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Size effects assessment of mine waste-rock shear strength combining numerical, laboratory and in situ approaches

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Abstract

With the aim of performing stability analyses of waste-rock (WR) piles, the critical shear strength of loose WR material must be characterised. However, due to the presence of oversized rock clasts, shear tests can only be carried out on small samples prepared using grading scaling techniques. In order to test samples similar to the field material, particle size reduction should be minimised. Considering a testing device able to handle samples of characteristic size X, the material should be scaled down to a maximum particle size d_{max}, given by the minimum sample aspect ratio $\alpha = X/d_{max}$ allowing a representative elementary volume (REV). However, worldwide geotechnical standards do not agree on minimum α values, and its effects on the mechanical behaviour of coarse samples remain poorly understood. Based on numerical, laboratory and large in situ shear testing approaches, this paper presents a comprehensive study on the effects of sample size and grading on the critical shear strength of WR materials. The main objectives are to analyse the minimum α required for a REV in shear testing and to study the suitability of the scalping technique to assess the critical shear strength of mine WR. We study this topic through three methodologies: (1) shearing simulations in the frame of the discrete element method, (2) experimental lab tests using medium to large triaxial devices, and (3) a large in situ direct shear test. We cover a wide range of α from 4 to 45. The results show that changes in grading through the scalping technique do not affect the critical shear strength for $\alpha \geq 12$, which is higher than some widely applied international standards requirements.

Keywords: *shear strength, waste rock, discrete element method simulations, laboratory triaxial tests, in situ testing*

1 Introduction

One of the most challenging tasks when performing stability analyses of high waste-rock (WR) piles is the material strength characterisation. Since coarse WR samples cannot be tested in standard laboratory devices, practitioners usually use data from reported compilations (Leps 1970; Barton & Kjærnsli 1981; Idraratna et al. 1993; Ovalle et al. 2020). Alternatively, small-scaled samples of WR material can be prepared using the scalping technique, which involves removing oversized particles that do not satisfy the minimum aspect ratio α recommended by geotechnical standards. This ratio is defined as $\alpha = X/d_{max}$ (where X is the characteristic sample size and d_{max} is the maximum particle size). However, information on the minimum α that is required for a representative elementary volume (REV) is limited. Moreover, as shown in Table 1, American, British and Japanese standards disagree on minimum α. For instance, the recommended values vary between 6 and 20 for triaxial testing.

Once a coarse granular material is scalped, the particle size distribution (PSD) changes to a more uniform distribution (i.e. lower C_u), which affects the mechanical behaviour. However, it has been systematically shown that these effects do not apply to the critical shear strength (Muir Wood & Maeda 2008; Voivret et al. 2009; Li et al. 2013; Yang & Luo 2018; Cantor et al. 2018; Polania et al. 2023). Therefore, the scalping

technique is an effective method for characterising critical strength, only if particle shape and mineralogy are constant along different grain sizes (Ovalle & Dano 2020; Carrasco et al. 2022, 2023).

In order to handle the coarsest possible sample, most authors that have reported data on coarse materials, such as rockfills and WR, tested samples having low α around 6, as recommended by the standard ASTM 7181 (ASTM International 2020; Marsal 1967; Marachi 1969; Linero et al. 2007, 2020; Bard et al. 2012; Ovalle et al. 2014). However, recent studies have shown that such values of α might be too low to ensure a REV (Cerato & Lutenegger 2006; Wu et al. 2008; Deiminiat et al. 2022).

Further research is needed to understand sample size effects and define size limitations for a REV. This paper presents a comprehensive numerical and experimental study on the effects of sample size and grading on the critical shear strength of WR material. We use the discrete element method (DEM) for numerical shear tests, and laboratory and in situ physical testing. The main objectives are to validate the minimum sample aspect ratio (α) required for a REV in shear testing, and to study the applicability and reliability of the scalping technique to assess the critical shear strength of mine WR.

