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Investigating economic and risk metrics using design of experiments in fully coupled caving geomechanics simulation

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Abstract

Technology for fully coupled simulation of the caving process, typically accounting for the flow of material using cellular automata (CA3D) and using non-linear stress-strain analysis (finite element method, FEM) for cave propagation, has been emerging over the last decade. The highest level of autonomy in this domain is achieved with automated model construction and meshing capabilities directly driven from data pertaining block model, geotechnical domains, drawpoints and production schedule, as is the case using a block cave model in the PCBC software from Dassault Systèmes.

This process automation enables encapsulating the simulation into advanced 'design space exploration' tools such as design of experiments (DoE) – referring to the ability to quantify the influence matrix of a large range of individual parameters and metrics and search the results space for behaviours of interest or optimise for desired outcomes.

Examples of such investigations can include understanding the mechanisms causing cave propagation to stall with the risk of creating an airgap or comparing alternative schedules with different directional scenarios for cave establishment and its impact both on geomechanics, such as fault activation, as well as project net present value (NPV). The encapsulation can extend to more complex downstream processing, such as detailed grade analysis, linking block model information to processing parameters or energy consumption. Importantly, integration with business drivers such as de-carbonisation and sustainability becomes possible.

Keywords: *caving geomechanics, cellular automata, finite element analysis, integrated simulation, optimisation, design of experiments, risk*

1 Introduction

This paper presents work in continuation of the methodologies for caving geomechanics simulation coupled with 3D cellular automata (Arndt et al. 2018), with further developments over the last four years contributing to the maturity of solutions. Experience was gained in different environments, ranging from early stage studies to established caving operations. The latter can offer significant amounts of historic data used for model calibration, important for production forecasts. Studies in greenfields projects combine a limitation in data with the aspiration to understand a broader variety of scenarios.

Three-dimensional non-linear stress-strain analysis using the finite element method has become a common workflow in block caving projects over the last two decades. Besides this coupling methodology, the analysis approach itself (Arndt et al. 2007; Beck et al. 2006) has not changed significantly other than in model size, complexity and available computational resources, whilst design space exploration tools (Koch et al. 2002) have been widely used in aerospace and manufacturing industries.

1.1 Background

The key drivers providing the foundation for the coupled caving solution as adopted in this work, GEOVIA PCBC and SIMULIA Abaqus, are summarised below. For a detailed description see (Arndt et al. 2018).

- The motivation for developing fully coupled simulation tools is derived from challenges integrating caving geomechanics (Brown 2002) into mine planning processes, to improve business key performance indicators related to safety, revenue maximisation (strategy on how best to exploit the mineral resource), and operation excellence (productivity).
- Automated processes will not just accelerate the cave modelling, often a niche task requiring a high level of expertise and exposed to scarcity of global resources, but also allow the use of results in case studies, sensitivity analysis and optimisation in an environment of uncertainty and constant changes to the available data.
- A workflow compatible with well-established production scheduling and geotechnical modelling frameworks, using industry proven technologies (PCBC and Abaqus) as the key building blocks, with rapid execution enabled by numerical efficiency and scalability of computer resources, supports confidence in simulation.
- Simulation of cave growth in complex geological settings designed to replicate realistic behaviour benefits from:
  - Three-dimensional representation of the problem geometry, geological and geotechnical data.
  - Representation of the in situ stress state.
  - Non-linear analysis of rock material exhibiting strain softening behaviour.
  - Direct use of production schedules for geomechanics simulation.
- Demonstration of complex caving propagation mechanisms through six selected benchmark cases including coupled cave propagation, inclined faults, overhanging domains, spill into an adjacent new mining block, multi-lift projects and preconditioning (hydraulic fracturing, HF) ensures coverage of key mechanisms observed in block caving.

1.2 Geomechanics risks

Why is geomechanics important in caving? Several challenges can affect safety, risk, cost, and production in a caving operation:

- Changes in stress and strain around undercut and extraction level of the cave drive ground support design and life-of-mine performance.
- The advancing caving zone can generate seismicity beyond the caving front and trigger events on critically stressed faults.
- The extent of the volume of broken rock can affect entry of waste material, rilling or the connection with other mining fronts (dilution).
- Cave back propagation and rate of production need to be managed to avoid development of an airgap (risk of air blast).
- The overall shape of the cave determines production outcomes (material flow and targeted grade) and ultimately recovery of ore for the life-of-mine.
- Timing of surface breakthrough and subsidence are important for operations, protecting investment and safety.

Managing major hazards has been discussed by (Flores 2019), providing detailed illustration of each scenario.

2 Workflow automation

The ambition to use design of experiments (DoE) within the complexity of fully coupled simulation requires a high level of automation. It is conceivable to achieve all the set goals with scripting alone and the choice of scripting language would usually be driven by native support in the applications, rather than functional coverage. Fish in FLAC3D (Vakili et al. 2020; Itasca Australia 2022) or Python in Abaqus (Arndt et al. 2009) are well-known examples for process and analysis automation. Beyond the workflow itself, which includes a significant amount of downstream processing, a graphical interface promotes user engagement and navigation of results. This is seen as a key enabler to broaden access to these tools from the small group of expert users to the broader community of mine planning engineers. The user interface used for this work is SIMULIA's Isight.

