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Improving the geostatistical modelling of geotechnical variables considering directional dependence

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Abstract

Together with geological and geometallurgical modelling, geotechnical modelling is one of the essential components for the planning and development of open pit and underground mining projects. A particular characteristic of many geotechnical variables is to be direction-dependent, i.e. the measurement of a core sample not only depends on the geographical position of this sample, but also its orientation. To account for this characteristic, it is proposed to regionalise such variables in a five-dimensional space corresponding to the product on the three-dimensional geographical space and the two-dimensional sphere, so that each measurement is indexed by its easting, northing, elevation, azimuth, and dip. Instead of making predictions and simulations conditioned to a particular direction, this new paradigm allows geotechnical variables to be interpolated at any place in the geographical space, for any direction. The spatial correlation structure can be inferred and modelled by using separable covariances or combinations of separable covariances, under an assumption of stationarity in the geographical space and isotropy on the sphere. Also, conditional simulation can be performed by turning bands or spectral methods, based on products of basic stationary random fields in the geographical space and isotropic random fields on the sphere. The proposed methodology is illustrated with the modelling of the linear discontinuity frequency (P_{10}) and the rock quality designation in two copper deposits.

Keywords: *geotechnical modelling, directionality, geostatistics*

1 Introduction

One of the main challenges in geotechnical engineering is to balance a detailed characterisation of geological formations that considers natural and induced stresses at the lowest possible cost. However, the techniques commonly used in the different mining operations are often limited to averaging the value of geotechnical parameters to characterise geotechnical domains and to design rock excavations. Thus, the geotechnical analysis does not provide enough detailed information because it does not consider that the geotechnical parameters are regionalised and structured in the geographical space.

The complexity of geotechnical problems requires the use of methods that reduce the uncertainty associated with geological and geotechnical variability. In this line, geostatistics can integrate spatial variability and heterogeneities in the modelling of the rock mass (e.g. Oh et al. 2004; Stavropoulou et al. 2007; Exadaktylos & Stavropoulou 2008; Choi et al. 2009; Ferrari et al. 2014; Hekmatnejad et al. 2017). However, the application of geostatistics in the modelling of geotechnical parameters faces several challenging issues, such as the lack of additivity of most regionalised variables, which complicates the upscaling or change of support (extension from a sample to a more voluminous block), and the directionality (the measurement depends on the sampling direction).

Additionally, given the limited measurements in the exposure area, the collected data come mostly from geotechnical boreholes. Like all types of one-dimensional measurements, the geotechnical parameters so measured, such as the discontinuity frequency (P_{10}) or the rock quality designation (RQD), are often directional. In practice, however, one does not discriminate the borehole samples according to their direction, which implies mixing different statistical populations of data. Besides, very few authors have considered the directionality of geotechnical parameters, so that the models developed lack this essential feature in the characterisation of rock masses.

The contribution of this research is focused on the development of new methods to improve the geotechnical characterisation of rock masses by the geostatistical modelling of their inherent mechanical properties and integrating uncertainty, spatial variability, and existing heterogeneity without implementing an extensive prospecting plan. A more detailed characterisation of the rock mass is a critical topic for the economic and operative improvement for different areas such as the mining (underground or open pit), hydrocarbon, geothermal, and nuclear industries.

2 Methodology

At this point, two critical aspects in the modelling of geotechnical variables are the directionality of the measurements and the change of scale or concept of extension (from a sample to a block).

2.1 Directionality

Our approach to solving directional dependence is to regionalise in five dimensions (5D), i.e. the geostatistics we know in three dimensions (3D) is extended to 5D for variogram analysis and simulation. The 5D space consists of the 3D geographical space (with east, north, and elevation coordinates) crossed with the two-dimensional (2D) sphere (with azimuth and dip angular coordinates) (Figure 1). To date, sphere–time models are used in environmental and atmospheric sciences to represent global phenomena on the surface of the earth and evolving through time. So, we propose to replace the 1D time axis with the 3D geographical space.