Table 1 Representative sample scales covering the standards from the American Society for Testing and Materials, the British Standards Institution and the Japanese Geotechnical Society

Shearing test	Standard	α
Direct shear (H and W correspond to height and width of the box)	ASTM D3080[1]	$W/d_{max} \geq 10$ and $H/d_{max} \geq 6$
	BS 1377–7[2]	$H/d_{max} \geq 10$
	JGS 0561[3]	$H/d_{max} \geq 23.5$
Triaxial (D corresponds to the diameter of the sample)	ASTM D7181[4]	$D/d_{max} \geq 6$
	BS 1377[2]	$D/d_{max} \geq 5$
	JGS 0520[5]	$D/d_{max} \geq 20$
	JGS 0530[6]	$D/d_{max} \geq 10$; for samples of D = 300 mm

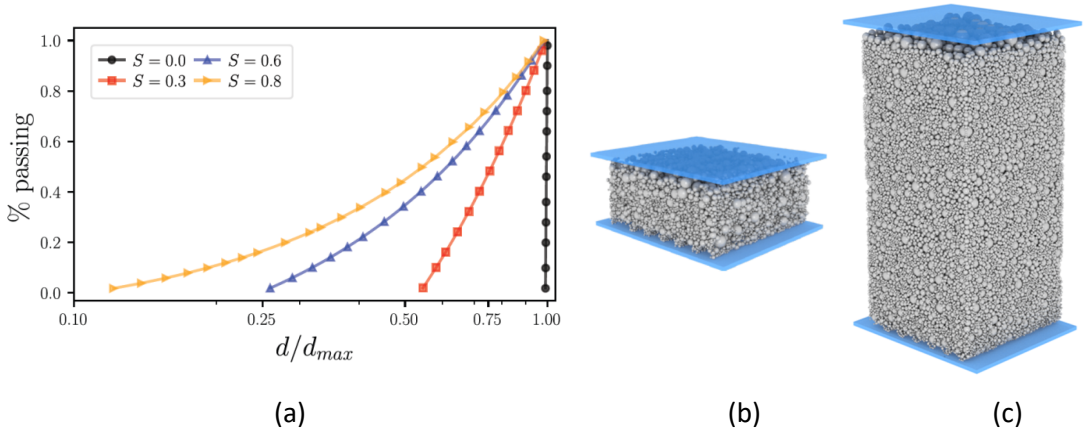
[1]ASTM International (2011), [2]British Standard Institution (1990), [3]Japanese Geotechnical Society (2015a), [4]ASTM International (2020), [5]Japanese Geotechnical Society (2015b), [6]Japanese Geotechnical Society (2015c)

2 Discrete element methods simulations

The DEM is a valuable method for studying the mechanical properties of granular media. It allows for a varying sample scale and PSD, and provides a detailed view into micromechanical sources of the macromechanical observations, which remains a great challenge in physical testing. It is worth noting that DEM simulations are used in this study as an indicative result, in order to estimate the effect of sample scale, which is later compared with tests on real waste-rock material. We used the non-smooth contact dynamics (CD) DEM method to simulate samples of spherical grains interacting via frictional contacts, implemented in the LMGC90 open-source platform (Dubois & Jean 2022). Compared to other DEM strategies, the CD method uses a mathematical framework in which the motion equation is implicitly integrated in a time-stepping scheme, without the need for regularisation parameters, such as contact stiffness or damping coefficients (Jean 1999). As a result, the CD method is unconditionally stable and can use larger time steps (Renouf et al. 2004).

Several numerical granular samples composed of spheres were generated. The grain size distributions were characterised using the grain size span $S = (d_{max} - d_{min})/(d_{max} + d_{min})$, where d_{min} is the minimum particle diameter. In all the cases, d_{max} was set to 2 cm. As shown in Figure 1a, we created samples having S values of 0, 0.3, 0.6 and 0.8. As S grows, the sample becomes better graded and $S = 0.8$ gives a ratio $d_{max}/d_{min} = 9$. When changing the height (H) of the simple shear box, the aspect ratio $\alpha = H/d_{max}$ varied between 4 and 45. Figure 1b illustrates numerical samples after depositing the grains into boxes.

The numerical assemblies were subjected to periodic simple shear up to deformation of 1,000% to be able to characterise the critical state properties over a large deformation interval. The critical state is defined at constant stress state and constant volume, under continuous shear deformation, which is consistently identified after a shear strain greater than 20%. Thanks to the periodic lateral boundaries, any particle reaching the lateral limit automatically reappears at the opposite extreme of the sample, thus allowing large deformations. Simple shearing was applied under constant vertical pressure $P = 10$ kPa. Quasi-static shearing conditions were ensured, keeping a constant inertial number $I = \dot{\gamma} d_{max} \sqrt{\rho / P} = 10^{-3}$, where $\dot{\gamma} = v/H$, v is the lateral velocity of the top and bottom walls, and $\rho = 2{,}600$ kg/m^3 (GDR-Midi 2004). The friction coefficient between grains was set to 0.4. More details about the simulations can be found in Cantor & Ovalle (2023).



