SSIM 2023 – PM Dight (ed.)

doi:10.36487/ACG_repo/2335_63

The case for using three-dimensional limit equilibrium stability analysis

IR Brown *TAGA Engineering Software Ltd, New Zealand*

PJ Wood *TAGA Engineering Software Ltd, New Zealand*

Abstract

At the previous Slope Stability in Mining conference, in 2021, Dr John Read claimed that there are geological and mathematical uncertainties in the method used for 3D limit equilibrium (LE) stability analyses. We acknowledge the valid issues raised by Dr Read, and our paper explains how we have developed a software system that provides a reliable 3D LE analysis for slopes in complex geology. Our experience with developing 3D LE software (TSLOPE) began in 1989 when we were able to show that the 3D LE analysis of the stability of a proposed canyon landfill in Southern California gave a much higher Factor of Safety than that calculated for a 2D section along the axis of the canyon. We have discarded many of the methods used in 2D LE software packages as they do not achieve full force and moment equilibrium and are a relic from a time when we were limited by computational ability. Our paper discusses the background and underlying theory supporting our 3D LE software and provides comparisons of 3D and 2D analyses carried out using the same geological model.

Keywords: *slope stability, 3D, limit equilibrium, Spencer's method, TSLOPE*

1 Introduction

At the previous Slope Stability in Mining conference in 2021, Dr John Read claimed that there are geological and mathematical uncertainties in the method used with 3D limit equilibrium (LE) stability analyses (Read 2021).

We have been involved with the development of 3D LE software TSLOPE (TAGAsoft 2023) for some 24 years, and we describe how the software has evolved in that time to satisfy some of the concerns raised by Dr Read.

In our paper we present two classes of slope problem that show why 3D LE analysis should be used and compare those with equivalent 2D LE results. The first example is a complex wedge as typically found at a bench scale in an open pit mine. The second example involves a weak layer that underlies a waste rock dump, and shows the effect of weak layers that are also present in other open pit settings.

2 Limit equilibrium analyses

The use of LE analyses has become common practice in civil and mining engineering slope design, and there is a long and well-documented history of its use. Early procedures predated the use of electronic computers, and hand calculations of simple slope geometries were carried out more than 100 years ago. As computer-based methods have developed, more complicated LE analysis procedures have been possible, particularly methods that involve iterative calculations.

One common feature of all LE analysis procedures is the definition of the Factor of Safety (FoS) defined with respect to the shear strength of the soil or rock as:

$$FoS = s/\tau \quad (1)$$

where s is the available shear strength and τ is the equilibrium shear stress.

To calculate the stability of a slope a slip surface is assumed, and equations of static equilibrium are used to calculate the stresses and FoS for the surface. The FoS is assumed to be the same at all points along the slip surface, however, this assumption is only valid when the slope is at the point of limiting equilibrium.

The soil or rock mass is divided into a number of vertical slices (2D) or columns (3D) that overlay the slip surface. Equilibrium equations are then solved for each slice or column. Three static equilibrium conditions are available for 2D analyses and this increases to six equilibrium conditions for 3D analyses.

The problem of computing a FoS is that it is statically indeterminate as there are more unknowns than the number of equilibrium equations. Therefore assumptions must be made to achieve a balance of equations and unknowns. Depending on the analysis procedure used, all conditions of static equilibrium may or may not be satisfied. Different procedures make different assumptions even when they satisfy the same equilibrium equations.

We prefer to use a procedure that satisfies full force and moment equilibrium, with few simplifying assumptions required to obtain a solution. We are no longer constrained by our ability to carry out complex calculations thanks to modern digital computers. Hence many of the 2D LE analysis methods that were developed before we had that capability are best discarded.

We have extended Spencer's method (Spencer 1967) for 3D LE by observing that the side-force inclination is fixed for all slices, effectively in a linear direction for 2D slopes, which in 3D becomes a plane. A minimisation routine is used to try to achieve full force and moment equilibrium of the forces: for 2D this involves three equations (two force and one moment) and for 3D this involves six equations (three force and three moment). For 2D analyses most slope cases achieve full force and moment equilibrium, although there are some slope examples where this is not achieved. While not common, when full equilibrium is not achieved in 2D analyses it is typically 1–2% of the total weight of the slope with respect to force equilibrium.

For 3D slopes, achieving full force and moment equilibrium using Spencer's method can sometimes be difficult. When there is symmetry in the geometry and material strengths on the assumed failure surface, TSLOPE usually calculates full equilibrium. However, when the geometry and/or the material strength distribution is unsymmetrical, full equilibrium is harder to achieve. In this case the equilibrium error, as a least-squares estimate for the three force and three moment equilibrium, gives an estimate to the reliability of the calculated FoS.