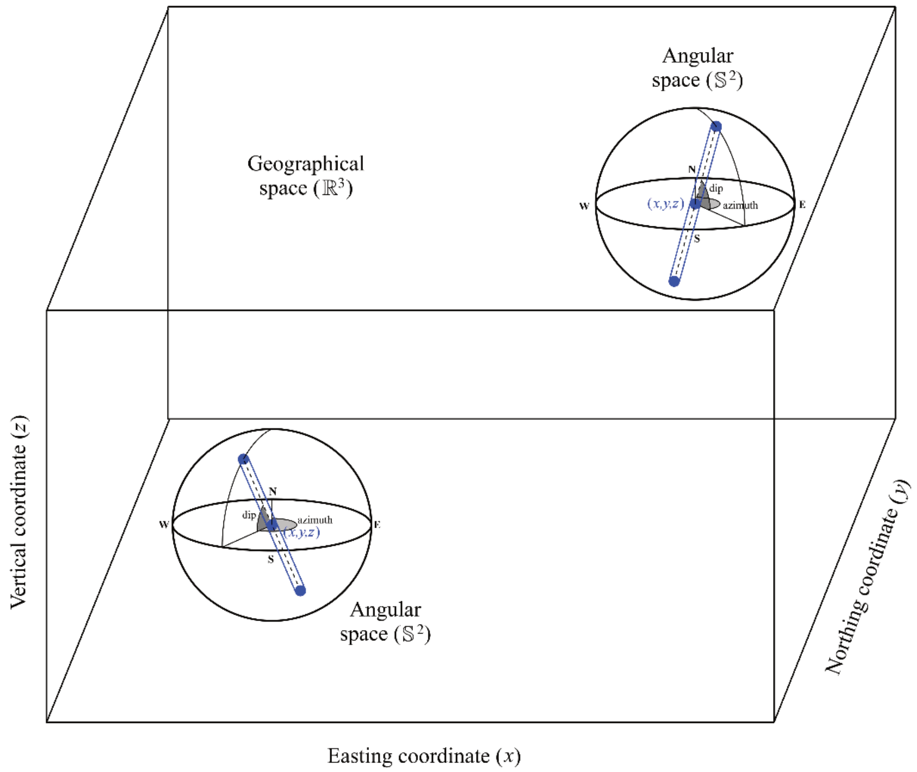

Figure 1 Geographical and angular spaces. Each core sample (blue cylinder) is indexed by the easting, northing and vertical coordinates of its gravity centre in the geographical space, as well as its azimuth and dip in the angular space, totalling five coordinates. That is, the measured FF or RQD values depend on both the geographical and angular coordinates

Given the complexity to model variables in such a 5D space, two assumptions are considered: the random field is assumed stationarity in the geographical space and isotropic on the unit sphere. They imply that the covariance or variogram between two data only depends on their geographical separation vector (denoted by h) and on their angular separation (denoted by δ) (Figure 2). These assumptions mean that the calculation of experimental variograms or covariances must be done for a fixed angular separation in addition to the geographical separation.

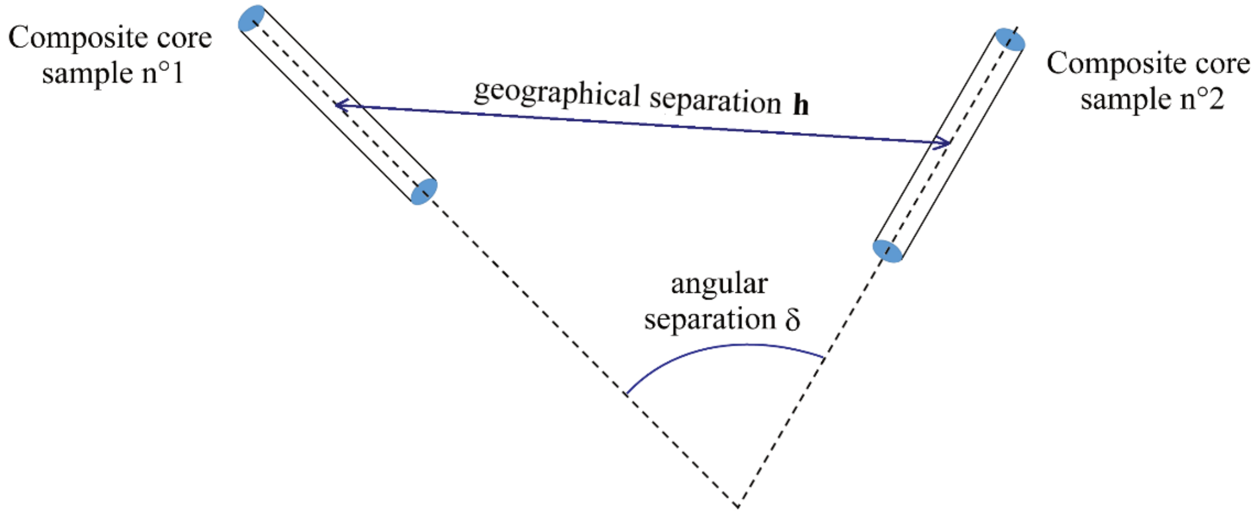

Figure 2 The geographical separation measures the distance between the gravity centres of the paired samples, while the angular separation measures the difference between their orientations

To fit a model, it is suggested to decompose the covariance functions as the product, or sums of products, of a spatial covariance depending on the geographical separation vector h (e.g. spherical or exponential model) and a directional covariance (Legendre polynomial) depending on the angular separation δ.

The regionalised variable can then be simulated by spectral or turning bands methods, consisting of adding and rescaling many independent repetitions of a basic random field with the prescribed correlation structure in the 5D space. This basic random field is obtained as the product of a geographical component (simulated with a piecewise linear function or a cosine wave) and a directional component (simulated with a spherical harmonics or a Legendre wave). The computational complexity of such a simulation (number of required floating-point operations) is proportional to the number of target grid nodes, which makes it extremely fast (Sánchez et al. 2019). Conditioning the sampling data is done by post-processing the simulation based on kriging or cokriging. To sum up, the geotechnical parameters are modelled and simulated in a 5D space by considering both geographical and directional components, as summarised in Figure 3.