2.1 Overview

Isight and the SIMULIA Execution Engine are used to combine cross-disciplinary simulation models in a single process flow, automate their execution, explore the resulting design space, and identify the optimal design parameters subject to required constraints. This technology has been used in aerospace and automotive applications for over a decade.

2.2 Workflow template

The workflow template to investigate economic and risk metrics shown in Figure 1, using the DoE as the main component. This is interchangeable with either a simple task or an optimisation method to replace the top component, depending on the targeted application.

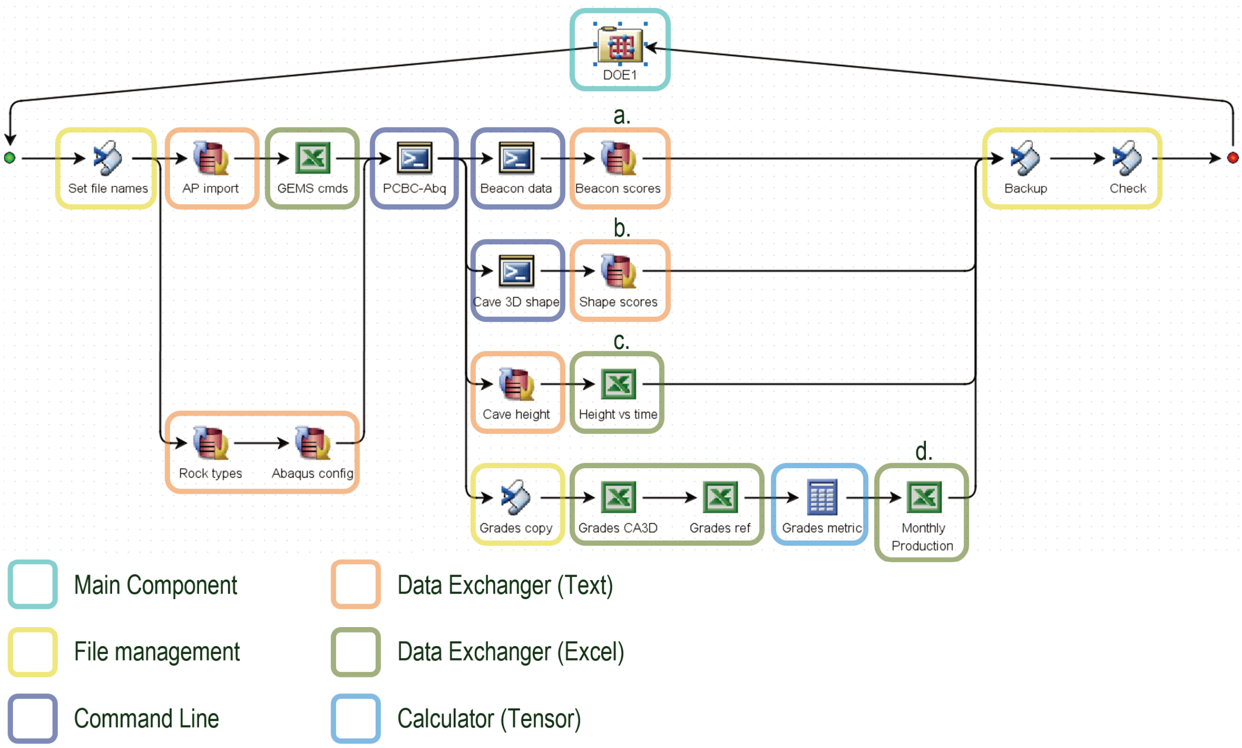

Figure 1 Workflow template, component categories and processing streams a–d

This can be divided in four main functional areas:

1. Management of input parameters, all model configuration and folder structure including efficient storage of large simulation files. This includes specific requirements for GEMS PCBC and SIMULIA Abaqus command line execution (without using the graphical user interface, GUI). These components are placed at the beginning and end of the workflow.

2. Data exchangers that are configured to place above parameters into the target configuration files for both GEOVIA PCBC CA3D (advanced profile, GEMS commands) and SIMULIA Abaqus (simulation configuration files, including in situ stress and the rock types parameter list or the number of CPUs for parallel execution as examples). Data exchangers can be applied to comma separated values (CSV) or Excel files and can perform read or write operations.
3. Command line execution of the coupled simulation process from GEMS PCBC.
4. Downstream processing. Here multiple streams are envisioned. For the scenario of calibration of an operating cave in particular, four metrics are suggested to be useful to evaluate model veracity:
   a. Beacon data – determining a score of the difference in time of onset of movement of beacons between simulation and measured.
   b. Cave shape – defining a metric for shape comparison is discussed under the next sub-heading, this score is evaluated at multiple key stages (manually selected, influenced by available data).
   c. Cave height – the evolution of cave height and airgap volume can be used for risk assessment, using a data exchanger cave height and volume can be transferred into Excel templates that contain graphs and calculate metrics.
   d. Production data (grades) – aggregated for a block or individually weighted by drawpoints, the monthly grades from CA3D simulation can be compared to grade sampling at different stages in the materials handling and processing. Here the calculation of metrics is using tensor operations in the built-in calculator component for performance reasons.

