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Probabilistic analysis of batter scale slopes: bridging the gap between Factor of Safety and Probability of Failure

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Abstract

Slope stability analysis in structurally controlled rock mass domains often relies on kinematic analysis, which overlooks the inherent variability of material strength parameters. Consequently, designs derived from this conservative approach frequently fall short of meeting the targeted design acceptance criteria when compared to actual field performance.

This study introduces an analytical solution for probabilistic analysis of batter scale slopes subjected to planar sliding, with consideration to the multivariate distribution of the input parameters. A comparative analysis between the results obtained from the proposed analytical method and conventional approaches reveals the conservative nature of the latter, leading to reduced design batter face angles.

By addressing the limitations of conventional kinematic analysis and highlighting the conservatism in design, this research contributes to the field of geotechnical engineering, offering valuable insights for improved slope stability assessment in structurally controlled domains.

Keywords*: slope optimisation, probabilistic analysis, planar sliding*

1 Introduction

Advancements in slope stability analysis methods have focused on overall and inter-ramp analysis, however, current industry standard methods of batter scale stability analysis still heavily rely on kinematic and deterministic limit equilibrium techniques (Teet et al. 2013; McQuillan et al. 2020). Moreover, observations of as-built batter slopes shows their performance exceeded design targets (Teet et al. 2013), suggesting that there is potentially some conservatism in the current design approach.

This conservative observation aligns with the behaviour of batter slopes in the Pilbara region of Western Australia, where slopes are typically structurally controlled and influenced by the orientation of the bedding planes. Within this structural setting, planar sliding is identified as the primary failure mechanism when the bedding planes daylight from the pit slope (Wyllie & Mah 2004).

Despite the common use of kinematic analysis for batter angle design, several limitations are noted for this method. The analysis only adopts the friction angle of the defect and does not consider other strength parameters such as cohesion. Similarly, the analysis doesn't consider other factors which can influence the batter stability such as batter height or unit weight of the rock mass. The most notable limitation of the analysis is its inability to model material strength variability. Without accounting for the variability of the material properties, the analysis represents the probability of occurrence (PoO) as opposed to the Probability of Failure (PoF).

Limit equilibrium techniques are also frequently used in the analysis of batter slopes; however, these analyses are predominately determinist (McQuillan et al. 2020) and generally do not consider bedding variability. Detailed probabilistic analysis of batter slopes can be carried out using geotechnical software packages, however, the calculation is carried out using the Monte Carlo type sampling method and iterative calculations need to be completed to determine design angles.

This paper discusses an alternate method for assessing the PoF of batter scale slopes subject to planar sliding. The method adopts limit equilibrium analysis to calculate deterministic Factor of Safety (FoS) and subsequently calculate the PoF by considering the multivariate distribution of all the relevant input parameters.

2 Method

The stability of rock batter slopes can be assessed using limit equilibrium analysis. The FoS of the slope is calculated as the ratio of the resisting forces to driving forces. For shear type failures, the defect shear strength can adopt a Mohr–Coulomb strength envelope which is expressed in terms of cohesion (c) and friction angle (ϕ). For planar sliding, the equation for FoS can be calculated using Equation 1 (Wyllie & Mah 2004), where ψ_p is the dip of the sliding surface, W is the weight of the sliding block and A is the area per unit width the sliding block, as shown in Figure 1 (based on Wyllie & Mah 2004).

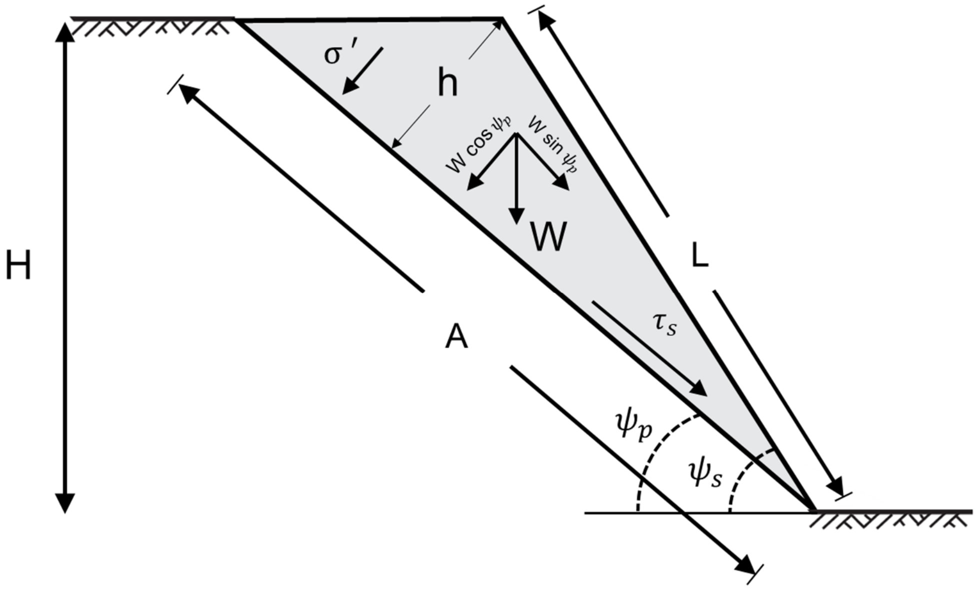

Figure 1 Resolution of W (weight of sliding block) into components parallel with and perpendicular to the sliding plane (ψ_p)

$$FoS = \frac{\mathrm{c\ A + W\ \cos}\psi_p\ \tan\phi}{\mathrm{W\ \sin}\psi_p} \tag{1}$$

Equation 1 can be expanded and expressed in terms of the basic slope input parameters. Considering ψ_s as the batter face angle, H as the batter height and ɣ as the rock unit weight, the parameters A and W from Equation 1 can be expressed using Equations 2 and 3.

$$A = \frac{H}{\sin\psi_p} \tag{2}$$

$$\begin{aligned}
W &= \text{Block Volume } \times \gamma \\
&= \frac{1}{2}\,A\,h\,\gamma \\
&= \frac{1}{2}\,\frac{H}{\sin\psi_p}\frac{H}{\sin\psi_s}\sin(\psi_s-\psi_p)\ \gamma \\
&= \frac{\gamma H^2}{2}\frac{\sin(\psi_s-\psi_p)}{\sin\psi_s\sin\psi_p} \\
&= \frac{\gamma H^2}{2}\frac{\sin(\psi_s-\psi_p)}{\sin\psi_s\sin\psi_p} \\
&= \frac{\gamma H^2}{2}\frac{\sin\psi_s\cos\psi_p - \cos\psi_s\sin\psi_p}{\sin\psi_s\sin\psi_p} \\
&= \frac{\gamma H^2}{2}\left(\frac{\cos\psi_p}{\sin\psi_p}-\frac{\cos\psi_s}{\sin\psi_p}\right) \\
&= \frac{\gamma H^2}{2}\left(\cot\psi_p-\cot\psi_s\right)
\end{aligned} \tag{3}$$

