of these analyses was developed by Hovland in 1977 for an arbitrary 3D curved sliding surface that corresponded to Bishop's (1955) 2D Ordinary Method of Slices (OMS). The next was that of Hungr (1987), who extended Bishop's 1955 2D OMS into a 3D algorithm that became the basis for the first commercial 3D LE computer program, CLARA-3 (Hungr 1987, 1988).

The distribution of the forces acting on an individual OMS slice and those on a column within an assemblage of columns in a Hungr 3D analysis are shown in Figure 2. Hungr's analysis neglects vertical intercolumn shear but not the intercolumn normal forces and horizontal shear forces (*P* and *T* in Figure 2).



(a) (b)

Figure 2 (a) Forces acting on an Ordinary Method of Slices slice; (b) Forces acting on a Hungr 3D column (Read 1987; Hungr 1987)

Hungr records that he developed the method independently in 1983 and extended it in 1989 to include the 3D equivalent of Janbu's simplified method (Hungr et al. 1989) but acknowledges parallel developments by Hutchison (1981) and Humphrey and Dunne (1982). He also acknowledges Hovland for the development of the true area of the column base (A).

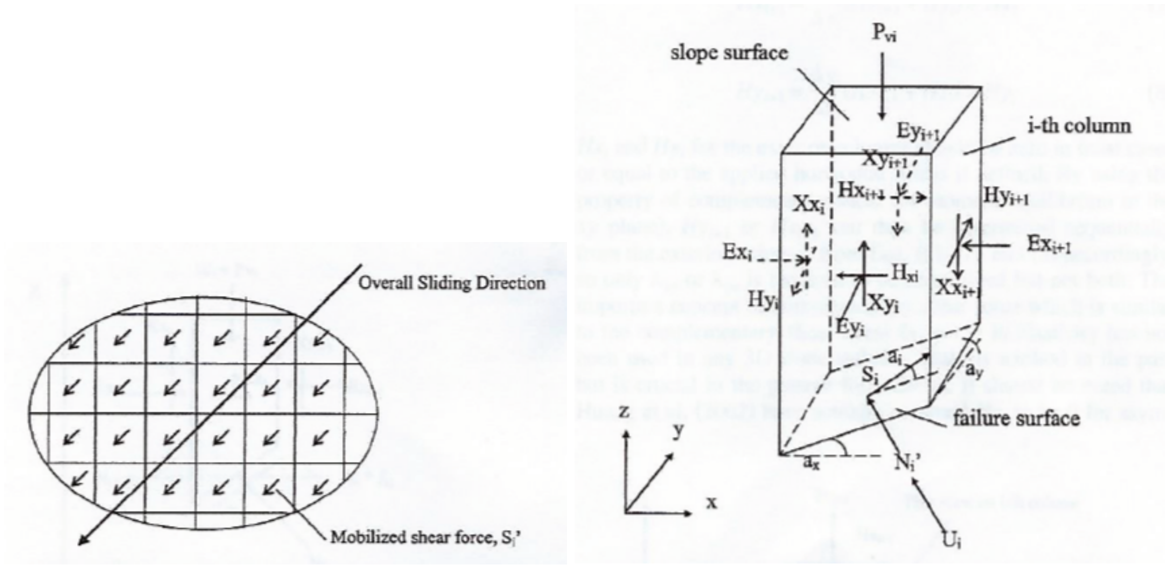
3 Reliability of three-dimensional limit equilibrium analyses

Hungr noted in 1987 that calculations were carried out with CLARA-3 to provide comparisons with the results of earlier 3D LE analyses by Chen and Chameau (1983) and Hovland (1977). As discussed in Section 4, there is a subsequent account of benchmarking by Fredlund et al. (2012) using later developments of the method, but otherwise there is little recorded verification and/or benchmarking of it. Rocscience Inc. (2017) suggests the method has been used mainly for back-analyses of known failures rather than searching for critical failure surfaces.

There is little to argue against that dividing a candidate sliding body into a series of square-shaped vertical columns provides a useful 3D model of an isotropic or symmetrical mass of soil such as London Clay. Whether it represents a closely jointed rock mass, such as occurs in any porphyry copper orebody, a complexly folded, banded iron formation in the Pilbara region in Western Australia, or an interbedded and folded coal sequence in Queensland's Bowen Basin, is questioned. A review of the information available in the public domain about analyses performed using 3D LE software raised three uncertainties about the reliability of the method.

First, and as questioned above, the process of dividing a complex, closely jointed or interbedded rock mass into a series of square-shaped vertical columns may be mathematically useful, but in terms of the geological and structural components of the geotechnical model, it is illogical.