Regarding the use of beacon data for calibration of models it is worth nothing that the timing was chosen as the subsequent path of beacons proved too difficult to match between observations and cellular automata. As this category is not commonly used no examples are shown in this work.

2.3 Cave shape metrics

A rather surprising observation for the authors was the difficulty in the comparison of cave shapes from a simulation to a defined target shape, which is a very different metric compared to the analysis of cave height over time. Here, target shapes can be either based on simple assumptions or detailed interpretation of the extent of the mobilised zone of operating block caves for the purpose of model calibration.

Several papers previously published in this series of proceedings provide details on inferred cave geometries, one set of references is included in a table of propagation rates in (Sainsbury et al. 2018). In many publications this comparison is done by showing a visual match – a task often perceived to not pose difficulties for the human brain. Lacking a mechanism for numerical quantification, visual assessments elude the automated processing required for DoE or optimisation.

Initially, the methods tested for comparing intersecting or overlapping volumes would not pass even the simplest visual judgements regarding their suitability, whilst methods focusing on the largest distances between the envelopes would be exposed to distortion of their metrics by cave shapes that are not smooth (in particular where a second panel would lead to two local maxima in the height of the cave). Consequently, the case for a distance weighed algorithm was made by (Weller 2019). Resources for this algorithm were found in the areas of digital image processing, shape matching, recognition, and classification (Neal 2012; Veltkamp 2001) and adapted for three dimensions. This provided a good fit in terms of computational simplicity and robustness, where the algorithm embodies the idea that:

- Both simulation and target geometries are mapped into a regular size (rectangular) block model.
- Small differences around the reference shape do not have a big impact on the overall shape.
- Conversely, differences that extend well beyond the reference boundary (either interior or exterior) imply shapes are less similar.

- Calculation of a numerical score (or metric) for each candidate quantifies its similarity to the reference shape and allows ranking of scenarios.

The method developed for cave shape comparison is outlined in Figure 2 using two-dimensional examples for presentation. Everything shown can be applied to the three-dimensional application in the same fashion. The process steps are:

1. Create a distance transform on the block model, based on the reference (template) cave boundary.
2. Find the symmetric difference between the candidate shape and the reference shape.
3. Calculate a score for the candidate as the weighted sum of the resulting blocks, weighted according to the distance transform.
4. Rank various candidates based on their score: the lower the score, the better the similarity to the reference.

Possible distance transform weights are shown in Figure 2. The Manhattan weighting was chosen for the comparison of cave shapes based on its ability to reduce the score for outliers. A numerical example for scores of different candidate shapes, each having identical non-weighted scores, is shown in Figure 3.

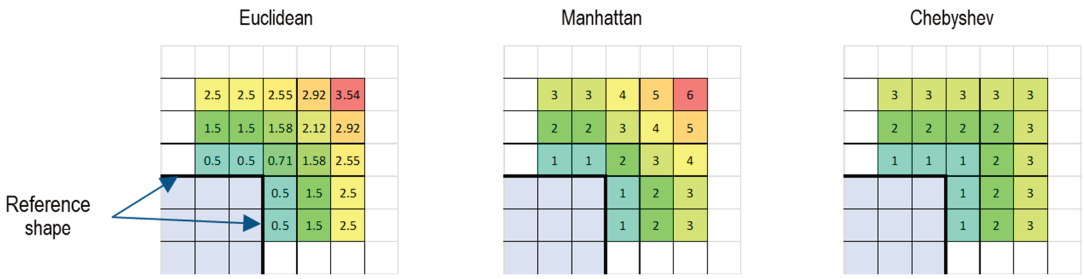

Figure 2 Distance transform weights

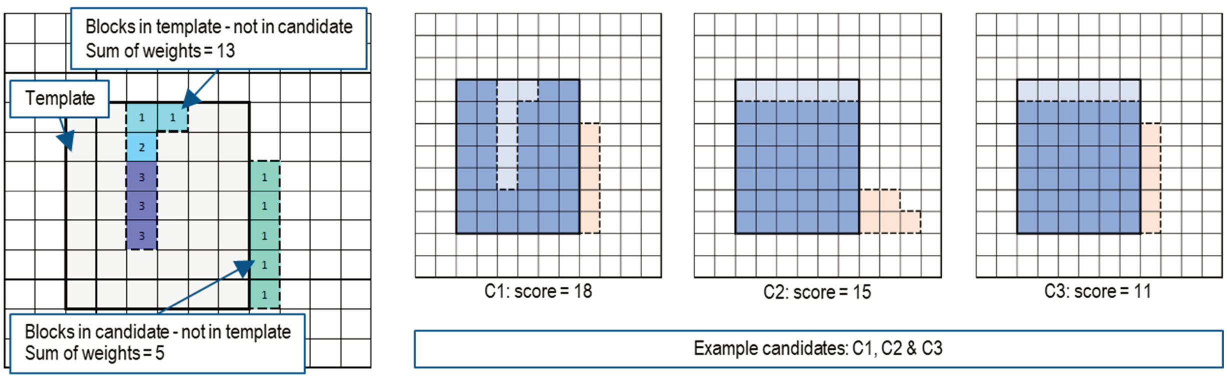

Figure 3 Score example (three candidate shapes with identical non-weighted scores)