Substituting Equations 2 and 3 into Equation 1, the FoS can be expressed as shown in Equation 4:

$$\begin{aligned}
FoS &= \frac{\mathrm{c\,A} + \mathrm{W}\ \cos\psi_p\ \tan\phi}{\mathrm{W}\ \sin\psi_p} \\
&= \frac{\mathrm{c\,A}}{\mathrm{W}\ \sin\psi_p} + \frac{\mathrm{W}\ \cos\psi_p\ \tan\phi}{\mathrm{W}\ \sin\psi_p} \\
&= \frac{\mathrm{c}\,\frac{H}{\sin\psi_p}}{\frac{\gamma H^2}{2}\left(\cot\psi_p-\cot\psi_s\right)\ \sin\psi_p} + \frac{\cos\psi_p\ \tan\phi}{\sin\psi_p} \\
&= \frac{2\,\mathrm{c}}{\gamma\ H\ \sin^2\psi_p\left(\cot\psi_p-\cot\psi_s\right)} + \frac{\tan\phi}{\tan\psi_p} \\
&= \frac{2\,\mathrm{c}}{\gamma\ H\ \sin^2\psi_p\cot\psi_p\left(1-\frac{\tan\psi_p}{\tan\psi_s}\right)} + \frac{\tan\phi}{\tan\psi_p} \\
&= \frac{2\,\mathrm{c}}{\gamma\ H\ \sin\psi_p\cos\psi_p}\left(1-\frac{\tan\psi_p}{\tan\psi_s}\right)^{-1} + \frac{\tan\phi}{\tan\psi_p} \\
&= \frac{4\,\mathrm{c}}{\gamma\ H\ \sin(2\,\psi_p)}\left(1-\frac{\tan\psi_p}{\tan\psi_s}\right)^{-1} + \frac{\tan\phi}{\tan\psi_p} \qquad \text{where } \psi_s > \psi_p
\end{aligned} \tag{4}$$

Equation 4 shows the relation between the FoS and the direct input parameters: batter geometry (H and ψ_s), rock density (γ), defect strength (c and ϕ) and dip of the sliding surface (ψp). It is noted that Equation 4 is only valid for cases where $\psi_s > \psi_p$ as the dip of the sliding surface needs to be shallower than the slope dip to meet the requirements for planar sliding. For cases where $\psi_s \leq \psi_p$, planar sliding is not expected as a valid failure mechanism.

2.1 Batter angle design

Using Equation 4, an analytical method for batter angle design is proposed. Two approaches, deterministic and probabilistic, are discussed in this paper and described in the following sections.

2.1.1 Deterministic approach

The design methodology for a deterministic approach is based on creating design charts for the slope angle for variable sliding surface dip angle. This is achieved by rearranging Equation 4 to express the slope angle ψ_s as a function of the dip angle ψ_p for a target FoS value as presented in Equation 5:

$$FoS = \frac{4\,c}{\gamma\,H\sin(2\,\psi_p)}\left(1-\frac{\tan\psi_p}{\tan\psi_s}\right)^{-1}+\frac{\tan\phi}{\tan\psi_p}$$

$$\frac{4\,c}{\gamma\,H\sin(2\,\psi_p)}\left(1-\frac{\tan\psi_p}{\tan\psi_s}\right)^{-1} = \left[FoS-\frac{\tan\phi}{\tan\psi_p}\right]$$

$$\left(1-\frac{\tan\psi_p}{\tan\psi_s}\right)^{-1} = \frac{\gamma\,H\sin(2\,\psi_p)}{4\,c}\left[FoS-\frac{\tan\phi}{\tan\psi_p}\right]$$

$$1-\frac{\tan\psi_p}{\tan\psi_s} = \frac{4\,c}{\gamma\,H\sin(2\,\psi_p)}\left[FoS-\frac{\tan\phi}{\tan\psi_p}\right]^{-1}$$

$$\frac{\tan\psi_p}{\tan\psi_s} = 1-\frac{4\,c}{\gamma\,H\sin(2\,\psi_p)}\left[FoS-\frac{\tan\phi}{\tan\psi_p}\right]^{-1}$$

$$\tan\psi_s = \frac{\tan\psi_p}{1-\frac{4\,c}{\gamma\,H\sin(2\,\psi_p)}\left[FoS-\frac{\tan\phi}{\tan\psi_p}\right]^{-1}}$$

$$\psi_s = \tan^{-1}\left(\frac{\tan\psi_p}{1-\frac{4\,c}{\gamma\,H\sin(2\,\psi_p)}\left[FoS-\frac{\tan\phi}{\tan\psi_p}\right]^{-1}}\right) \tag{5}$$

Equation 5 is only valid when the denominator of the expression on the right-hand side is positive. This is an artifact of the conditions required for planar sliding to be a viable failure mode, which is where the defect dip angle must be less than the slope angle and greater than the defect friction angle: $\phi < \psi_p < \psi_s$. Where this condition is not met, the slope is not susceptible to planar sliding and ψ_s of 90° can be adopted as summarised in Equation 6.

$$\psi_s = \begin{cases} \tan^{-1}\left(\frac{\tan\psi_p}{1-\frac{4\,c}{\gamma\,H\sin(2\,\psi_p)}\cdot\left[FoS-\frac{\tan\phi}{\tan\psi_p}\right]^{-1}}\right) & where\ \ 0 \le \frac{4\,c}{\gamma\,H\sin(2\,\psi_p)}\cdot\left[FoS-\frac{\tan\phi}{\tan\psi_p}\right]^{-1} \le 1 \\ 90 & otherwise \end{cases} \tag{6}$$

Equation 6 can be plotted by substituting values for the defect strength (c and ϕ) and rock density (ɣ), and considering the height and target FoS as shown in Figure 2. For the example, the following values were used: c = 8 kPa, ϕ = 25°, ɣ = 33 kN/m^3, H = 10 m and a target FoS of 1.1.