It becomes even more illogical with the adoption of the Cheng and Yip (2007) concept that, as illustrated in Figure 3, when forces are applied, all the columns move together in a solid body until the point of failure when they separate and move away from each other. That in no way represents any of the likely mechanism of failure of a closely jointed or a folded, interbedded rock mass.



(a) (b)

Figure 3 (a) Unique sliding direction for all columns; (b) External and internal forces acting on a typical column (Cheng & Yip 2007)

Finally, Cheng and Yip are the source of the algorithms used in Slide3 (Rocscience 2017). In Figure 3, the forces shown by Cheng and Yip are for determining force and moment equilibrium for Morgenstern and Price. In their paper, Cheng and Yip note that, for simplicity, the weight of the soil and vertical load are assumed to act at the centre of each column. They also note that this is:

> *"Not exactly true but good enough if the width of each column is small enough, and the resulting equations will be highly simplified and should be sufficient for practical purposes."*

What is 'not exactly true' and how small is 'small enough'? The questions are not answered.

For Morgenstern and Price, Cheng and Yip also note that by using the property of complementary shear (or moment equilibrium in the *xy* plane), Hy_{i+1} or Hx_{I+1} can be determined sequentially from the exterior columns using Equations 5, 7, and 8 in their paper, so that only λ*xy* or λ*yx* in their Equation 5 is required to be determined, not both. They go on to note that:

> *"The important concept of complementary shear force which is similar to the complementary shear stress ($\tau_{xy} = \tau_{yx}$) in elasticity has not been used in any 3D slope stability analysis method in the past but is crucial in the present formulation."*

They then add:

> *"Although the concept of complementary shear stress is applicable only in the infinitesimal sense, if the size of the column is not great this assumption will greatly simplify the equations."*

Why is the concept of complementary shear force crucial and what size is not great? Questions that again are not answered. To increase the uncertainty, Cheng and Yip make an additional note in a later section of their paper that the effect of λ*xy* or λ*yx* is small and the error in this assumption is actually not important. Cheng and Yip at least state their assumptions, but that they have been utilised in a widely used stability analysis seemingly without query is surprising.

4 Three-dimensional limit equilibrium design acceptance criteria

Because of the different ways each of the 2D LE methods of analysis achieve limiting equilibrium, for any given geometry, the estimated FoS will be similar but not always precisely the same. This is understood and accepted across the industry, as reflected by the FoS in the range of 1.2 to 1.3 becoming the commonly accepted design acceptance criteria (DAC) for 2D LE stability analyses, although there are few published references that explain why this is so. Since the definition of the FoS by Taylor (1984), typical values in the range of 1.0 to 1.5 have been set by observation and trial-and-error experience, taking into account issues such as the reliability of the data, the type of analyses utilised, and the simplifying assumptions made.

Current tolerable and acceptable levels of the FoS in civil and mining engineering, along with attendant values for the Probability of Failure (PoF), are given in Table 1. The table was prepared during the first of the Large Open Pit (LOP) research programs by a committee comprised of the editors of *Guidelines for Open Pit Design* (Read and Stacey 2009), together with representatives from Barrick, BHP, DeBeers, and Rio Tinto. The table is under review by a research committee from the current LOP program (LOPIII, www.lopproject.com).

Table 1 Typical Factor of Safety and Probability of Failure acceptance criteria values (Wesseloo & Read 2009)

		Acceptance criteria		
Slope scale	**Consequences of failure**	**FoS (min) (static)**	**FoS (min) (dynamic)**	**PoF (max) P[FoS ≤ 1]**
Bench	Low-high	1.1	n/a	25–50%
Inter-ramp	Low	1.15–1.2	1.0	25%
	Medium	1.2	1.0	20%
	High	1.2–1.3	1.1	10%
Overall	Low	1.2–1.3	1.0	15–20%
	Medium	1.3	1.05	5–10%
	High	1.3–1.5	1.1	≤5%

In a study of 3D failure, Kenney (1956) indicated that the magnitude of error involved in applying 2D rather than 3D analyses to slope stability problems was conservative, and unlikely to exceed 10%. Hungr et al. (1989) found 'very good' correspondence in cases of rotational and symmetrical sliding surfaces such as ellipsoids.