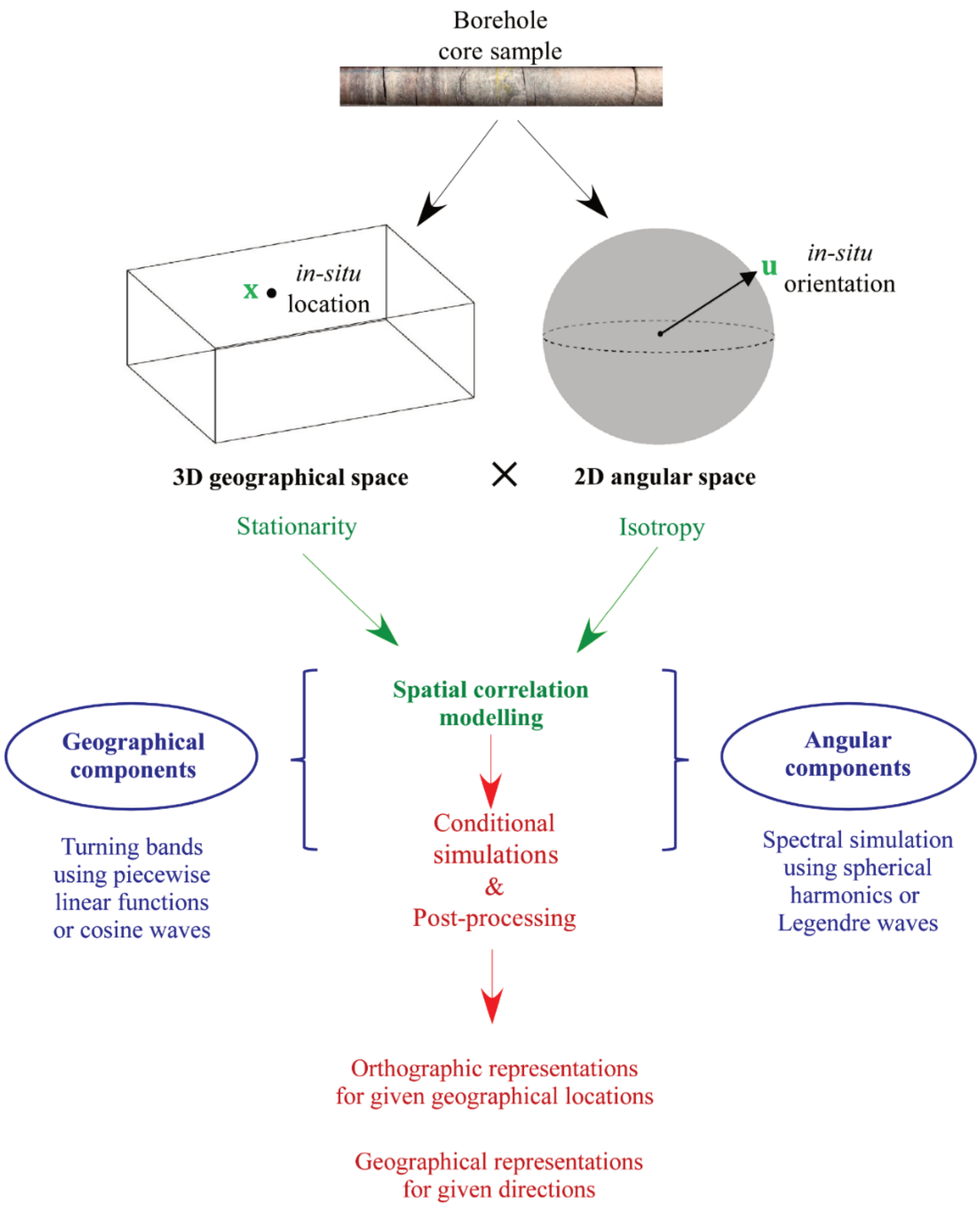

Figure 3 Methodology for the geostatistical modelling of geotechnical variables

2.2 Upscaling

Another critical aspect of the determination of the directional variable is the upscaling to large blocks. In practice, the characteristics of rock masses change gradually with the increasing sample size. For example, the rigidity and strength of a region decrease as the size of this region increases. When the size is larger than a critical value called the representative volume element (RVE) size, the characteristic values remain unchanged. The concept of RVE is introduced to the abovementioned size effect (Esmaieli et al. 2010; Zhang et al. 2012). The REV is the basis to determine a rock mass mechanics model (Zhang et al. 2017).

The traditional geostatistical approach that regionalises variables in the 3D geographical space only allows calculating values that mix different sampling directions within the block support. In contrast, we propose an upscaling strategy that only mixes values measured or simulated along the same direction (Figure 4). This directional approach supposes a further advantage as it is not needed for the data to be measured in the same direction because simulation can be done for each geographical coordinate and each direction.

Our strategy is two-fold: on the one hand, an upscaled directional variable is obtained by block-averaging the point-support values associated with the same direction. For a given block, the value of the resulting variable depends on the direction, as it happens with the original (not upscaled) variable. On the other hand, since practitioners are often interested in characterising each block with a single value, an upscaled non-directional variable is defined by considering, among all the directional values, the one corresponding to the maximum fracturing direction (maximum discontinuity frequency, or minimum RQD) (Figure 4). In this case, for a given block, the value of the resulting variable represents the maximum fracturing direction, as a 'summary' of all the directions.

All the calculations can be made on each simulation separately or averaged over all the simulations to obtain a prediction.