(a) (b) (c)

Figure 1 DEM numerical samples: (a) Particle size distributions sample screenshots with grain size span S = 0.8; (b) α = 5; (c) α = 25 (not at scale)

Figure 2 presents the critical internal friction angle (ϕ_{cr}) for all the numerical samples tested. ϕ_{cr} is between 19° and 21°, which are typical values reported after laboratory physical tests on glass beads with different gradings (Liu et al. 2014; Hazzar et al. 2020; Polania et al. 2023). The results indicate that strength is significantly affected by the sample scale for $\alpha < 15$. Namely, higher shear strength is obtained at low α. In samples having $\alpha > 15$, shear strength stabilises and becomes independent of the grain size distribution, which is consistent with reported data on the effect of the PSD on critical strength.

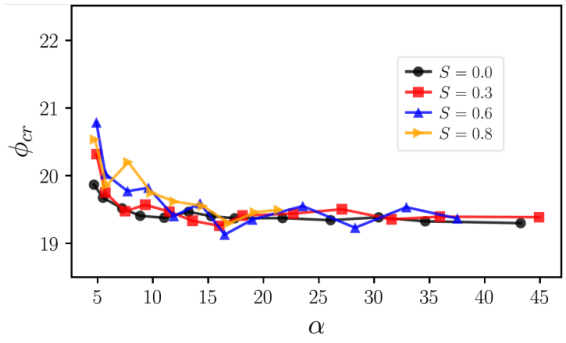
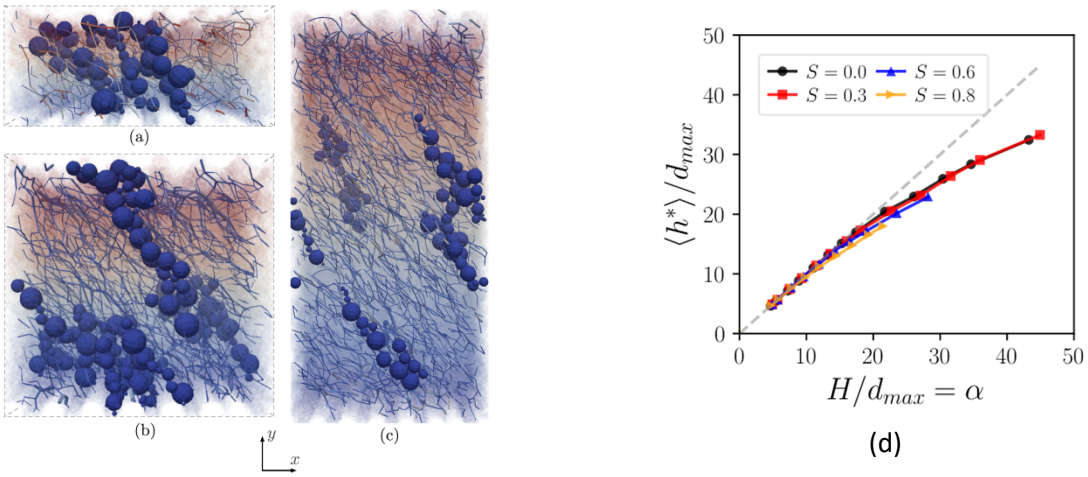


Figure 2 Results of DEM numerical simple shear tests: critical friction angle

Micromechanical analyses of our DEM simulations indicate that size effects are due to large heterogeneity of stresses within the sample for low values of α. We have identified column-like structures that could be responsible for stress heterogeneity within the samples. This was done by setting an interparticle force cutoff, allowing the granular structure to form individual series of grains in contact – in other words, detecting the set of largest forces capable of joining grains only through single contacts (see details in Cantor and Ovalle, 2023). The results are shown in Figures 3a, 3b and 3c for samples of α varying from 5 to 43, where the local rigid structures detected during critical state shearing are highlighted in dark blue. These structures carry a high proportion of the external forces and, at low α (Figures 3a and 3b), are

capable of connecting the boundary walls by forming column-like configurations. However, if the size of the sample is large enough (Figure 3c), the columns formed are not able to join the boundary walls, resulting in lower stress concentration on them. Figure 3d shows a comparison between the average height of the column-like structures at critical state (h^*) and the height of the sample (H). It can be observed that h^* is comparable to H up to α~15, which is around the same value for REV conditions (i.e. stable macroscopic critical shear strength in Figure 2).