The direct application in a calibration exercise using data from producing block caves is straightforward. In green fields projects comparison to an 'ideal cave' assumption could be used to reduce geomechanics risks, such as identification of wedges and mobilising blocks. This in turn could lead to fragmentation problems. Careful study of results would certainly expose these events but in an automated process with larger numbers of iterations (DoEs with 100 individual simulations are practical in this context) above process provides significant time-savings.

2.4 Grade comparison metrics

In production, reconciliation of forecasts with data from grade sampling, either early in the flow of material at the drawpoint or later at crusher or plant is important for operations. The strength of the cellular automata approach with its simulation of material movement inside the muck pile can be realised in the detailed output of drawpoint tons and grades for each interval. This provides an opportunity to use this granularity for financial analysis in all scenarios and in particular for operating block caves to test and refine model calibration.

Real-life data from drawpoint sampling is subject to several challenges. First, the frequency of samples can be anywhere from zero to dozens of samples per month. Data formats do not seem to be standardised and most likely information is listed in a simple list (CSV). Drawpoint name and calendar date need to be transformed into a regular format that suits comparison (scoring value or metric) with simulation output. To accelerate this process, BIOVIA Pipeline Pilot's machine learning toolset was used to perform data cleaning and aggregation. The data pipeline for this process is shown in Figure 4.

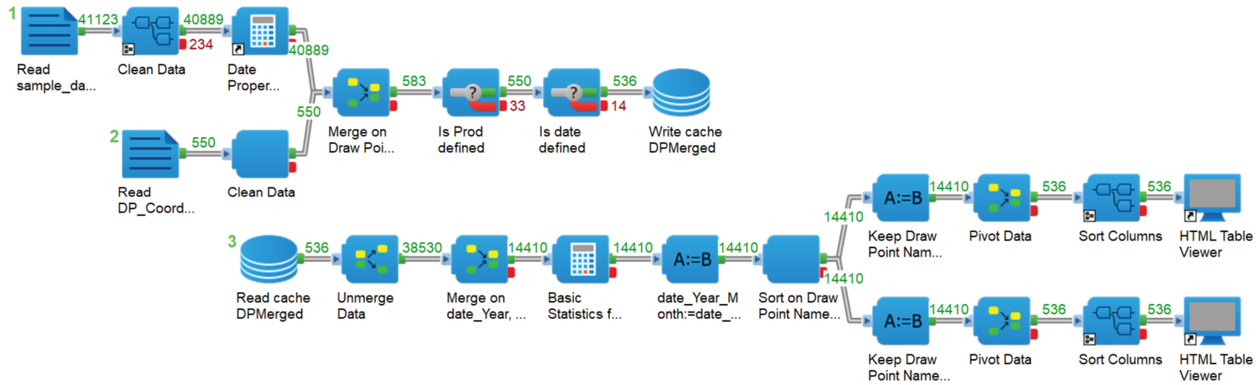

Figure 4 Pipeline merging sampling data by month for each drawpoint

Typically, several hundred data points out of 10 thousands would be rejected. The remainder is placed into monthly buckets for each identified drawpoint for analysis. To reduce the impact of variability, the resulting table is weighted by sample frequency for the calibration metric, whilst the tons produced (monthly at each drawpoint) can be used as a weight for assessing the quality of matching the downstream material flow.

Combined, this contributes one of the weighted scores in the workflow shown in Figure 1 (DoE workflow).

2.5 Material modelling

Constitutive behaviours for block caving have been the topic of intense discussion in the industry. Whilst the Mohr–Coulomb (MC) constitutive model used in this work could be regarded as a simple choice, two extensions of the model behaviour provide sufficient flexibility to cover realistic caving behaviours. The reason for this compromise is computational efficiency, where, first, using a material model native to the code is highly optimised internally and, second, handling nonlinearity inside the constitutive model adds the potential for numerical instability. Using a Generalised Hoek–Brown envelope as starting point, the MC material is extended by using:

1. Strain softening. After reaching a peak strength the material gradually decreases the cohesion as a function of plastic strain. The friction angle remains constant in this case, see Figure 5a. Although the bi-linear option has been used throughout the applications, more points (multi-linear) can be used to refine the softening stages. It is worth noting that at some stage the second or third stage damage competes with criteria for material to be considered part of the muck pile (failure strain), which for the material translates into the governance of the cellular automata domain (CA3D).