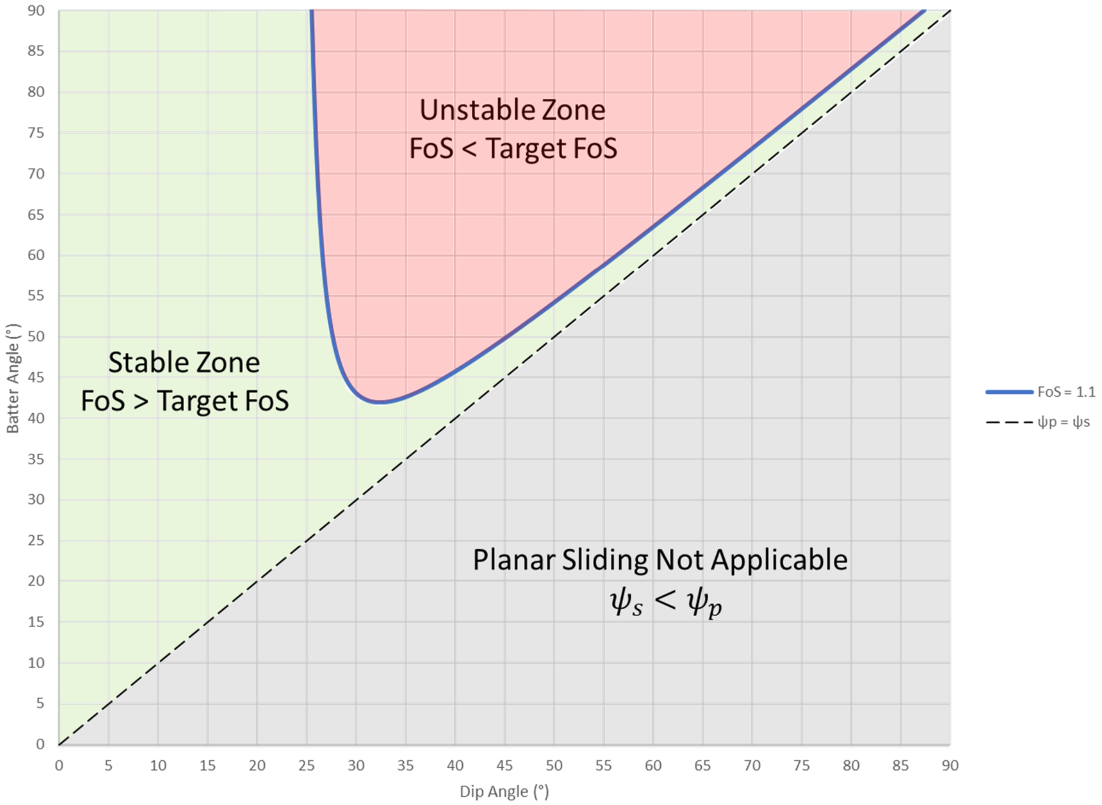

Figure 2 Batter design chart using deterministic analysis

2.1.2 Probabilistic approach

The probabilistic approach calculates the PoF for a given batter angle (ψ_s) based on the variability of the input parameters: c, ϕ, ɣ and ψ_p. This method is based on the assumption that the input parameters (c, ϕ and ɣ) are normally distributed as it is the most common distribution function adopted in most geotechnical engineering applications (Wyllie & Mah 2004). Distribution of the defect dip angle ψ_p is considered based on the observable variability of the parameter.

To calculate the PoF, the distribution of the FoS is determined for a given value of ψ_p and ψ_s. As the input parameters for the FoS are non-negative and normally distributed, the resultant FoS will also be normally distributed as shown in Figure 3 and Figure 4. Based on the distribution of the input parameters, the mean FoS (FoS_μ) and standard deviation (FoS_σ) can be determined. Using these parameters, the PoF is calculated for the condition FoS is less than 1. The calculated PoF ($PoF_{\psi s|\psi p}$) will be for a given set of ψ_p and ψ_s.

Finally, the PoF for a given batter angle ($PoF_{\psi s}$), will be the integration of $PoF_{\psi s|\psi p}$ multiplied by the probability of the occurrence of ψ_p across the range from 0 to 90°. These steps can be repeated at different interval ranges for ψ_s to develop a PoF design chart.

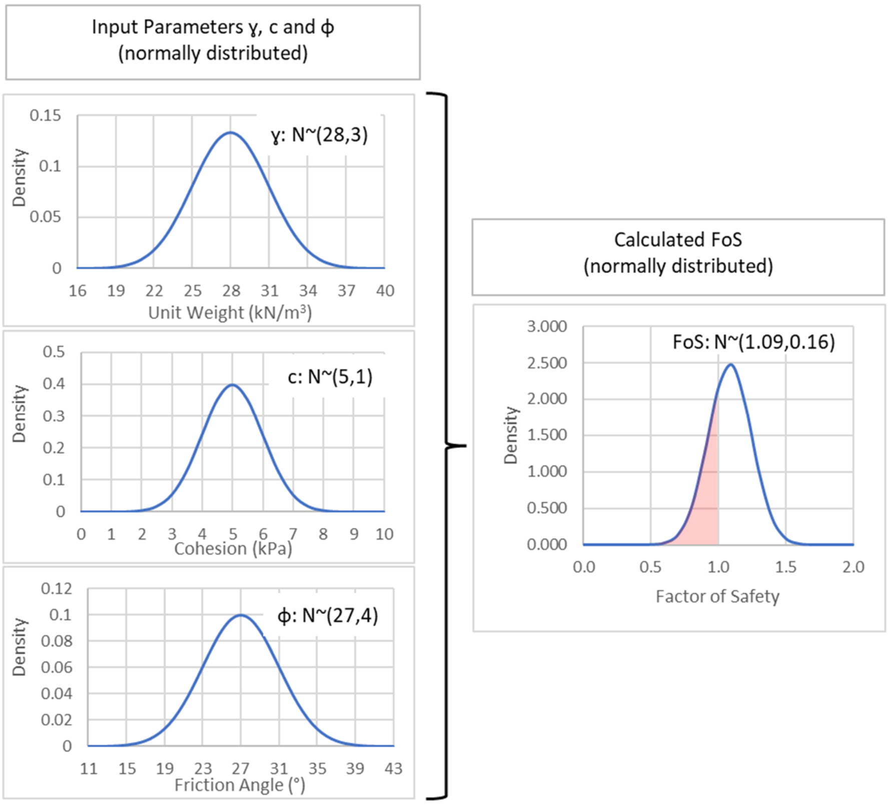

Figure 3 Resultant distribution of FoS based on normal distribution of input parameters c, ɣ and ϕ

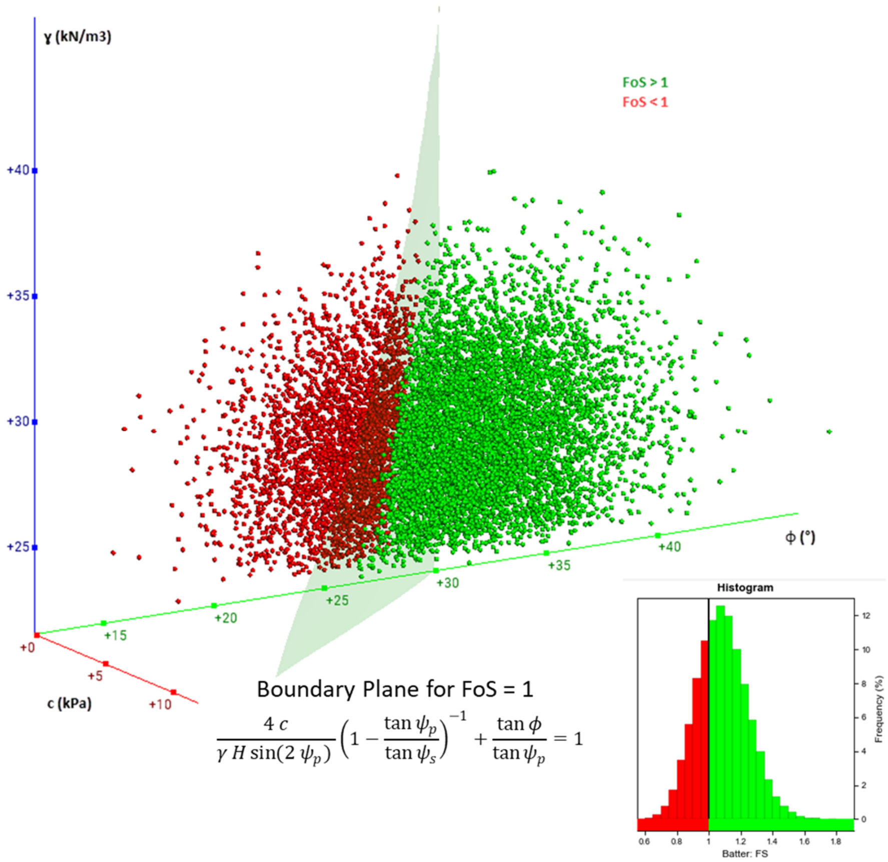

Figure 4 3D illustration of multivariate distribution of FoS based on input parameters c, ɣ and ϕ