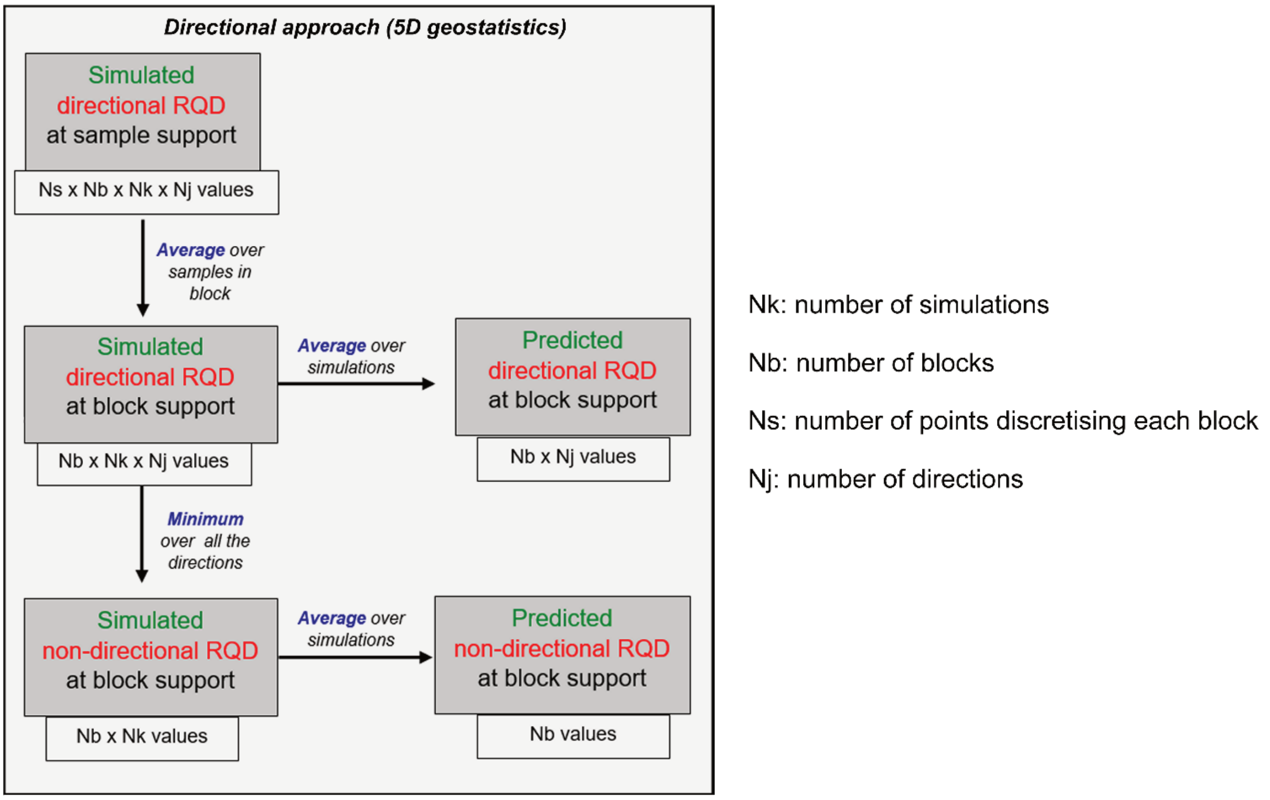

Figure 4 Synthesis of the upscaling proposal. Example of rock quality designation (RQD) for which the upscaled non-directional value is defined as the minimum RQD over all the directions

3 Case studies

3.1 Linear discontinuity frequency

To illustrate the described concepts, we present an application to a dataset from El Teniente copper deposit in Chile, where the importance of accounting for directionality is demonstrated. The database used consists of borehole data taken from the surface or from underground drilling galleries. For each borehole, several intervals (with a length generally comprised between 20 and 30 m) were logged. The logging information recorded the positions of 'weak veins' (i.e. veins with a weak mineral assemblage, identified as a critical factor in the fragmentation process during caving) intersecting the borehole and the angles between their poles and the borehole axis. Based on this information, the linear frequency of weak veins P_{10} was calculated for 10 m-long composites along the boreholes, yielding a set of more than 3,500 data points distributed in a volume of 1,800 × 2,100 × 950 m^3 (Figure 5).

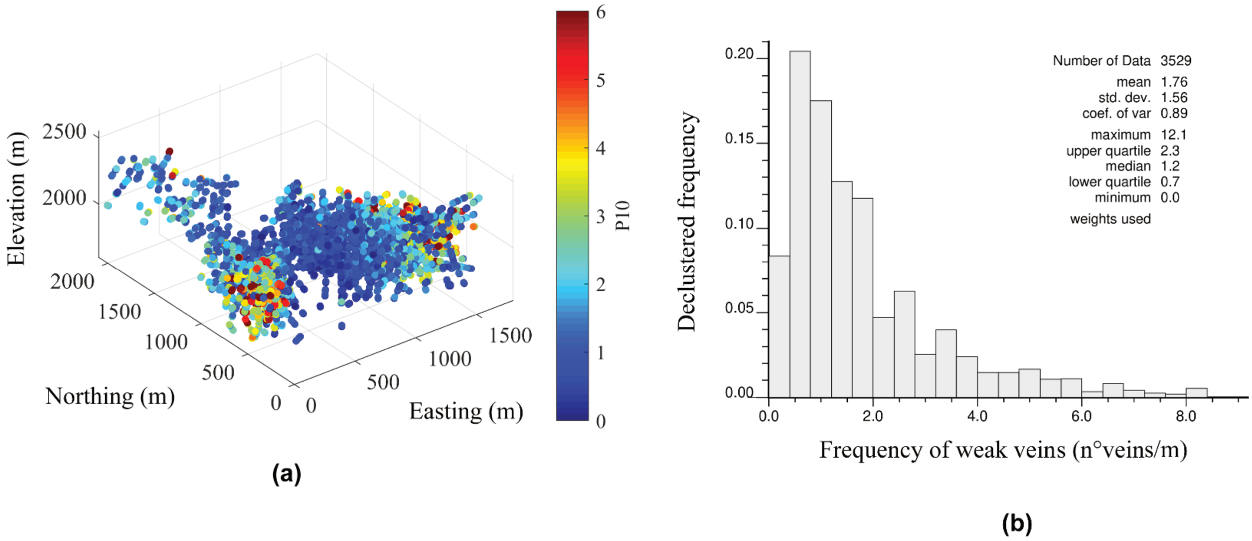

Figure 5 **(a) 3D location map; (b) Histogram of weak vein frequency of borehole data**

Prior to simulation, the data are normal-scored transformed, then their experimental variogram is calculated for different geographical separation vectors (along the horizontal plane and vertical direction) and different angular separations (Figure 6). The experimental variograms are finally modelled by considering a nugget effect and basic separable structures (product of spherical models for the geographical components, and Legendre polynomials for the directional components).