As an example, if the equilibrium error is 5%, the error in the FoS is approximately of the same order. For engineering purposes, this is generally adequate, given the uncertainties in material strength and how it is distributed within a slope. A high equilibrium error may also indicate that the assumed failure surface needs to be adjusted, or the assumed material strengths may need modification. Selecting a suitable failure surface for 3D problems is significantly more difficult than for 2D analyses. Currently TSLOPE does not implement a search routine for 3D slope cases; instead it requires the user to carefully evaluate the geology of the slope to determine possible failure surfaces.

3 TSLOPE evolution

The first implementation of TSLOPE (then known as TSLOPE3) used a Fortran code developed by Dr Robert Pyke and his colleagues in 1989. The program was required to show adequate stability of a proposed canyon landfill in Southern California (Puente Hills).

TSLOPE3 discretised the municipal refuse into vertical columns overlying the landfill liner. The liner was to be constructed in zones with different shear strength properties that were assigned to the base of the columns. Figure 1 shows the completed landfill with a vertical range of about 150 m and an area of about 27.5 ha.

A critical section was analysed using 2D LE calculations and an unsatisfactory FoS was calculated (Figure 2). Using the 3D model, Dr Pyke was able to show that the shape of the base of the landfill and the spatial variation of liner shear strength were important considerations when assessing stability leading to a LE 3D FoS that was significantly higher than 2D (Figure 3).

At this stage of software development, TSLOPE3 used only the ordinary method of columns (OMC) as the method of analysis. This method is the 3D extension of the ordinary method of slices: a procedure of slices that neglects the forces on the sides of the slices or, in 3D, the sides of the columns. In effect each column is considered to be independent of its neighbours; it does not provide a solution that satisfies full force and moment equilibrium.

We have repeated the slope stability analyses using the current version of TSLOPE and in Table 1 we compare the FoS calculated using OMC and Spencer's methods.

One of the features of TSLOPE3 was the ability to calculate the direction of sliding that provided the lowest FoS. Other 3D LE software at that time required the user to input the direction of sliding at the start of the analysis.