FoS_μ and FoS_σ are calculated by considering the joint distribution of the input parameters c, ϕ and ɣ. First the expected value (mean) and standard deviation for each of these parameters is calculated using Equations 6 and 7 (Wasserman 2011). Based on the location of each parameter in Equation 4, the mean and standard deviation need to be calculated for the expressions c, tan(ϕ) and 1/ɣ.

$$x_\mu = \int_{x_{min}}^{x_{max}} \frac{f(x)\,p(x)}{p(x_{min} \leq x \leq x_{max})} dx \tag{7}$$

$$x_\sigma = \sqrt{\int_{x_{min}}^{x_{max}} \frac{(f(x) - x_\mu)^2\,p(x)}{p(x_{min} \leq x \leq x_{max})} dx} \tag{8}$$

where:

f(x)	=	c, tan(ϕ) or 1/ɣ.
p(x)	=	probability mass function of x.
$p(x_{min} \leq x \leq x_{max})$	=	probability of the cumulative distribution function between x_{min} and x_{max}.

The mean value of the FoS is calculated by substituting the mean values of the expressions calculated earlier as shown in Equation 6:

$$FoS_\mu = \frac{4\,c_\mu\,\left(\frac{1}{\gamma}\right)_\mu}{H\sin(2\,\psi_p)}\left(1 - \frac{\tan\psi_p}{\tan\psi_s}\right)^{-1} + \frac{(\tan\phi)_\mu}{\tan\psi_p} \tag{9}$$

where c_μ , $(1/ɣ)_\mu$, $(\tan ϕ)_\mu$ = mean value of the variables c, 1/ɣ and tan ϕ (calculated using Equation 8).

To calculate the standard deviation of the FoS, the standard deviation of c/ɣ needs to be determined first. This can be calculated using Equation 10, which is based on the forumula to determine the standard deviation of the product of two normally distributed variables (Wasserman 2011).

$$\left(\frac{c}{\gamma}\right)_\sigma = \sqrt{\left(\left(\frac{1}{\gamma}\right)_\mu c_\sigma\right)^2 + \left(c_\mu\,\left(\frac{1}{\gamma}\right)_\sigma\right)^2 + \left(\left(\frac{1}{\gamma}\right)_\sigma\,c_\sigma\right)^2} \tag{10}$$

where σ_c , $\sigma_{1/ɣ}$ = standard deviations of the variables c and 1/ɣ (calculated using Equation 8).

Finally, FoS_σ can be calculated using Equation 11 which is the square root of the sum of the variences of each term from Equation 4 (Wasserman 2011).

$$FoS_\sigma = \sqrt{\left(\frac{4\,\left(\frac{c}{\gamma}\right)_\sigma}{H\sin(2\,\psi_p)}\right)^2\left(1 - \frac{\tan\psi_p}{\tan\psi_s}\right)^{-2} + \left(\frac{(\tan\phi)_\sigma}{\tan\psi_p}\right)^2} \tag{11}$$

where:

$\sigma_{\tan ϕ}$	=	standard deviations of the variables c, 1/ɣ and tan ϕ (calculated using Equation 9)
$\sigma_{c/ɣ}$	=	standard deviation of c/ɣ (calculated using Equation 10).

Using FoS_μ and FoS_σ, the PoF is calculated for FoS < 1 as shown in Equation 11. Where the criteria for planar sliding is not met (i.e. bedding dip is greater than or equal to the slope dip), the PoF is equal to zero.

$$PoF_{\psi_s|\psi_p} = \begin{cases} 0 & \psi_p \geq \psi_s \\ p\left(N\sim\left(FoS_\mu, FoS_\sigma\right) < 1\right) & \psi_p < \psi_s \end{cases} \tag{12}$$

The PoF determined in Equation 12 is for a given slope angle (ψ_s) and dip angle (ψ_p). Considering the distribution of ψ_p, the PoF for each slope angle can be calculated using Equation 13. Plotting Equation 13 for ψ_s ranging from 0 to 90 against PoF will produce the batter angle design charts as shown in Figure 5.

$$PoF_{\psi_s} = \int_0^{90} PoF_{\psi_s|\psi_p} \cdot p(\psi_p)\ d\psi_p \tag{13}$$

where p(x) is the probability mass function of ψ_p.

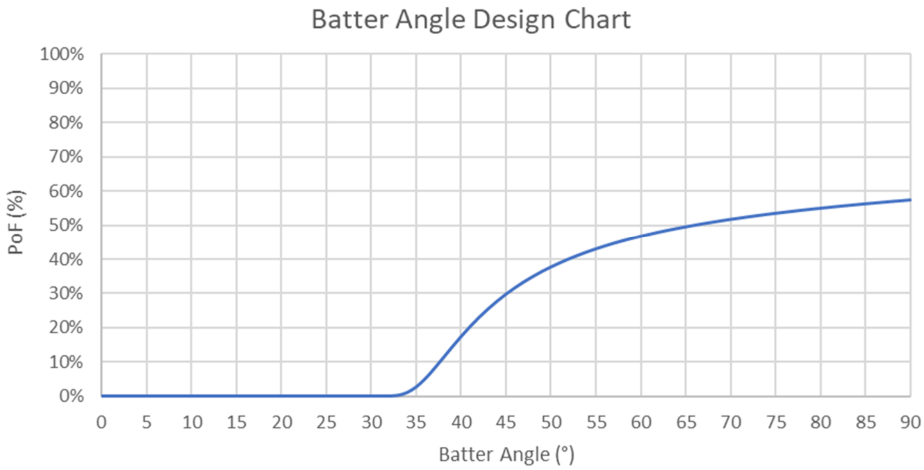

Figure 5 Batter design chart using probabilistic analysis

The calculations can be easily completed in a spreadsheet or simple code. Example script in Python is provided in Appendix 1.

3 Worked example and results

Use of the equations listed in Section 2 will be demonstrated using a worked example. Design charts from both deterministic and probabilistic methods will also be created. Finally, a comparison between the results of the proposed methodology against the output from kinematic analysis will be presented and discussed.

The parameters adopted in this example are summarised in Table 1.

Table 1 Input parameters

Parameter	Symbol	Units	Distribution	Mean value	Standard deviation	Min	Max
Slope height	H	m	Constant	10			
Unit weight	γ	kN/m^3	Normal	33	2	26	38
Cohesion	c	kPa	Normal	8	3	0	13
Friction angle	ϕ	°	Normal	25	2	18	29
Dip angle	ψ_p	°	Normal	30	5	0	90
Dip orientation	-	°	Constant	0			

3.1 Kinematic analysis

As a baseline assessment, kinematic analysis was carried out on the dataset in Table 1. A dataset was of 5,000 points and assessed using dips for planar sliding using the distribution specified in Table 1. It is considered that all dip angles are unfavourable (i.e. dipping out of the slope), and this approach will also be adopted in the analysis to follow.

Figure 6 represents the stereonet from the analysis using Dips (RocScience 2023a) and Figure 7 shows a plot of batter angle versus PoO, typically adopted as the PoF, which suggests a design batter angle of 29°.