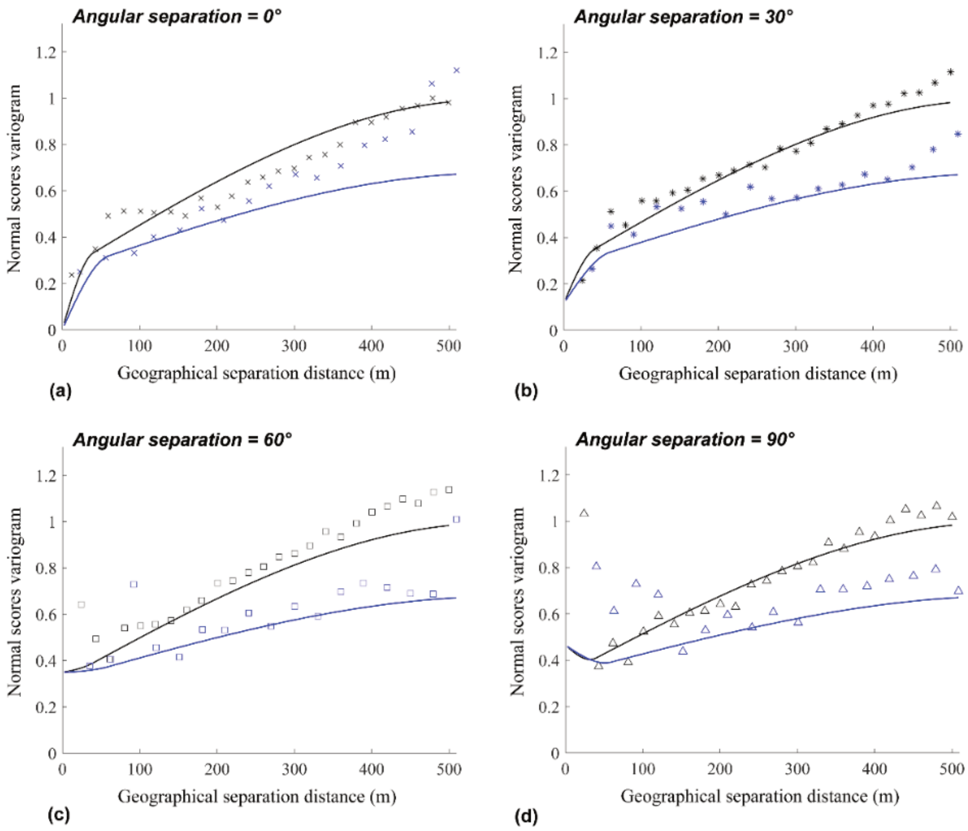

Figure 6 **Experimental and modelled (solid lines) variograms of the normal scores data along the horizontal (black) and vertical (blue) directions for angular separations between paired data ranging from (a) 0° to (d) 90°**

Even if the vein stockwork is isotropic, considering the directionality of the vein frequency has a strong impact on the simulation results because the correlations between measurements depend on their directions. Indeed, a measurement is highly informative on the values at surrounding locations along the same direction, as indicated by the low variogram values at short separation distances and small separation angles but brings much less information on the vein frequency along a perpendicular direction.

In addition to the standard tools used for the 3D representation of regionalised variables, new visualisation tools can be of interest to structural geologists and geotechnicians, such as 'regionalised azimuthal projections' that map the directional variations of the rock mass properties at given locations in the geographical space (Figure 7). In these projections, the half-sphere is represented on a disc, the parallels appearing as concentric circles and the meridians as line segments radiating from the centre. Figure 7 shows that the veins oriented in a vertical north–south plane are abundant near the locations in the upper and lower right side, as the weak vein frequency in the east direction is the highest, whereas they are scarce near the locations in the lower-left corner and in the centre on the right side.