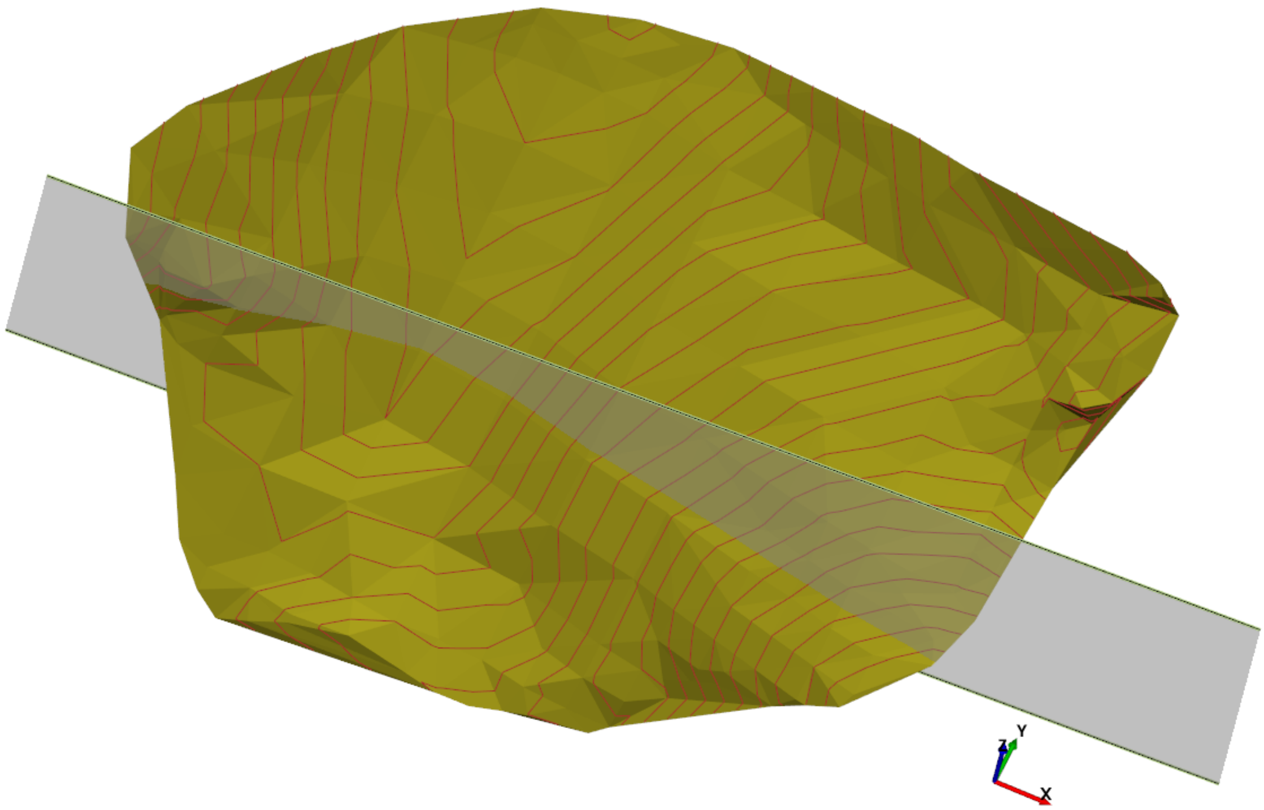

Figure 1 Puente Hills landfill showing a 3D design surface for the top of the fill (section shown as the grey panel in Figure 2); contour interval of 6 m

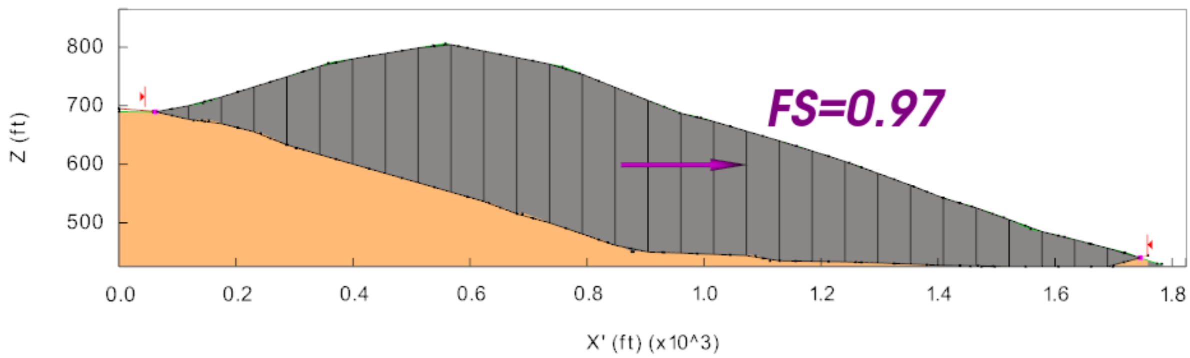

Figure 2 Puente Hills landfill critical 2D section showing slices through the refuse with potential slide surface on the landfill liner; Factor of Safety (ordinary method of columns) of 0.97