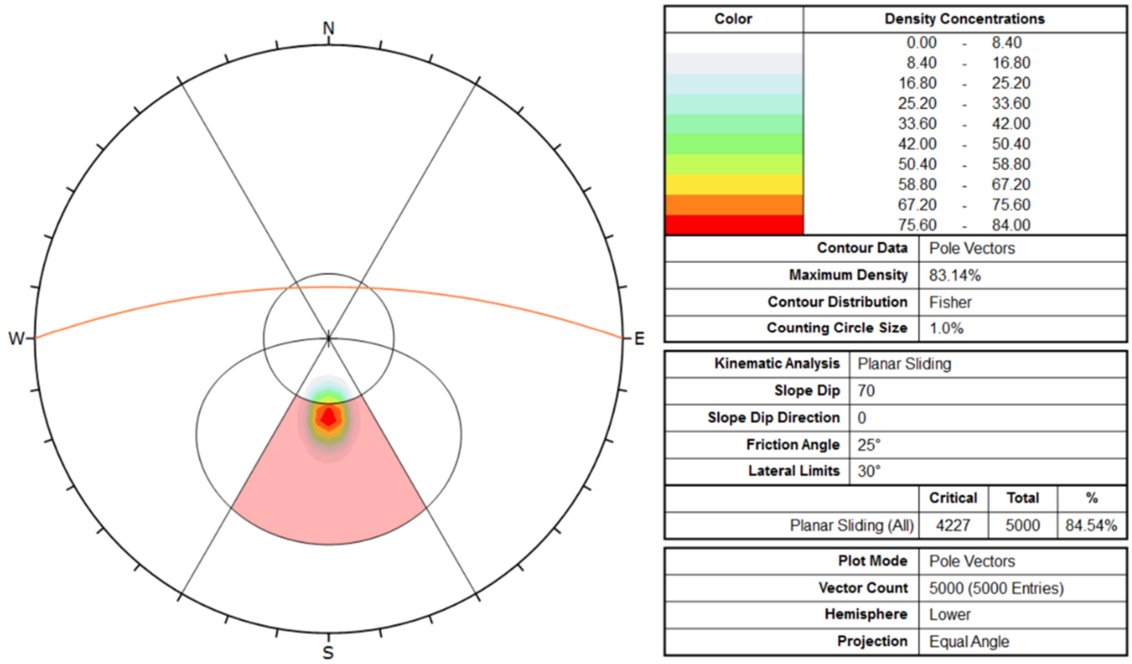

Figure 6 Kinematic analysis results overlaid on a stereonet

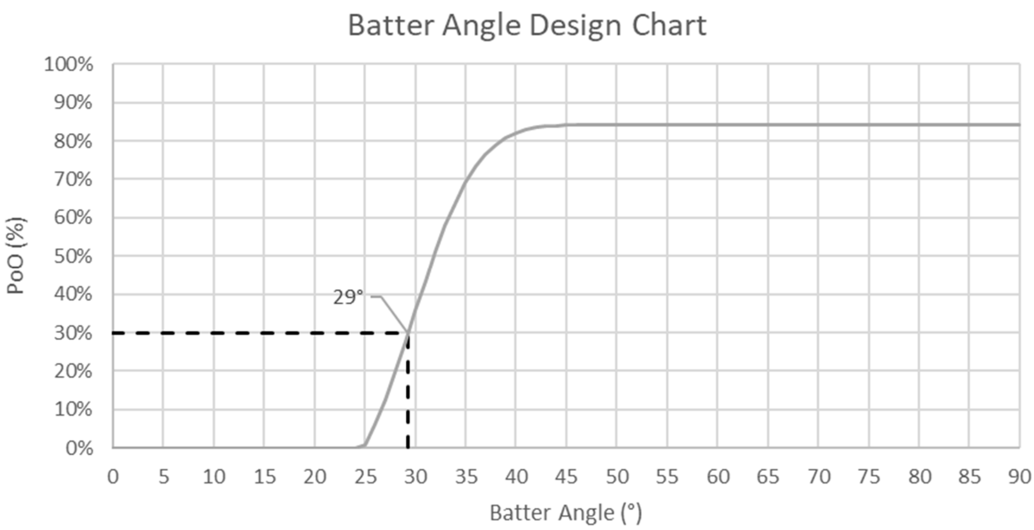

Figure 7 Batter angle versus PoO design chart using kinematic analysis

3.2 Deterministic analysis

Using the mean values from Table 1, the deterministic design chart for this geotechnical domain can be plotted using Equation 5 as shown in Figure 8. Plots have been created for different FoS values ranging from 1.0 to 1.2.

From the design chart, for a mean dip angle of 30° targeting an FoS of 1.1, a batter angle of 43° is required. Alternatively, a plot of batter angle versus FoS can also be plotted as shown in Figure 8, which indicates that a batter angle of 43° is required for FoS = 1.1.

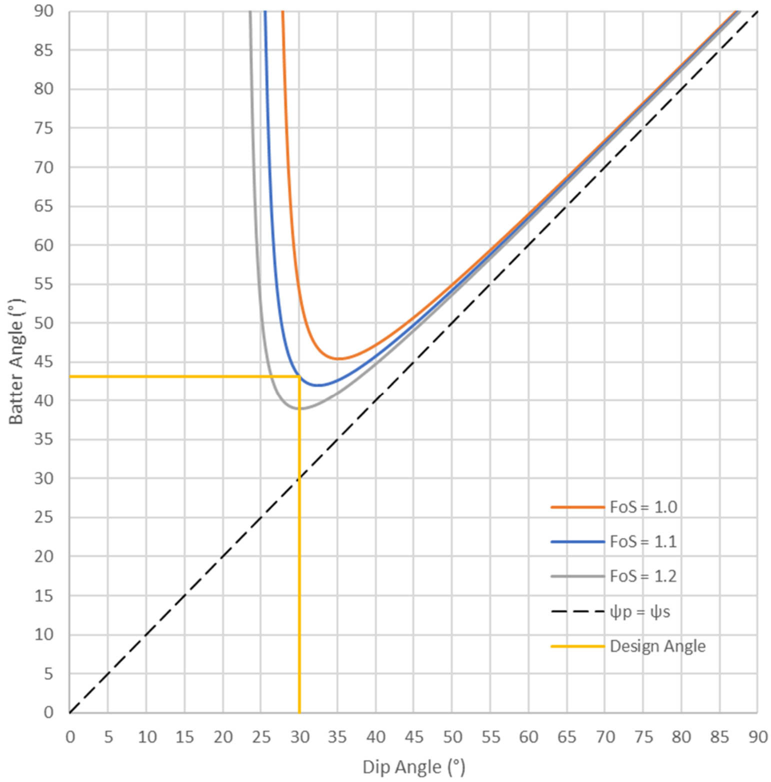

Figure 8 Dip angle versus batter angle design chart (limit equilibrium – deterministic analysis)

3.3 Probabilistic analysis

To calculate the PoF based on the variability of the input parameters, the mean FoS and associated standard deviation need to be calculated using Equations 6 and 7. First the mean and standard deviation of c, 1/ɣ and tan ϕ need to be determined using Equations 8 and 9. Standard deviation of c/ɣ is determined using Equation 10.

Results of the calculations are summarised in Table 2.