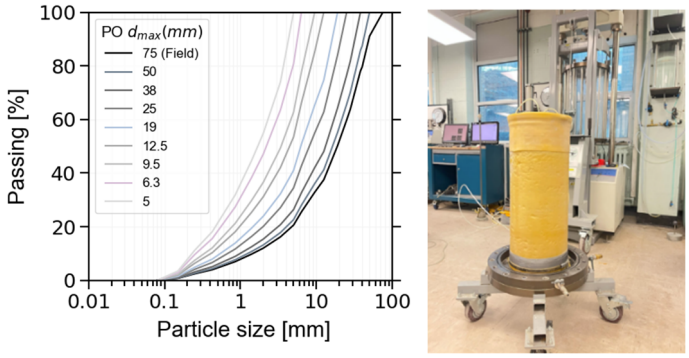


(d)

Figure 3 Screenshot of samples with S = 0.8 and sample scale (a) α = 5, (b) α = 20, and (c) α = 43, highlighting in dark blue the local rigid structures detected during critical state shearing; (d) Evolution of the normalised average height of the local rigid structures as a function of the sample scale α

3 Laboratory triaxial tests

A WR material called porphyry (PO) from a gold rock mine was used for laboratory testing. PO has a uniaxial compression strength value of 250 MPa. The sample had $d_{max} = 75$ mm and was scalped several times down to $d_{max} = 5$ mm, as presented in Figure 4. For more than 45 particles per each particle size fraction, we have characterised particle shape before scalping to verify whether they remained unchanged along particle sizes. We have computed sphericity (S = particle width/particle length) and roundness ($R = \sum(r_i/N)/r_{in}$, where r_i is the radius of circles fitting particle concave corners, N the number of the fitted circles, and r_{in} the radius of the largest inscribed circle), using the algorithm proposed by Zheng and Hryciw (2016). PO material presented ranges of $R = 0.3$-0.4 and $S = 0.6$-0.7 for all particle sizes, which can be considered in the same range concerning the grain shape description.



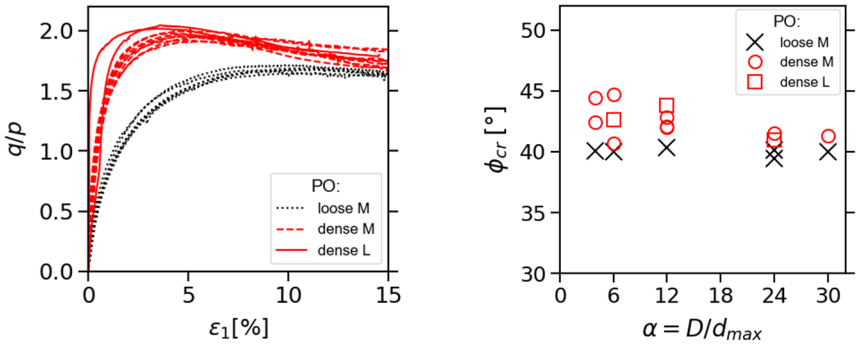
(a) (b)

Figure 4 WR samples: (a) Particle size distributions of the scalped materials; (b) Photos of a large sample L of D = 300 mm

Two triaxial cells were used for medium (M) samples of $D = 150$ mm and large (L) samples of $D = 300$ mm. In total, 19 drained triaxial shear tests were carried out on dry, loose and dense samples. Loose samples were prepared without compaction by simply pouring layers of homogeneous material. Dense samples were prepared using 25 standard Proctor hammer blows uniformly distributed over the surface of each layer. Dry densities (γ_d) obtained varied for each case, since gradings were different due to scalping. γ_d of loose samples varied between 18.05 and 18.90 kN/m^3, while dense samples varied between 21.45 and 25.62 kN/m^3 (specific gravity of solids is $G_s = 2.69$). A confinement pressure of 150 kPa was used for all tests.

Figure 5a presents the stress–strain behaviour in terms of the stress ratio q/p versus axial strain (ε_1) for all tests carried out, where q is the deviatoric stress and p is the mean stress. As expected, loose samples presented monotonic stress incrementally up to their maximum strengths around $\varepsilon_1 = 10\%$, which remained almost constant up to $\varepsilon_1 = 15\ \%$. However, dense samples attained their peak strengths around ε_1 ~3%, followed by post-peak strength softening towards critical strength at large strains. In this paper, the mobilised critical shear strength (ϕ_{cr}) is assumed at $\varepsilon_1 = 15\ \%$, for comparisons. The results on loose samples exhibited mean $\phi_{cr} = 40°$. Dense samples showed average peak strength around $\phi_p = 48°$. However, their mean $\phi_{cr} = 42.4°$ was slightly higher than loose samples. This might be related to strain localisation and heterogeneous strain field after peak strength. The obtained peak and critical state values are in the range of data reported by previous studies on WR materials (Linero et al. 2007; Valenzuela et al. 2008; McLemore et al. 2009; Ovalle et al. 2014, 2020).