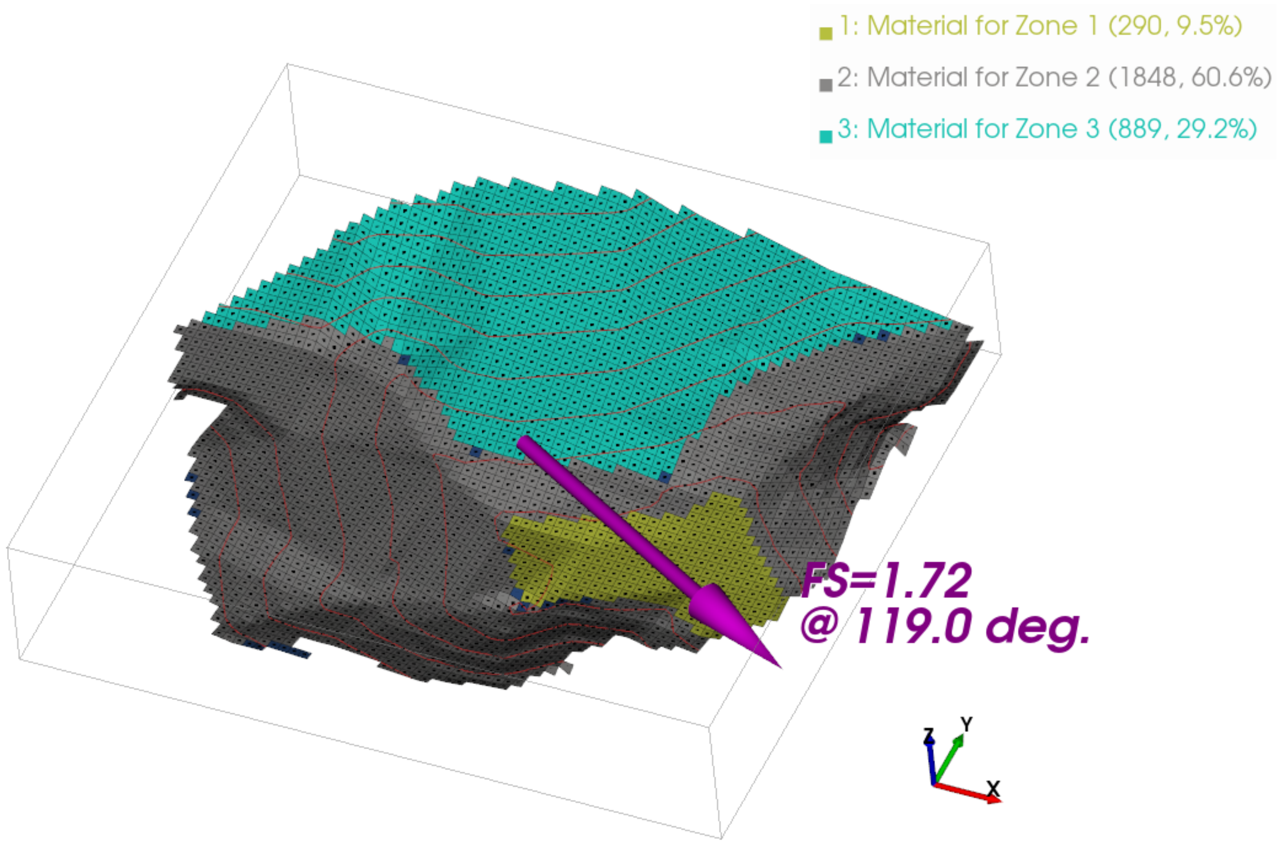

Figure 3 Puente Hills landfill liner surface showing the base and number of columns in each material zone; Factor of Safety (ordinary method of columns) of 1.72

Table 1 Results of stability analyses (Factor of Safety), Puente Hills landfill

	Ordinary method of columns	Spencer's
2D	0.97	0.99
3D	1.72	1.97

In the early 1990s, the OMC procedure of TSLOPE3 was implemented in TECHBASE® (TECHBASE International, Ltd 2007): a relational database manager designed for use with engineering and other technical data. This version of TSLOPE3 was labelled TSLOPE3D. It uses the 3D database structures to access fields with relevant data for each column in the model, replicates the original Fortran algorithm, and provides direct access to a geological model included in the TECHBASE database structure. TECHBASE graphics programs provide an enhanced graphic display of results.

TECHBASE was used to manage and model all geological and geotechnical data during construction of a large hydropower project in New Zealand. There were many active landslides in the reservoir area that required extensive investigation and remediation. TSLOPE3D provided an effective check on the labour-intensive 2D slope stability analyses and was able to provide insight into the areas where slope drainage would have greatest effect (Gillon et al. 1992).

In 2003, we carried out the first analyses using Spencer's method as currently implemented in TSLOPE.

4 Comparison of 2D and 3D limit equilibrium analyses

4.1 Complex wedge

This example shows a wedge of sandstone in an open pit bench. The wedge shown in Figure 4 is defined by the excavated surface (grey), a low-angle shear zone dipping out of the slope (purple) and three joints (green). The data were obtained from PointStudio, a Maptek application. The bench topography was derived from a point cloud, and the geological structures were identified from the point cloud using software tools and recorded as simple planar discontinuities. We then constructed a composite slip surface using the convex hull of the three joints and the shear zone, and assigned appropriate shear strengths to each component of the slip surface.