Table 2 Calculated mean and standard deviation

Function	Mean value	Standard deviation
c	7.72	2.657
1 / ɣ	0.03	0.002
tan ϕ	0.46	0.040
c / ɣ	–	0.082

Using the calculated parameters, for a batter angle of 45° and the mean dip angle of 30°, the mean and standard deviation for the FoS are equal to 1.06 and 0.113 respectively, which in turn gives a PoF of 29.3%. The PoF can be validated against the output from a planar stability limit equilibrium analysis software tool such as RocPlane (RocScience 2023b), as shown in Figure 9. The slight variation in the results can be accounted for in the method adopted by the software, which is based on Monte Carlo type simulation.

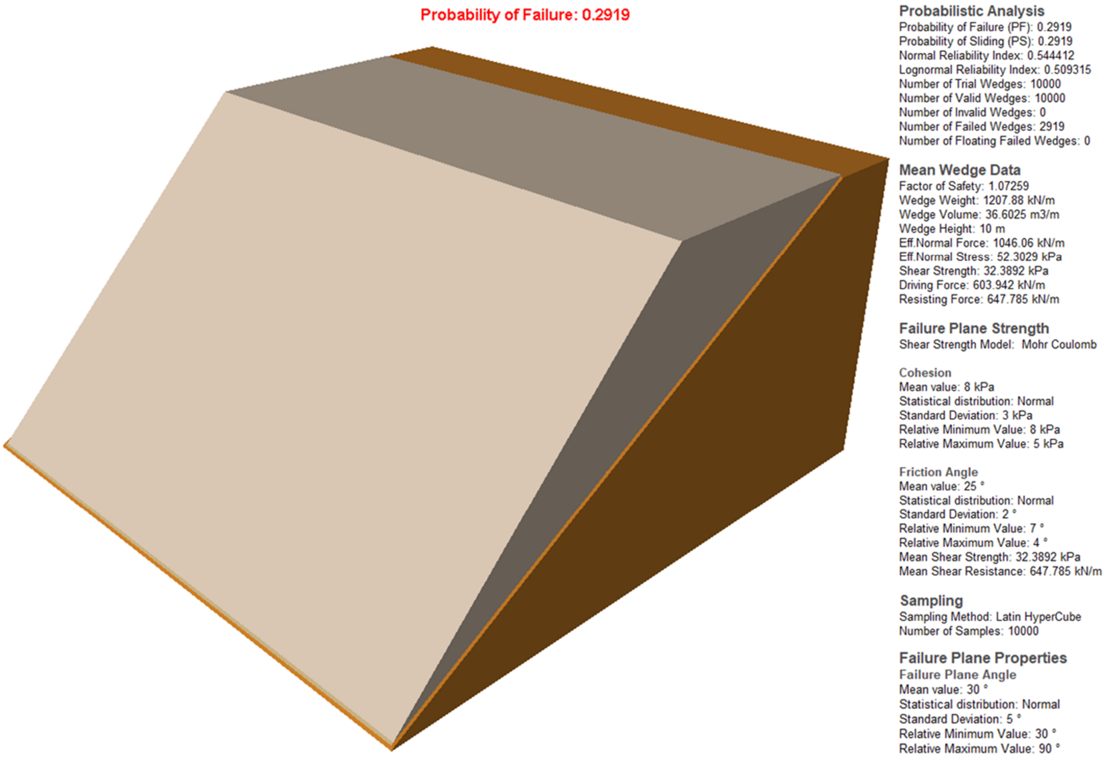

Figure 9 Comparison between calculated PoF (29.3%) versus simulated PoF (29.2%) in RocPlane

Considering the distribution of the dip angle, the PoF for each batter angle can be calculated using Equation 13. For a batter angle of 45°, the PoF is calculated as 26.0%. This is comparable to the result obtained from RocPlane as shown in Figure 10.

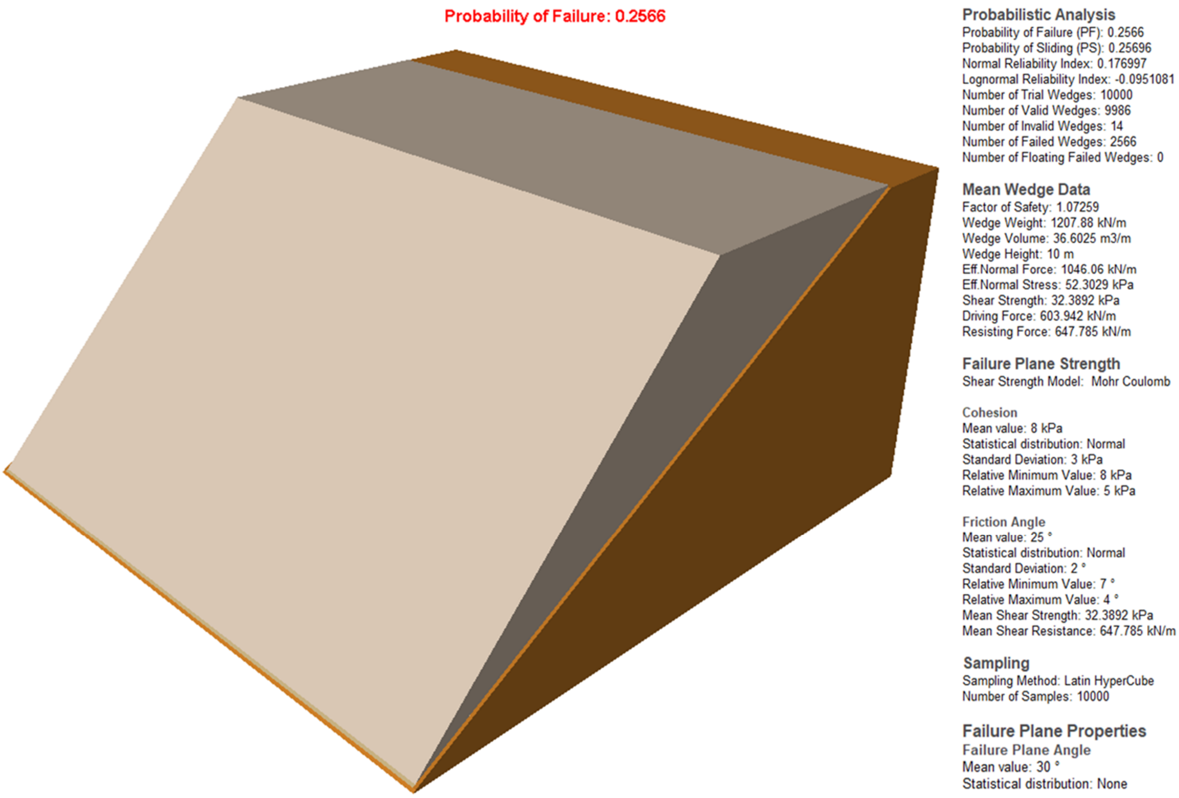

Figure 10 Comparison between calculated PoF (26.0%) versus simulated PoF (25.7%) in RocPlane

Plotting the PoF from Equation 13 against the slope angle provides a design chart for a batter angle within the geotechnical domain. Plot for this example is shown in Figure 11, which shows that a target design acceptance criteria of 30% corresponds to a batter angle of 46°. The plot in Figure 11 is a direct output from the code provided in Appendix 1.