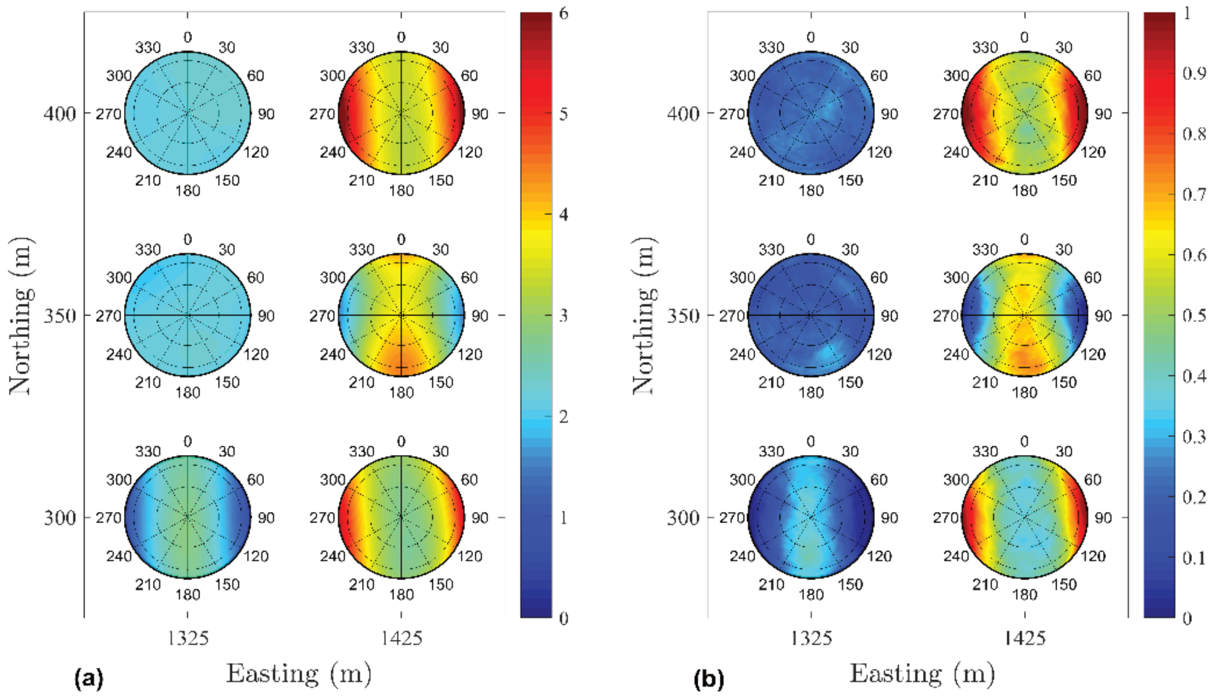

Figure 7 (a) Regionalised azimuthal projections showing the average simulated weak vein frequency and (b) the probability that the true frequency is greater than three veins per metre, at six particular locations in the geographical space

3.2 Rock quality designation

We now illustrate the proposed approach with an application to the prediction of RQD and geotechnical zoning in a copper deposit and compare to the results of the traditional approach where RQD is regionalised in the geographical space only. The database used consists of boreholes drilled from the surface and from underground drilling galleries, from which geologists have measured RQD. The sampled volume is about $350 \times 500 \times 660$ m^3. Again, the RQD data are normal-scored transformed prior to simulation.

The experimental variogram (Figure 8) presents a discontinuous behaviour at the origin (nugget effect). The low variogram values at short separation distances and small separation angles indicate that an RQD measurement is more informative on the values at surrounding locations along with the same directions than along a perpendicular direction. The experimental variogram increases until it reaches a sill at 60–100 m.

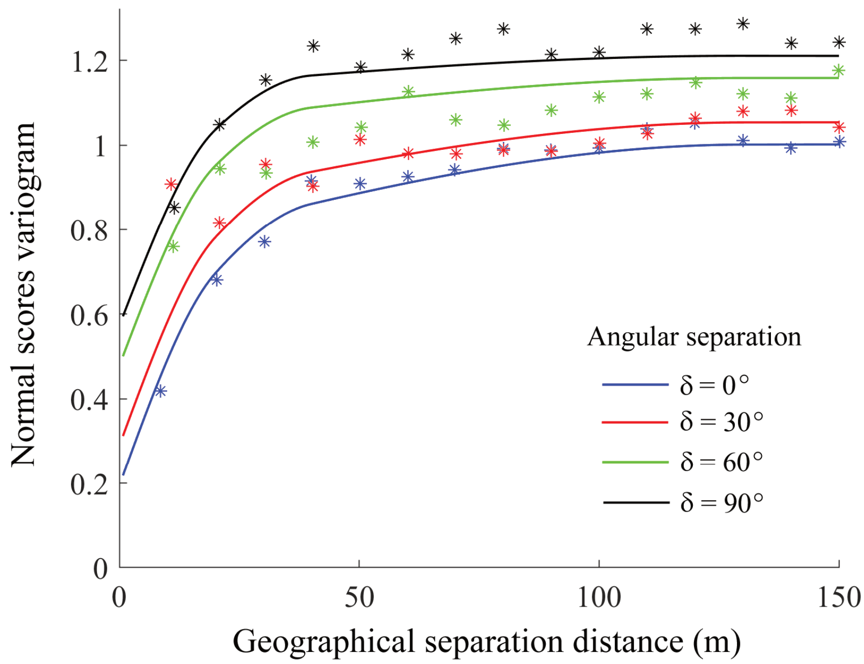

Figure 8 Experimental (asterisks) and modelled (solid lines) variograms of the normal scores data for angular separations between paired data from 0–90°

In this work, a directional approach is proposed to address the change of support (upscaling) of the simulated RQD values. The traditional way to conduct a change of scale from the sample support (cylindrical borehole composite) to a block support implies averaging the simulated values in the geographical space. For this change of scale to be meaningful, a single direction must be chosen, which raises difficulties as it is necessary to find a representative direction for (a) measuring and (b) averaging the values on the block (Figure 9a).

The new directional approach overcomes these limitations by defining a direction-dependent upscaled RQD value in each block, by averaging the RQD values measured along the same direction at different points within the block. Therefore, this strategy does not mix values measured along different directions. Additionally, it supposes a further advantage as it is not needed for the data to be measured in the same direction because the simulation of RQD can be done for each geographical coordinate and each direction (Figures 9d, 9e, and 9f). In this way, it is possible to know the directional variability of RQD in each block, reflecting the anisotropy of the rock mass and giving insights into the geometry of the fragments formed by the intersection of joints in a rock mass.

The comparison between the simulation results in the absence (Figure 9a) or the presence of a directional component (Figures 9d, 9e, and 9f) allows visualising the spatial variability to be expected in the field for a given direction. The latter is further useful to quantitatively define favourable conditions for the advance of the excavation from a non-qualitatively point of view, as is currently done in Bieniawski's rock mass rating classification.