Figure 5b illustrates ϕ_{cr} for all tests against α. It can be observed that loose samples presented almost constant strength over α varying from 4 to 30 (scatter is less than 1°). This observation suggests that the scalping technique is appropriate to assess the critical shear strength of loose samples and confirms that the PSD does not affect the critical strength. However, stable values of ϕ_{cr} obtained from dense samples can be observed only in samples with $\alpha > 12$, possibly because the critical state is unlikely to be reached in dense materials under triaxial conditions. Nevertheless, ϕ_{cr} in samples having $\alpha = 24$ and 30 are significantly stable for loose and dense samples, including tests on M and L samples. This last result is consistent with DEM numerical tests, where ϕ_{cr} was stable only for $\alpha > 15$.

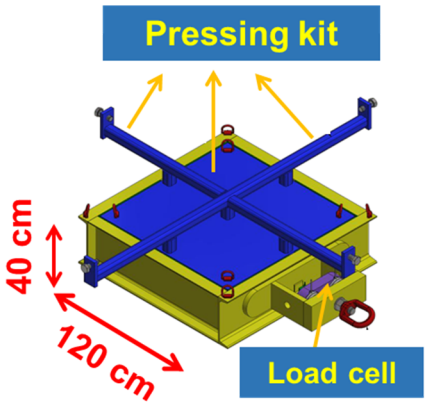


(a) (b)

Figure 5 Triaxial tests results ($\sigma_3 = 150$ kPa): (a) Stress–strain curves; (b) Critical friction angle

4 In situ tests

Based on the approach of Matsuoka et al. (2001), a large in situ direct shear test was developed. As shown in Figure 6a, the device consists in a large rigid metallic box of square section of $L = 120$ cm and $H = 40$ cm. Five WR-fill platforms of sections 8 × 4 m were prepared on top of a WR pile (Figure 6e). The material of each platform was composed of loose PO WR having a d_{max} of 75 mm, allowing $\alpha = H/d_{max}$ ~5.



(a)

(b)

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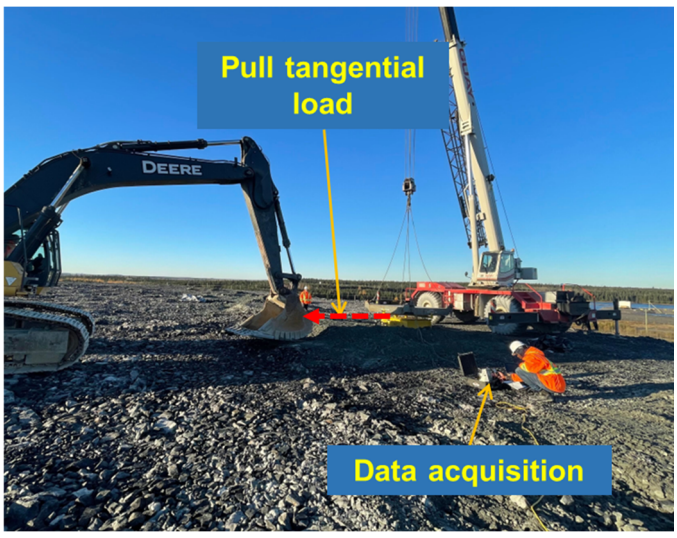
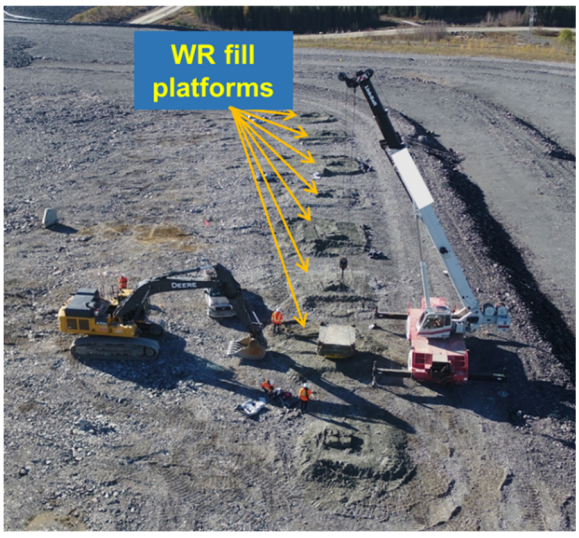