2. Material layering using a Hoek–Brown (HB) fitting envelope as shown in Figure 6. This addresses the challenge that in realistic caving behaviours the confining stress changes the equivalent friction angle (and cohesion) in a non-linear relationship and became apparent when calibrating models with cave heights exceeding 1,000 m. This also provides better comparison with more realistic models such as demonstrated by the Itasca constitutive model for advanced strain softening for caving (Ghazvinian et al. 2020) (Figure 5b).

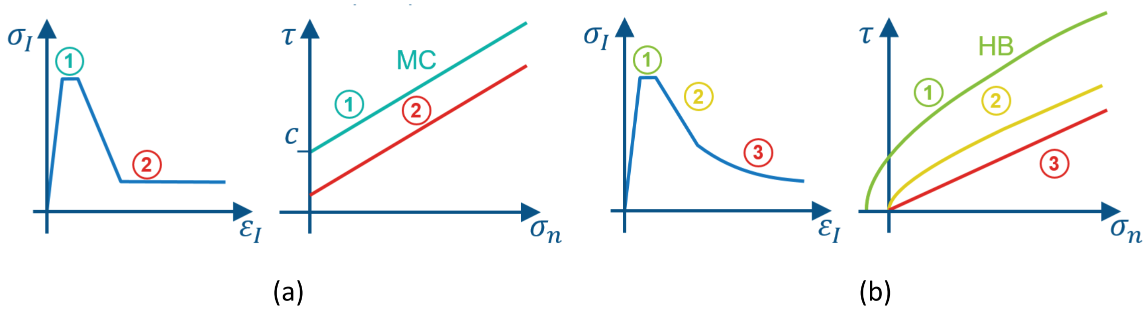

Figure 5 Strain softening (a) Mohr–Coulomb (MC) compared to (b) IMASS model (Ghazvinian 2020 et al.)

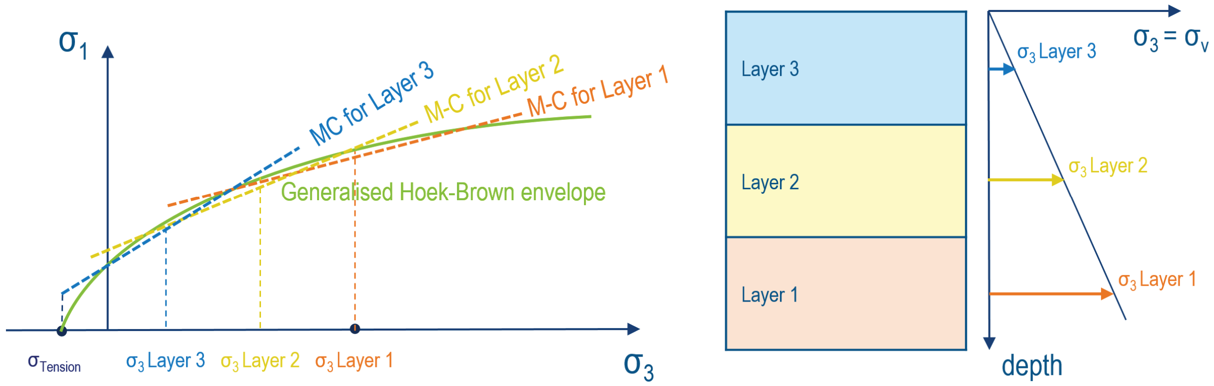

Figure 6 Material layering using a Hoek–Brown (HB) fitting envelope

2.6 Summary of the developments

The journey from integrated simulation and optimisation tools for production scheduling to heavily automated workflows for DoE established a wide ranging functionality in an interesting toolset. Many details of the above discussion might seem rather academic in nature but enable essential operations to be performed efficiently. Discussions with industry partners reveal the concerns about using the increasing amount of data acquired at block caving operations and the challenge to understand model input, where the list of key parameters (some of which are impossible to determine accurately) can exceed well over one hundred. A non-exhaustive list of example parameters would include strength of each domain (translated into constitutive model parameters), swell factor, stress anisotropy and orientation, and flow and mixing parameters.

3 Application in economic and geomechanics risks

The Regal deposit (Bui 2014; Villa 2014) is a fictitious orebody modelled as a massive porphyry copper deposit similar to many of the large block cave mines. This dataset has been used to evaluate economic and risk metrics for the work shown in this chapter. This is mainly driven by time constraints in gaining approvals to publish real-life project data from current block cave projects. Doubling as a test model and training dataset, a small footprint was chosen to enable rapid execution with no unusual hardware requirements.

Assumed block value contours (NSR) are used to illustrate basic model characteristics as shown in Figure 7, with a middle section of blocks coloured on the left (a) and blocks exceeding a higher threshold on the right (b). A simple production schedule is used as the base case with a diagonal drawpoint opening sequence starting in the back right corner in this view.