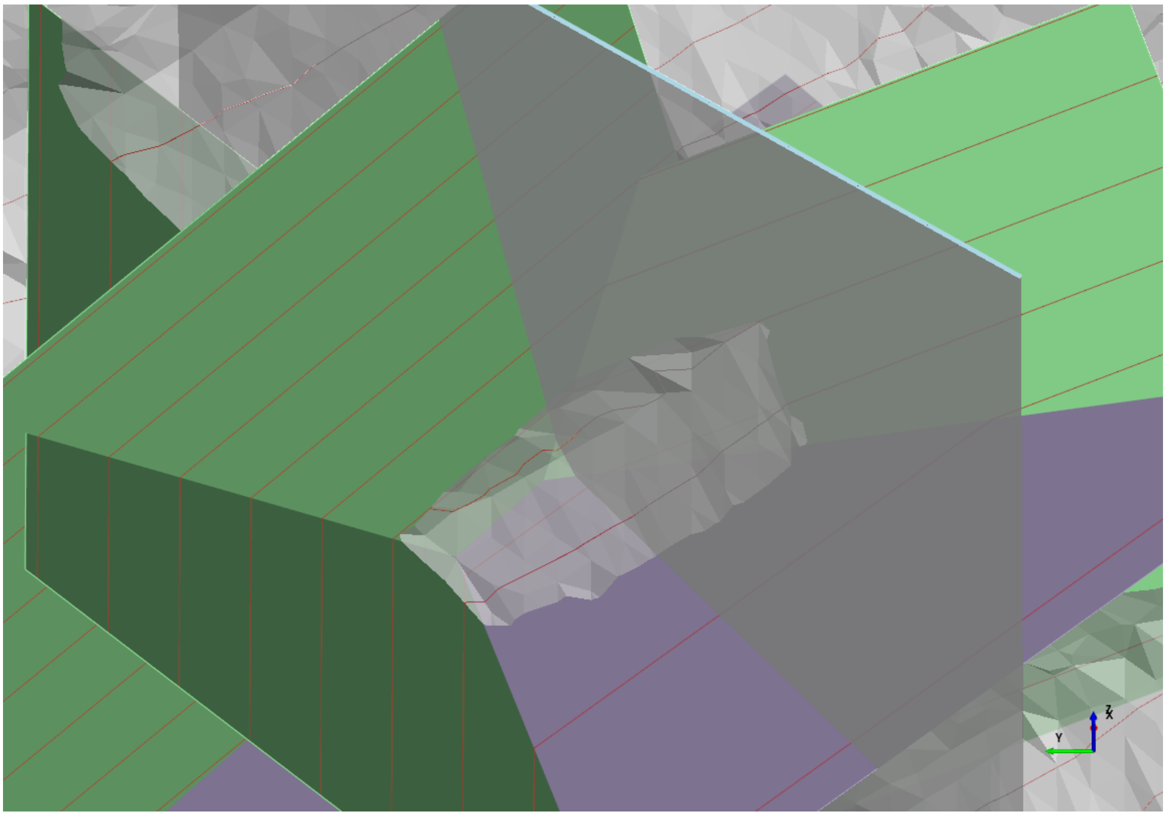

Figure 4 Perspective view of quarry bench with joints (green), shear zone (purple) and section location (grey panel)

The results of 2D and 3D analyses using Spencer's method are shown in Figures 5 and 6. For the 3D analysis we have used a small column size (0.10 m) resulting in 5,006 columns above the slip surface.

This complex wedge example shows a lower FoS for 3D than the equivalent 2D slope case. This result confirms our experience that there can be significant differences in calculated FoS between 2D and 3D analysis when there are no obvious geometric constraining factors such as in the Puente Hills landfill example. In many cases the 3D FoS is greater than 2D, but the difference can be hard to judge ahead of carrying out the analysis.

The explanation for a lower 3D FoS can be attributed to the distribution of effective normal stresses at the base of each column. Note that the zone of highest effective normal stress shown in Figure 7 is restricted to a part of the shear zone component of the slip surface. The area of effective normal stress greater than 30 kPa is only a small part of the overall slip surface, however, when we compare the effective normal stress distribution at the base of slices in the 2D slope case about 25% of the slices show effective normal stresses greater than 30 kPa.