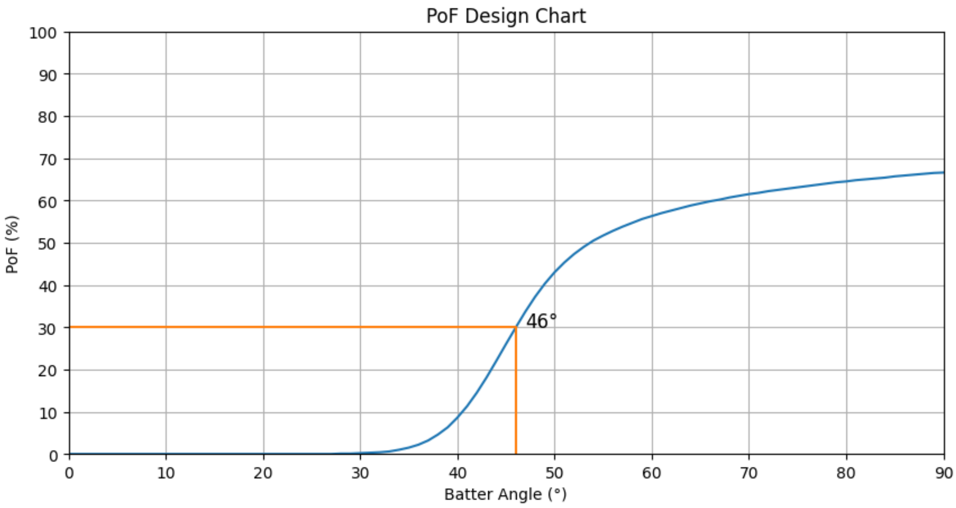

Figure 11 Batter angle versus PoF design chart (limit equilibrium – probabilistic analysis)

3.4 Summary of results

The *Guidelines for Open Pit Slope Design* by Read and Stacey (2010) provides guidance on typical FoS and PoF acceptance criteria values. For analysis of bench scale slopes, the acceptance criteria for FoS is a minimum of 1.1, and for PoF a maximum of 25–50%. The results of the analysis methods will be compared against a target FoS of 1.1 and PoF of 30%.

Figure 12 shows a comparison of the results of the three design charts developed for the same set of parameters and the selected batter angle for each method. Figure 12 shows that the kinematic analysis method underestimates the batter angle by more than 10° in this example, which can be mainly attributed to the exclusion of cohesion in the modelling of the defect strength.

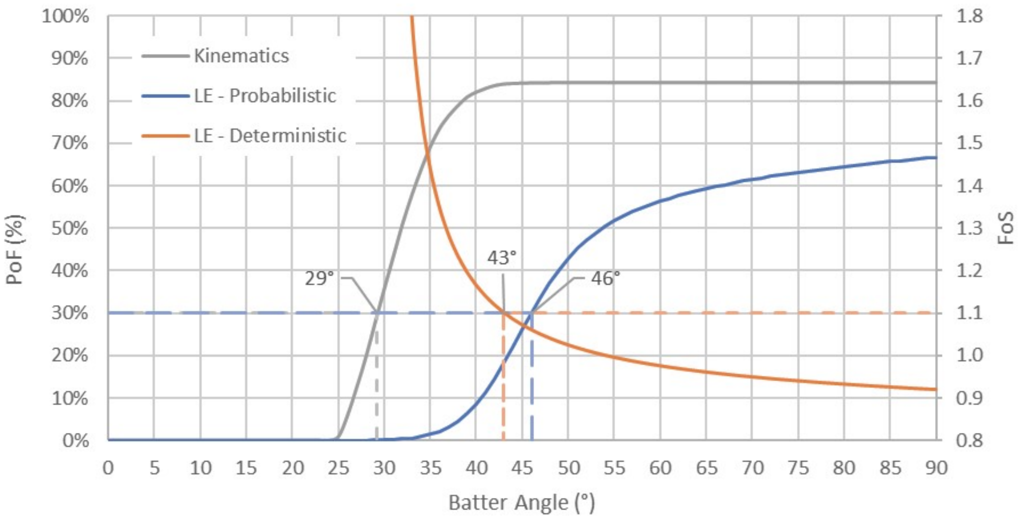

Figure 12 Comparison of different analysis methods and results

Comparison of the results between deterministic and probabilistic analysis show that there is little variation between the two methods. In a practical approach, batter angles will be designed to the nearest 5° and both results may essentially provide the same value. This theme was observed for different sets of defect strength and batter height, suggesting that adopting mean values and a target FoS of 1.1 may be sufficient for the analysis batter scale slopes subject to planar sliding without the need for additional probabilistic analysis.

A comparison between the calculated FoS and PoF for pairs of ψ_s and ψ_p for the parameters defined in Table 1 is presented in Figure 13. FoS values of 1.0 and 1.1 can be seen to correspond closely to PoF values of 50 and 30%, respectively. Variation in degree of conformance is, however, noted to change based on the distribution of the values in Table 1.