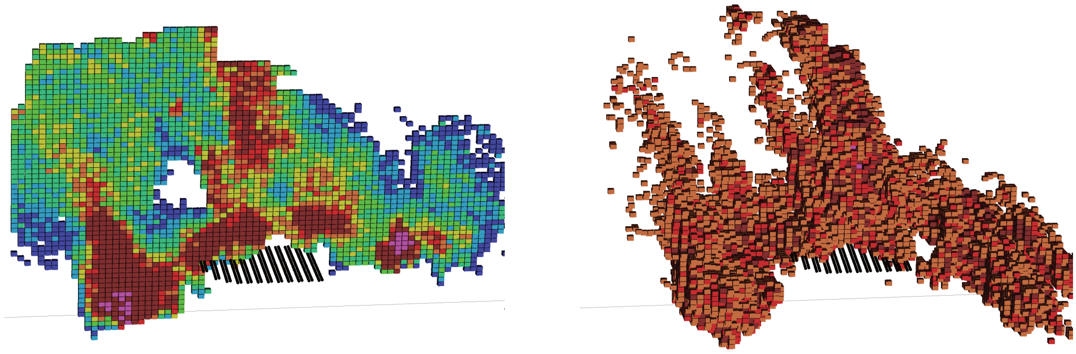

Figure 7 PCBC CA3D model of the Regal training dataset with (a) NSR as contour plot, (b) blocks >AUD 40

3.1 Financial risks

Generally, a coupled simulation will tend to have a thinner (dome shaped) cave and access materials higher up in the central region earlier than predicted from a template mixing (unconstrained) simulation, and materials at the fringes of the cave enter the muck pile later than expected. Over a four year period, with the production schedule assumed for the example to provide enough time for the cave to break through to surface, and using a constant discount rate of 7%, the net present value (NPV) after these 48 monthly steps will differ by more than 10% between an unconstraint cave flow simulation and the coupled simulation.

More interesting is the observation that small changes in rock mass strength parameters (friction angle and cohesion ±5%) can lead to more than 5% difference in NPV in the first five years, shown in Figure 8. This is caused both by cave shape (changes in width and height) and by a small airgap developing that can induce rilling on the surface of the muck pile (Haflil et al. 2014), possibly leading to unintended vertical and horizontal material movement.

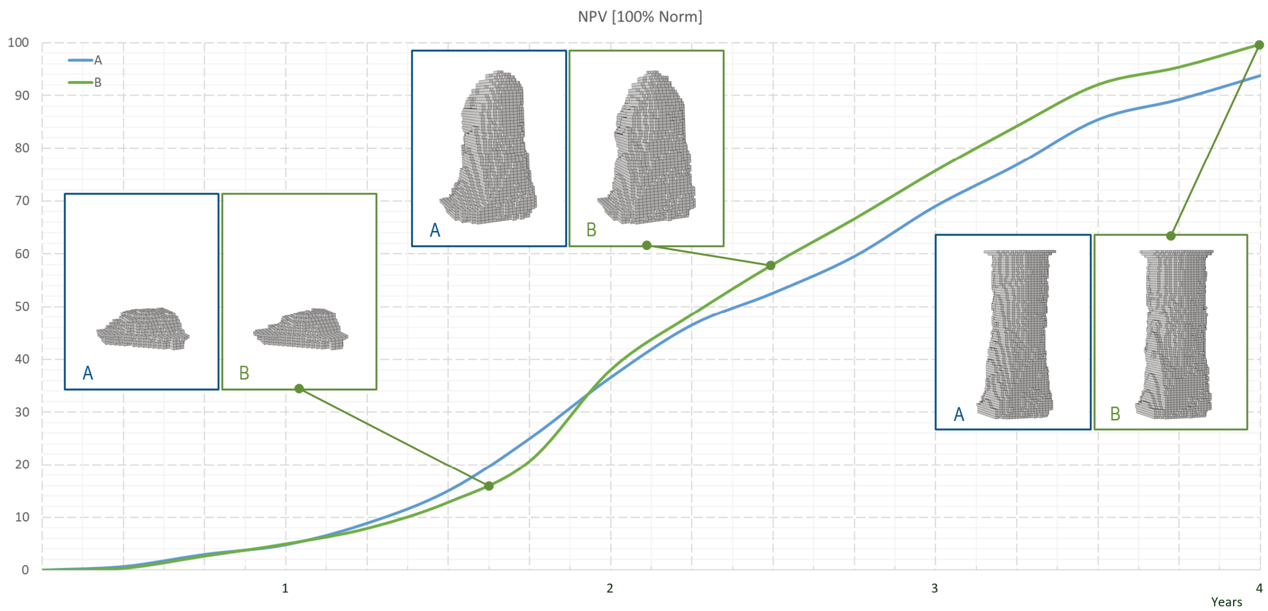

Figure 8 Relative NPV scaled to 100% for two scenarios selected from a DoE over four years

Given that it is common in the industry to perform extensive analysis and optimisation using the assumption of unconstrained flow, this observation provides a clear business case to include geomechanics earlier in the design work, where adjusting ore reserves based on geomechanics is not a new concept (Burgio & Diering 2008).

3.2 Geomechanics risks

This section investigates the sensitivity of the caving simulation to produce an airgap. From the above workflow, the metrics for cave growth in height over time and airgap volume are analysed using DoE. The three variables that are scaled using factors in a design matrix are:

1. Swell factor (all materials).
2. Cohesion scaling factor (all materials).
3. Friction angle scaling factor (all materials).