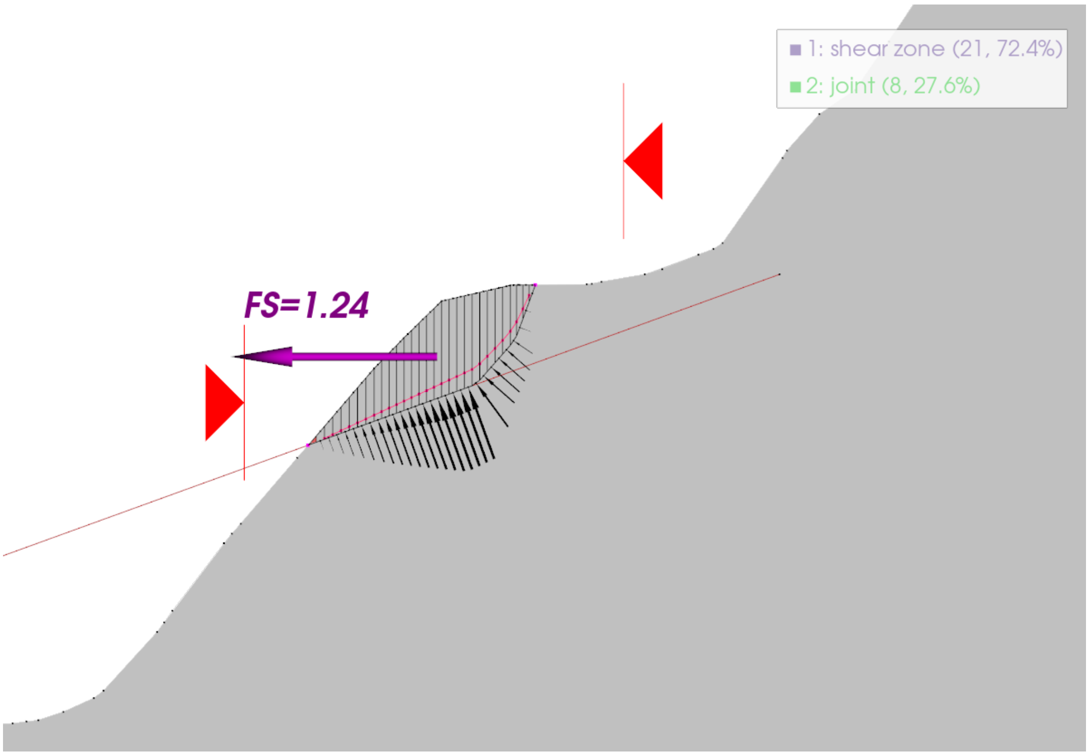

Figure 5 2D slope case: arrows show effective normal stress on base of slice, with the line of thrust in red

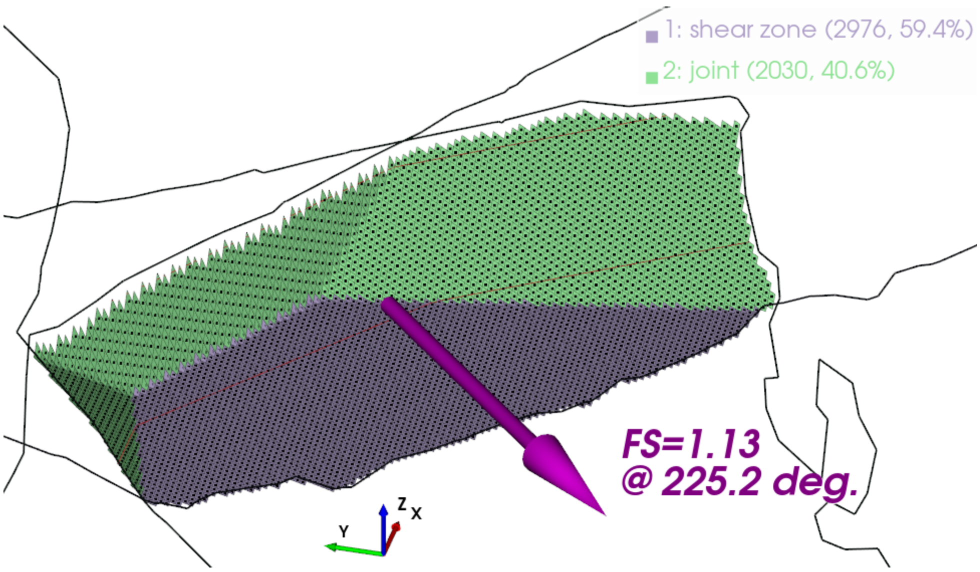

Figure 6 3D slope case with excavated surface removed to show underlying composite slip surface colour-coded for material