The apparent ability of 3D LE methods to provide higher FoS values than 2D methods was taken by many to be a step in the right direction, as potentially it offered an economically enhanced design for the slope (Cheng et al. 2005), a view still held by some consultants and practitioners. Subsequent reviews of 3D LE methods of analysis show much wider differences can occur (Kalatehjari & Ali 2013), with Cavounidis (1987) and Mowen et al. (2011) reporting differences as large as 30% in some cases. Giterana et al. (2008) found differences of 15–50%. These differences emphasise the lack of verification and benchmarking studies that address the potential range in differences in the FoS that may occur between different 3D software packages. An 'extensive' comparison of benchmarked results were reported during the development of the Soilvision Systems SVSLOPE-3D software package. However, only three examples, each of which is soil mechanics-based and benchmarked only against CLARA/W, were described and the accompanying list of comparative references was only five (Fredlund et al. 2012). The need for such studies and reviews of how the algorithms in the available 3D LE software packages are applied is emphasised by the emerging evidence that analyses of the same geometry by and within different packages can and have provided extremely variable FoS values.

Initially it was thought that the higher FoSs were due to the 3D effect of spherical failure surfaces. Subsequent to the creation of CLARA-3, views were expressed that the additional strength came from the internal support of the columns for each other, a view promulgated in the questionable mathematical assumptions of Cheng and Yip, as encapsulated in Slide3 (Section 3), which has all the columns moving together in a solid body in a unique sliding direction towards the point of failure, when the columns separate and move away from each other.

In interbedded coal sequences, such as those mined by companies in the Bowen Basin and elsewhere, where failure has occurred by sliding along a weak seam within the layered sequence, three other factors need to be taken into account. One is associated with the change from 2D slices to 3D columns along the failure surface, one with the Cuckoo search routine utilised by Slide3, and the other with plane strain considerations.

As discussed, in 3D LE, the 2D slices along the failure surface become columns. Prior to failure, the developing 3D failure surface must first pass through overlying higher strength materials where there are likely to be a significant number of columns with higher strength than at the base. Hence, potentially, the 3D FoS may always be unrealistically higher than the 2D FoS.

This effect may be exaggerated by the Cuckoo search, which is described as a stochastic, nature-based global search algorithm inspired by the natural parasitic but successful behaviour of the Cuckoo species of laying their eggs in the nests of other host birds of other species (Wu 2012). Wu notes that weak layers capable of dictating how slopes will fail present a challenge for stochastic global optimisation algorithms as these layers are unlikely to be found given random chance. This challenge can be addressed in Slide2 using the block search method that utilises a 'window' or a 'polyline'. The block search method is not available in Slide3, however, which only provides the Particle Swarm search as an alternative to Cuckoo for the four analyses available in SLIDE-3 (Spencer 1967; Morgenstern and Price 1965/general limit equilibrium; Bishop 1955; Janbu 1957).

Finally, 2D LE stability analyses are plane strain analyses and their vagaries are well understood from decades of observation and trial-and-error experience. Given that the high walls in most interbedded coal sequences have a plane strain geometry that can extend over kilometric distances, it seems reasonable to ask why it is thought necessary to introduce 3D analyses, which do not have a lengthy period of observation and trial-and-error experience? Based on the available evidence, it would seem it is only because the apparent ability of 3D LE methods to provide higher FoS values than 2D methods would be useful in accepting highwall designs where a 2D LE analysis had provided an FoS that was less than that required by the DAC (generally 1.2).

5 Conclusion

From all of the information currently available in the public domain, it is concluded that the assumption a 3D LE analysis of a plane strain geometry will provide a 'better' outcome than a 2D LE analysis is unsustainable; the geological and mathematical assumptions in the software being used are impracticable and not well enough understood with respect to the slope configurations they are being applied. Consequently, it is considered that establishing a DAC for a 3D LE stability analysis is clearly beyond the capability of the currently available 3D LE software packages. This does not exclude the use of numerical 3D finite element or finite difference analyses, which should be used to determine the stress/strain/deformation attributes of any section of a wall that has not met the 2D LE DAC. They should also be used in areas where the geometry becomes more complex, such as at the junctions of high walls and end walls, and/or where there are features such as access road re-entrants.

6 Recommendation

The conclusion reached should not be left where it is. The opinions expressed in the paper are those of the author and there may well be differences of opinion. Because the reliability of 3D LE analyses is causing debate within the industry and the potential for differences of opinion to create further uncertainties, it is suggested that the topic be taken up as a Research Project by a team that is led by an experienced practitioner, includes input from recognised practitioners, developers and research groups with a stake in the outcome, and is commissioned to report on the Method and Design Acceptance Criteria uncertainties associated with 3D LE stability analyses to industry within 18 months.

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