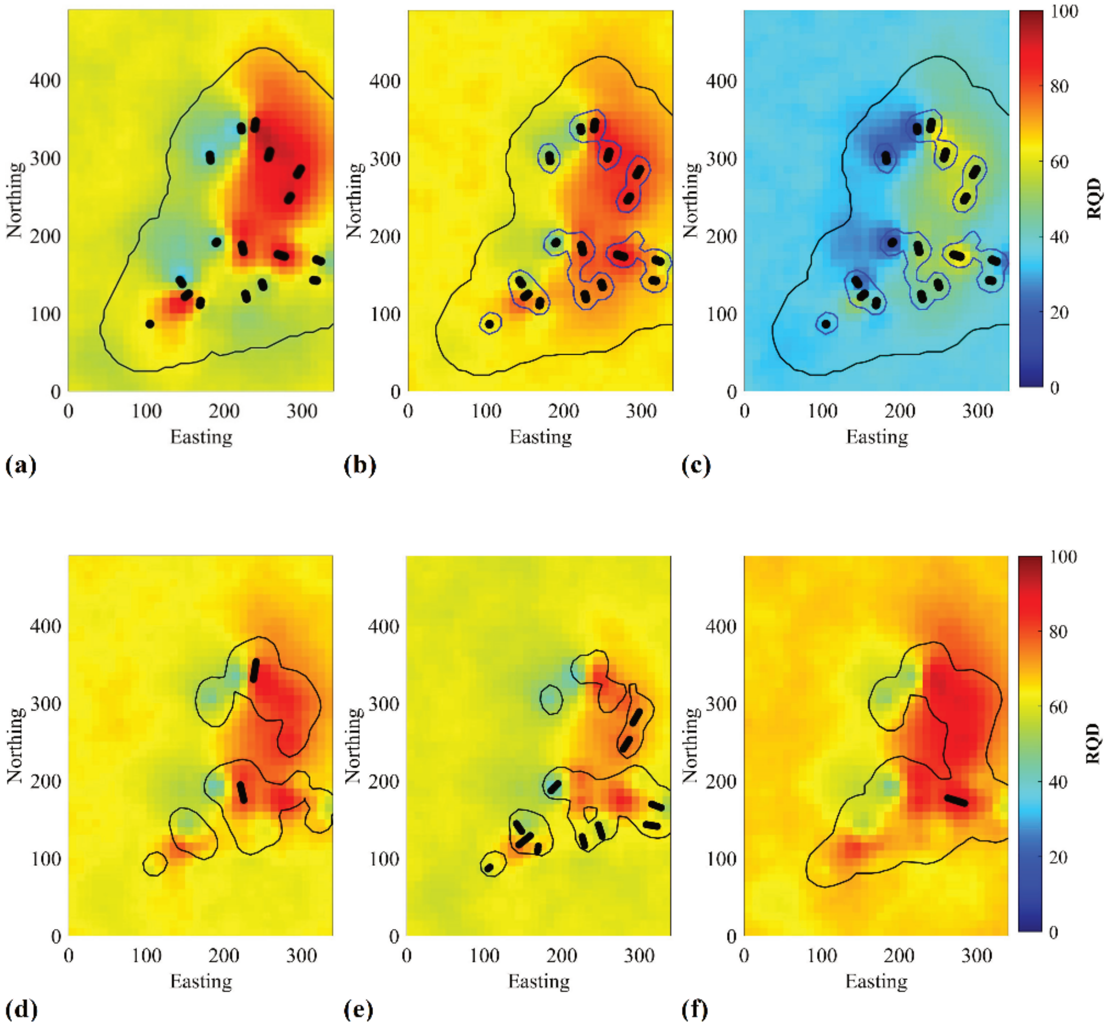

Figure 9 Average of simulations of block support rock quality designation (RQD) using (a) the three-dimensional traditional approach and (b) the proposed 5D directional approach: average RQD over all the directions; (c) Minimum RQD over all the directions; (d) Directional RQD along the north, (e) east, and (f) and vertical directions. The solid lines represent the contour of the 'confidence region', where the kriging error variance is less than 90% of the variogram sill (region enclosing the sampling data, which are superimposed in the maps; only the data close to the target direction are represented in the last three maps)

The risk of misinterpretation is minimised when RQD is simulated along different directions and one can be aware of the variations of RQD not only in the geographical space but also in the angular space. For example, there is a greater similarity between the map for the average of the simulations using the traditional (Figure 9a) and the directional approaches when RQD is simulated along the north direction (Figure 9e). This reveals a bias of the RQD obtained with the traditional approach, which should not be used for the following stages of the geotechnical design since, locally, it may be conditioned to a particular sampling direction and cannot be extrapolated to other directions. Thus, a geologist or a mining engineer could interpret favourable conditions towards the east of the study area only based on Figure 9a (traditional approach). Even though it is true that a better rock quality is present in that sub-area, this interpretation is conditioned by the RQD values measured along a specific sampling direction, which is likely to be locally biased concerning the RQD measured in other directions. In contrast, using the directional approach, the better rock quality towards the east evidenced in all the maps can be confidently interpreted as the real behaviour of the rock mass. Since the directional block support RQD is an unbiased representation for a specific direction, it can be used to evaluate the impact of the advance of the rock excavation in this direction in the mechanical behaviour of rock mass.

On the other hand, in addition to this 'directional' block support RQD, we also obtain a 'non-directional' block support RQD by selecting the minimum RQD value (not the average) across all the directions of the 2D angular space represented by a unit sphere, which is deemed the most representative of the real jointing degree of the block (Figure 9c). The RQD minimum is expected to occur along the direction perpendicular to the fracture planes. Furthermore, considering this non-directional RQD as part of a risk analysis leads to a single, non-directional map corresponding to the worst-case scenario for each block. In contrast, the average scenario (Figure 9b) (average of RQD over all the directions on the sphere and over all the simulations) may be too optimistic and a misleading representation of the actual rock quality.