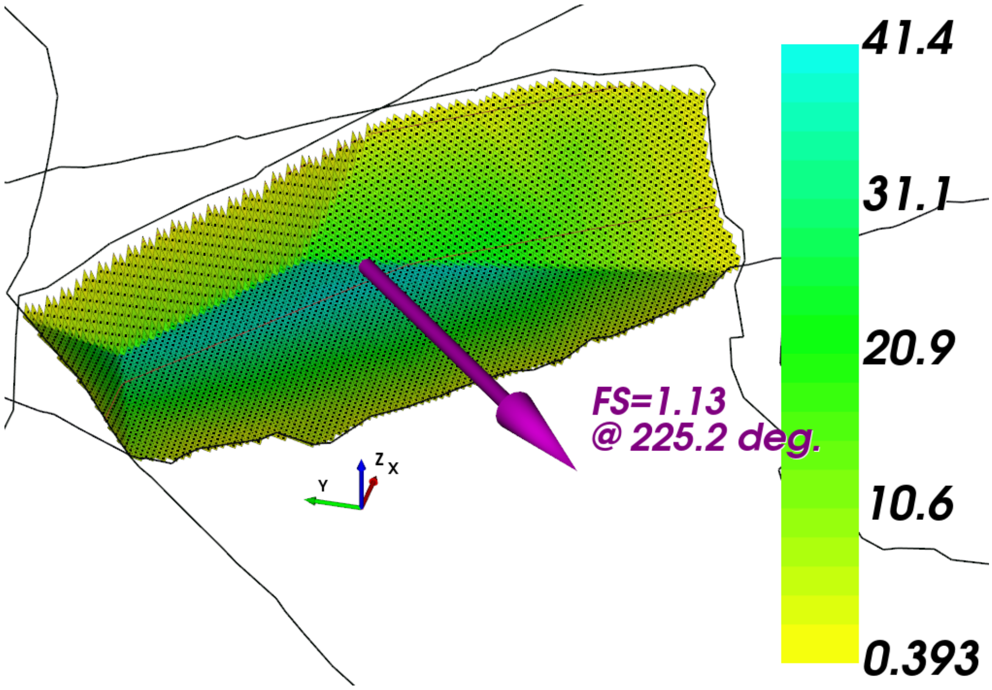

Figure 7 3D slope case with composite slip surface colour-coded for effective normal stress at the base of each column

4.2 Weak layer

Conventional 2D LE analysis software has strategies to make sure that a slip surface is confined to a weak layer and is then extrapolated through a path of least shear resistance to an exit point on the ground surface. We have taken an example project file saved from a 2D LE program and automatically extruded the model into the out-of-plane direction. We make the assumption that this is a plane strain problem; in reality it is unlikely to be as the surfaces shown in the section view are expected to vary away from the section line.

This weak layer example is from an open pit coal mine in New South Wales where a waste rock dump has been constructed over a gently dipping shear zone in the footwall (Figure 8). Our 2D FoS calculated using Spencer's method (1.08) is close to the FoS calculated in another 2D program. The slip surface has followed the low strength shear layer (green) for 52% of the slip surface length. The circular search algorithm has estimated the exit parts of the slip surface through higher shear strength material at the toe and head of the slope.

In 3D the circular part of the slip surface that is shown in Figure 8 is part of a sphere. We can then use that sphere to derive a 3D failure surface to investigate the effect of the high strength Spoil Category 2.5 on a potential 3D slip surface. The extruded model, with sphere dimensions calculated from 2D LE analysis, is shown in Figure 9.

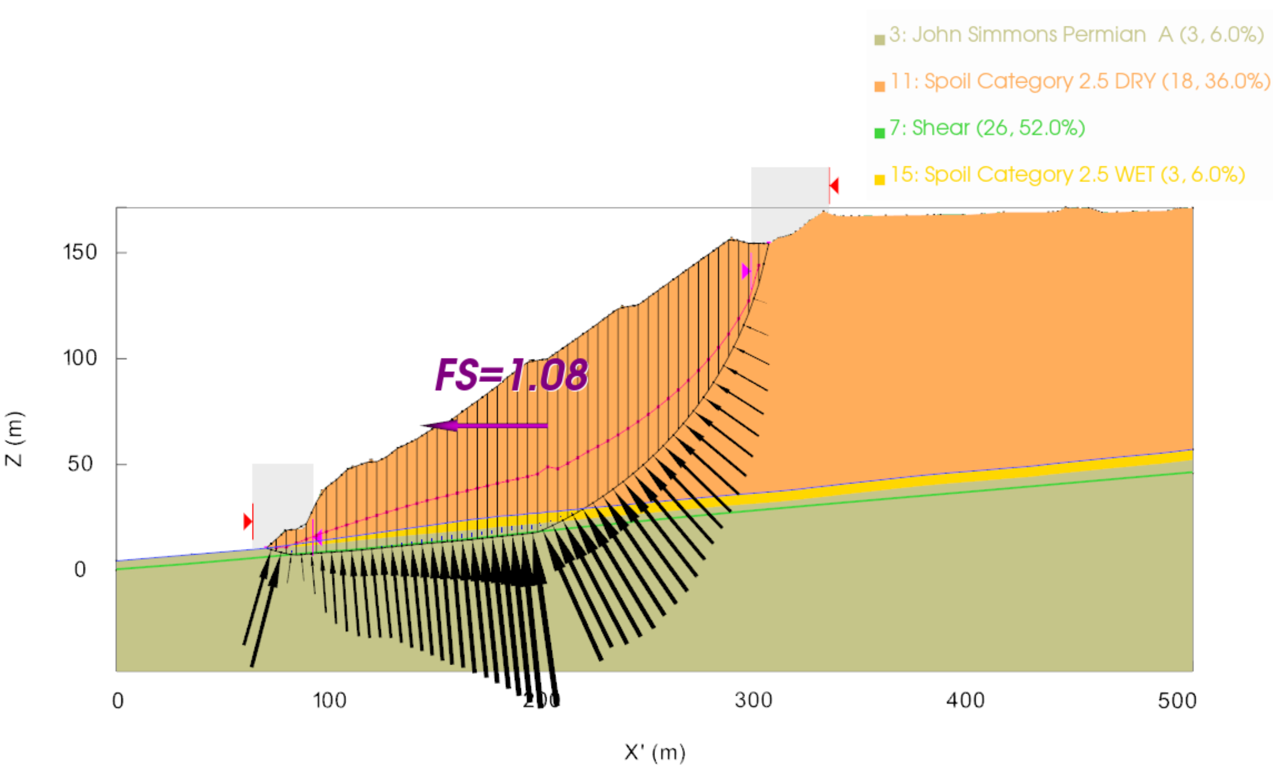

Figure 8 2D slope case with the slip surface constrained to a weak layer: arrows show effective normal stress at the base of each slice; red line is the line of thrust