The variety of outcomes, each plotting a graph for cave height over time and airgap volume, is shown as an overview of all scenarios in Figure 9. The arrangement of scenarios in the DoE, the way the parameter permutation is created (known as a 'Latin Hypercube'), is provided in Table 1.

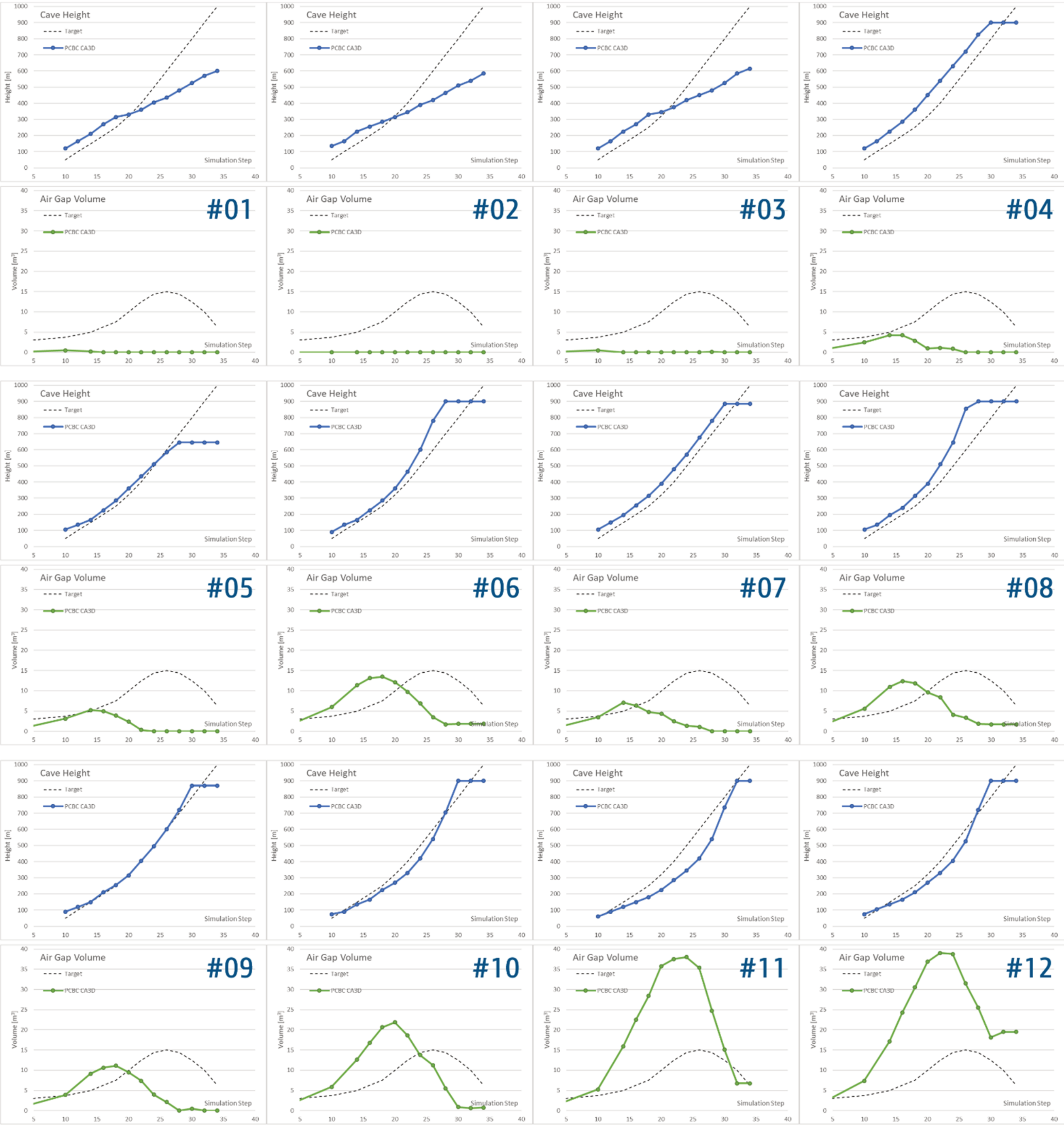

Figure 9 DoE scenarios of cave propagation (m) (blue), and airgap (m³) (green)

Table 1 Design matrix (list of parameters for DoE – Latin Hypercube)

#	Cohesion	Friction angle	Swell factor	#	Cohesion	Friction angle	Swell factor
1	1.0	1.055	1.123	7	1.218	1.123	1.140
2	1.036	1.096	1.157	8	1.255	1.082	1.200
3	1.073	1.109	1.174	9	1.291	1.068	1.217
4	1.109	1.041	1.166	10	1.327	1.136	1.191
5	1.146	1.014	1.132	11	1.364	1.027	1.149
6	1.182	1.0	1.183	12	1.4	1.15	1.208

To illustrate the variety of above 12 scenarios, an early stage visualisation for the cave and airgap is shown in Figure 10. The cave shape (grey) and airgap (black outline) is shown in combination with principal stress tensors in a vertical section.