Since the use of a single RQD value to represent the degree of fracture in a block is less informative than a direction-dependent RQD (given the high variability in the angular space), we propose to complement the non-directional minimum RQD with the anisotropy index (AI) of jointing degree for rock masses, following Zheng et al. (2018). The AI measures the spread or dispersion of the RQD for each block within the directional space and is defined for each block and each simulation as:

$$A_{Index} = 100\left(\frac{RQD_{max}-RQD_{min}}{RQD_{max}}\right) \quad (1)$$

Figure 10 shows that the anisotropy of the jointing degree is as high as 40–60%, with a marked contrast in the eastern part of the map close to the sampled area. This anisotropy of the jointing degree for the rock mass (directional variability of RQD) is not negligible and cannot be detected when RQD is regionalised in the three-dimensional geographical space and the directional component is discarded (i.e. using the traditional approach).

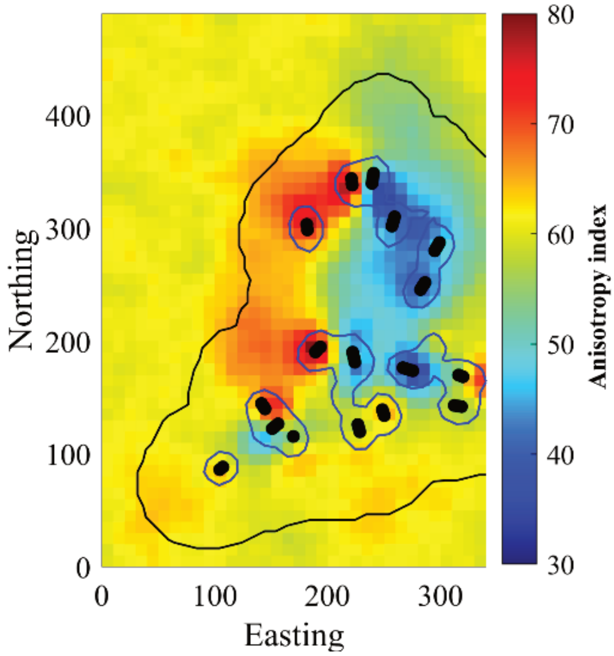

Figure 10 Anisotropy index (AI) of jointing degree using the directional approach (average AI over all the simulations)

The simulation can be used for geotechnical zoning. For each block and each simulation, the simulated block support RQD can be assigned one of five classes (very good, good, fair, poor, very poor), then the class that most frequently appears across the simulations is retained as the final classification of the block. The zoning maps so obtained strongly differ. Figure 11a ignores the directionality and mixes RQD measurements made in different drilling directions. A similar mixing arises with the map in Figure 11b. Although RQD is regionalised in a 5D space, the simulated values are averaged over all the directions. In contrast, the map in Figure 11c only considers one direction per block, the one associated with the lowest RQD and yields a more conservative definition of the geotechnical domains. These results highlight the advantages of considering the angular space when modelling directional-dependent variables and hold up the beneficial impact of this approach in the geotechnical zoning, knowledge of spatial behaviour of rock masses, and management of uncertainties in underground projects.

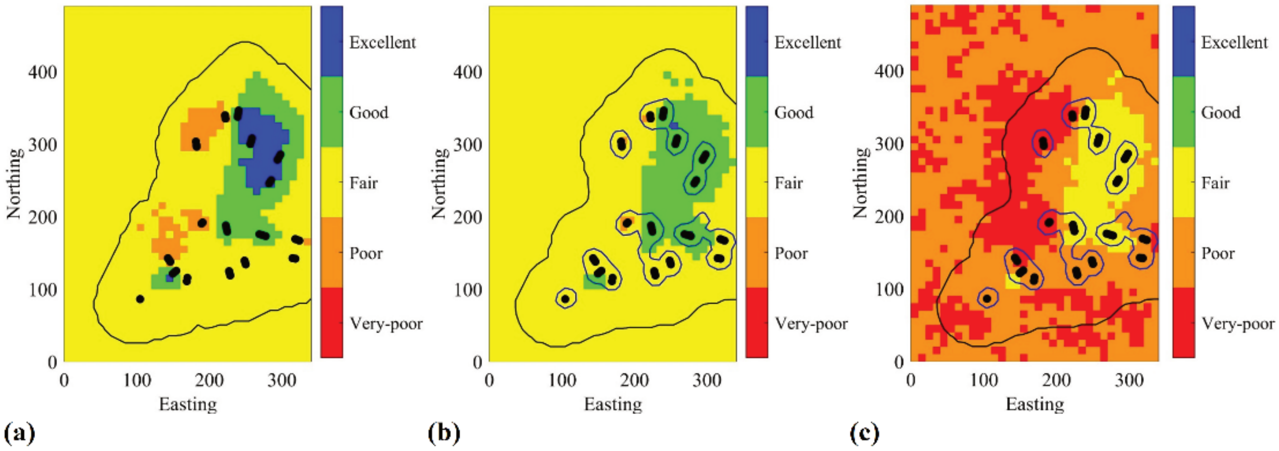

Figure 11 Geotechnical zoning maps using block support simulated RQD. (a) Traditional approach; (b) and (c) Directional approach: average RQD over (b) all the directions and (c) minimum over all the directions

4 Conclusion

The values of geotechnical variables such as P_{10} or RQD, measured on borehole core samples, depend not only on the in situ geographical position of the samples but also on their in situ directions. Our proposal consists in regionalising such geotechnical variables in a 5D space and accounts for directionality. In this sense, their spatial correlation structure depends only on the geographical separation vector and on the angular separation between measurements, which facilitates the calculation of experimental covariances or variograms, the fitting of a model through separable nested structures, as well as the simulation through products of basic random fields defined in the 3D geographical space and on the 2D sphere. The first case study has demonstrated the applicability of the tools and algorithms with the modelling of the discontinuity frequency in the El Teniente deposit (Chile), whereas the second case study dealt with the prediction of the rock mass quality and geotechnical zoning using RQD.

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