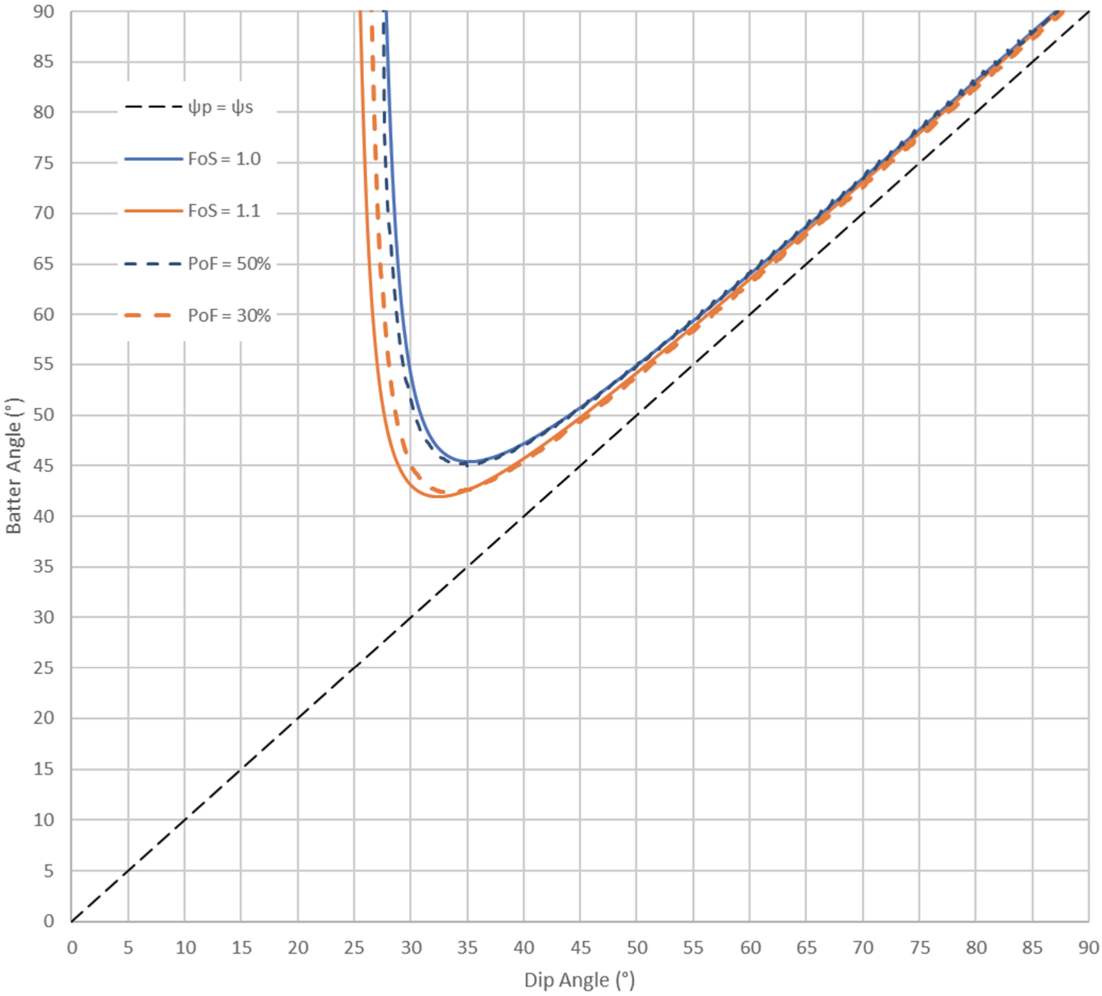

Figure 13 Comparison between Factor of Safety and Probability of Failure results

4 Conclusion

This paper introduced an alternative approach for assessing the PoF in batter scale slopes affected by planar sliding. The method utilises limit equilibrium analysis to determine a deterministic FoS and incorporates the multivariate distribution of relevant input parameters to calculate the PoF. It was noted, however, that comparison between the deterministic analysis targeting FoS of 1.0 or 1.1 and adopting mean values corresponded closely to probabilistic analysis targeting a PoF of 50 and 30%, respectively.

Using the analytical method proposed, design charts (deterministic or probabilistic) can be created for use in different geotechnical design domains. Appendix 1 provides a Python script which can be used to calculate the PoF and generate the design charts based on user-set input parameters.

It was noted that kinematic analysis results in conservative batter design angles due to its limitations in modelling all the input properties for the stability analysis.

Appendix 1 Python script for calculating PoF

```
import numpy as np
from scipy import integrate, stats
import matplotlib.pyplot as plt
import math

# Define parameters (mean, std dev, min, max)
y = (33, 2, 26, 38)
c = (8, 3, 0, 13)
phi = (25, 2, 18, 29)
wp = (30, 5, 0, 90)

# Define slope height
H = 10

# Define target PoF as decimal e.g. 0.3
PoF_target = 0.3

ws = np.arange(91)  # Batter angles from 0 to 90 deg

# Initialize an empty array to store PoF results
PoF = np.zeros((len(ws), 2))
PoF[:, 0] = ws

# Calcualte PoF for each batter angle
for i, ws in enumerate(ws):

    # Calculate mean and standard deviation of input parameters

    c_range = stats.norm.cdf(c[3], c[0], c[1]) - stats.norm.cdf(c[2], c[0], c[1])
    c_u = integrate.quad(lambda x: x * stats.norm.pdf(x, c[0], c[1])/c_range, c[2],
c[3])[0]
    c_o = np.sqrt(integrate.quad(lambda x: (x - c_u)**2 * stats.norm.pdf(x, c[0],
c[1])/c_range, c[2], c[3])[0])

    phi_range = stats.norm.cdf(phi[3], phi[0], phi[1]) - stats.norm.cdf(phi[2],
phi[0], phi[1])
    tan_phi_u = integrate.quad(lambda x: math.tan(math.radians(x)) *
stats.norm.pdf(x, phi[0], phi[1])/phi_range, phi[2], phi[3])[0]
    tan_phi_o = np.sqrt(integrate.quad(lambda x: (math.tan(math.radians(x)) -
tan_phi_u)**2 * stats.norm.pdf(x, phi[0], phi[1])/phi_range, phi[2], phi[3])[0])

    y_range = stats.norm.cdf(y[3], y[0], y[1]) - stats.norm.cdf(y[2], y[0], y[1])
    y_inv_u = integrate.quad(lambda x: 1 / x * stats.norm.pdf(x, y[0],
y[1])/y_range, y[2], y[3])[0]
    y_inv_o = np.sqrt(integrate.quad(lambda x: (1 / x - y_inv_u)**2 *
stats.norm.pdf(x, y[0], y[1])/y_range, y[2], y[3])[0])
```

```
    c_y_o = np.sqrt((y_inv_u * c_o)**2 + (c_u * y_inv_o)**2 + (y_inv_o * c_o)**2)

    # Function to calculate mean of FoS
    def calculate_FoS_u(c_u, y_inv_u, tan_phi_u, wp):
        FoS_u = (
            4 * c_u * y_inv_u / (H * np.sin(math.radians(2 * wp))) /
            (1 - math.tan(math.radians(wp)) / math.tan(math.radians(ws))) +
            tan_phi_u / math.tan(math.radians(wp))
        )
        return FoS_u

    # Function to calculate standard deviation of FoS
    def calculate_FoS_o(c_y_o, tan_phi_o, wp):
        FoS_o = np.sqrt(
            (4 * c_y_o / (H * np.sin(math.radians(2 * wp))) /
            (1 - math.tan(math.radians(wp)) / math.tan(math.radians(ws))))**2 +
            (tan_phi_o / math.tan(math.radians(wp)))**2
        )
        return FoS_o

    # Function to calculate PoF_ws|wp
    def calculate_PoF_ws_wp(wp, ws, c_u, y_inv_u, tan_phi_u, c_y_o, tan_phi_o):
        if wp >= ws:
            PoF_ws_wp = 0
        else:
            FoS_u = calculate_FoS_u(c_u, y_inv_u, tan_phi_u, wp)
            FoS_o = calculate_FoS_o(c_y_o, tan_phi_o, wp)
            PoF_ws_wp = stats.norm.cdf(1, FoS_u, FoS_o)
        return PoF_ws_wp

    PoF_ws = integrate.quad(lambda x: calculate_PoF_ws_wp(x, ws, c_u, y_inv_u,
tan_phi_u, c_y_o, tan_phi_o) * stats.norm.pdf(x, wp[0], wp[1]), wp[2], wp[3])[0]

    PoF[i,1] = round(PoF_ws*100,1)

lower_idx = np.where(PoF[:, 1] <= PoF_target*100)[0][-1]
upper_idx = np.where(PoF[:, 1] >= PoF_target*100)[0][0]

# Perform linear interpolation
x1, y1 = PoF[lower_idx]
x2, y2 = PoF[upper_idx]
ws_design = x1 + (x2 - x1) * (PoF_target*100 - y1) / (y2 - y1)

# Plot PoF design curve
x = PoF[:, 0]
y = PoF[:, 1]

# Create a figure and axis
fig, ax = plt.subplots()
```

```
ax.plot(x, y)
ax.plot([0,ws_design,ws_design],[PoF_target*100,PoF_target*100,0])

ax.set_xlim(0, 90)
ax.set_xticks(np.arange(0, 91, 10))
ax.set_ylim(0, 100)
ax.set_yticks(np.arange(0, 101, 10))

ax.set_xlabel('Batter Angle (°)')
ax.set_ylabel('PoF (%)')
ax.set_title('PoF Design Chart')

plt.text(ws_design + 2, PoF_target*100 - 5, str(int(ws_design)) + '°', fontsize=12,
color='black')

ax.grid()

plt.show()
```

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