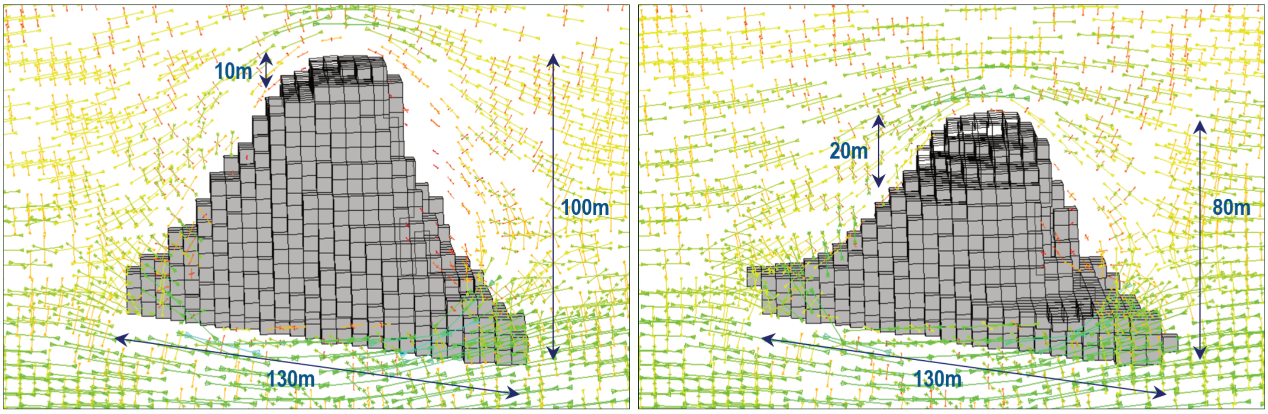

Figure 10 Early stage (1.5 years) cave and airgap development for scenarios (a) #05 and (b) #10

An additional outcome of DoE is the quantification of the influence of each parameter to the chosen metrics, which can be shown in a Pareto plot (or sensitivity matrix) as shown in Figure 11. In the case of larger numbers of variables with uncertainty, this can support prioritisation of further efforts in either data acquisition or subsequent analyses.

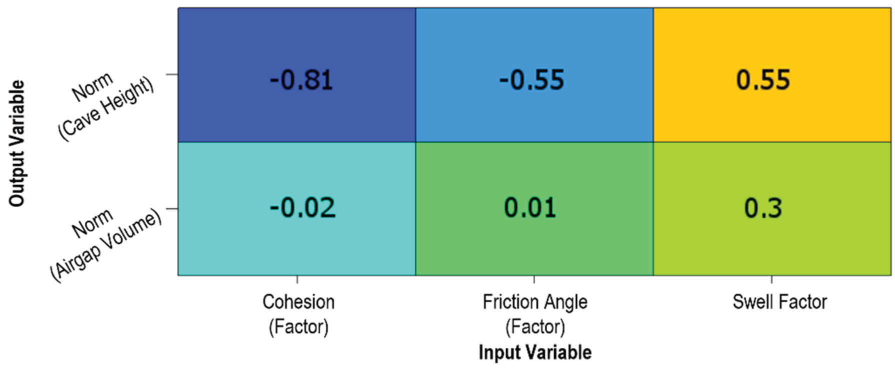

Figure 11 Pareto plot (sensitivity matrix) of design of experiments input and output parameters

The discussion about allowable airgap size from simulation results might be a difficult one, as the industry is very much aware of the associated risks. Returning to the conceptual model of caving (Duplancic 2001; Cumming-Potvin et al. 2018) it is necessary to appreciate that the caving process zone requires a certain amount of space for material to be able to move downwards. This is more black and white in a simulation, where blocks with air have (approximately) zero stiffness and drive the tensile and low confinement failure modes in the constitutive model required for cave propagation.

Whilst the choice of variables here, swell factor, cohesion and friction angle, are obviously not under anyone's control, the sensitivity to these and other variables such as direction of the major principal stress (often associated with significant uncertainty and a low number of available measurements) can support making more robust decisions in the design stage.

4 Conclusion

The methodology and workflows using DoE allow the mine design, planning and scheduling engineers to generate and analyse a variety of life-of-mine production scenarios in the study phase to derive plausible business risk profiles (central, upside and downside) that match industry benchmark, improve mine design resilience and adaptability with the improving orebody knowledge. As a further benefit critical infrastructure location and its offset distance from the cave can be optimised to minimise abutment and cave induced stress interaction. Cave shapes and propagation from the DoE provide a more realistic approach to assess scheduling, metal recovery and value impacts. This is a risk adjusted approach to vertical free flow cave propagation and can be applied to trade-off studies between a single-lift footprint versus multi-lift caving scenarios.

Workflow automation and methods like DoE combine two powerful tools to better understand the range of outcomes, important where we are exposed to aleatoric uncertainty (randomness in the physical world) and epistemic uncertainty (lack in our knowledge of the physical world).

Acknowledgement

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