(e)



(f)

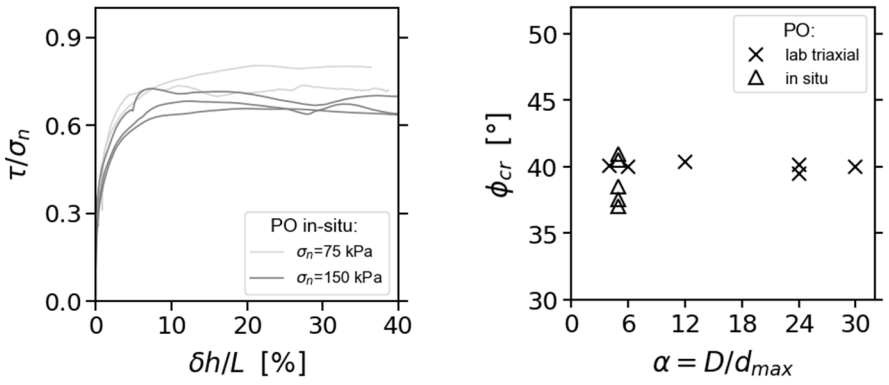
Figure 6 In situ large direct shear tests: (a) Scheme of the 120 × 120 × 40 cm shear box; (b) The box placed on a WR platform and being filled with loose material; (c) The box filled with WR material; (d) Placing counterweights (10 tons each); (e) Lateral pulling with an excavator shovel and data acquisition system; (d) General view of several platforms where tests were carried out

To carry out a test, the box was placed on top of a platform and then filled with the same material as the platform (Figure 6b). Then, a pressing kit acting as the loading plate (Figure 6a) covered the sample contained in the box, over which 10-ton counterweights were placed by a crane (Figure 6d). Each load

generated a normal stress of 75 kPa on the WR sample; different tests were done at normal stresses of $\sigma_n = 75$ and 150 kPa. The test set-up was instrumented with two load cells calibrated at 100 kN each, and these were installed on the front side of the box (Figure 6c). Two potentiometers extending up to 1.5 m each were referred to an external fixed point for horizontal displacement measurements. All the instruments were connected to a data acquisition system shown in Figure 6e.

Once the counterweight load was placed, the box was linked to an excavator hook using a flexible chain and then laterally pulled at a rate of 3 cm/s. The test was carried out until horizontal displacement of around 40% of the box size L was reached.

Figure 7a presents the stress–strain behaviour in terms of the shear ratio τ/σ_n versus normalised horizontal displacement for all in situ tests carried out. It can be observed that large displacements allow critical shear strength reaching beyond 20% of L. The results on in situ loose samples exhibited a mean value of $\phi_{cr} = 39°$, which is very similar to the triaxial mean value obtained after triaxial tests on loose samples of $\phi_{cr} = 40°$. In terms of size effects, Figure 7b compares laboratory and in situ results, indicating that data scatter is still too high at α used for in situ of 5.



(a) (b)

Figure 7 In situ tests results: (a) Stress–strain curves; (b) Critical friction angle and comparison with triaxial tests

5 Conclusion

In this paper, we have presented a numerical and experimental study on sample size effects on critical shear strength of coarse granular materials. DEM simulations of simple shear indicate that a REV for critical strength can be obtained only for aspect ratio $\alpha \geq 15$. On the other hand, laboratory triaxial tests on WR material showed that REV conditions are attained independently of α if the samples are initially loose. However, dense samples do not exhibit representative critical strength for sample scales $\alpha < 12$, while results for $\alpha = 24$ and 30 are systematically stable. The latter results suggest that α for dense triaxial samples should be between 12 and 24, which is consistent with $\alpha \geq 15$ found in DEM simulations. Large in situ direct shear tests result in similar critical internal friction angle as laboratory triaxial tests, but scatter is significant probably due to a low value of $\alpha = 5$.

Further studies on different WR materials are needed to confirm these results and to determine a precise minimum value of α for samples having REV. In the meantime, it seems clear that the recommendations of some widely used standards are not satisfactory and should be revisited.

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