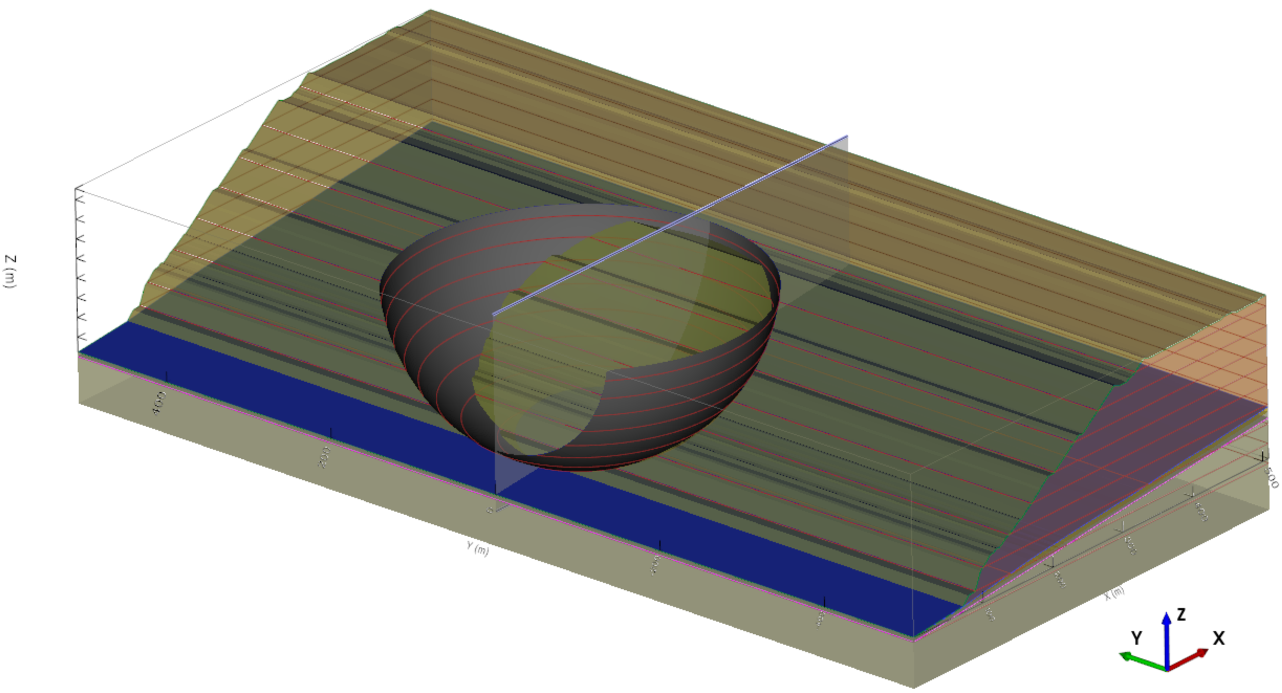

Figure 9 Extruded model with sphere calculated from the critical surface search shown in Figure 8

Figure 10 shows the FoS of the 3D slope case with the composite slip surface that includes the sphere obtained from the 2D analysis, and the weak layer. We then modified the sphere to form an ellipse with radius in the Y direction and half of the radius in the X direction, and used that to construct the composite slip surface. Figure 11 shows the FoS of this 3D slope case.

This example shows that there can be a significant increase in calculated FoS when a 3D analysis is carried out with a weak layer present. In this case the FoS increased between 2D, the sphere and the ellipse, as shown in Table 2.

The weak layer (shear) has Mohr–Coulomb cohesion of zero and a 15° angle of friction. In the 2D slope case the weak layer strength was applied to the base of 52% of the slices. The 3D slope cases have the weak layer strength acting on the base of 21.0% (sphere) and 21.3% (ellipse) of the columns.

Table 2 Results of stability analyses (Factor of Safety) of a waste rock dump over weak layer

2D	3D sphere	3D ellipse
1.08	1.38	1.58

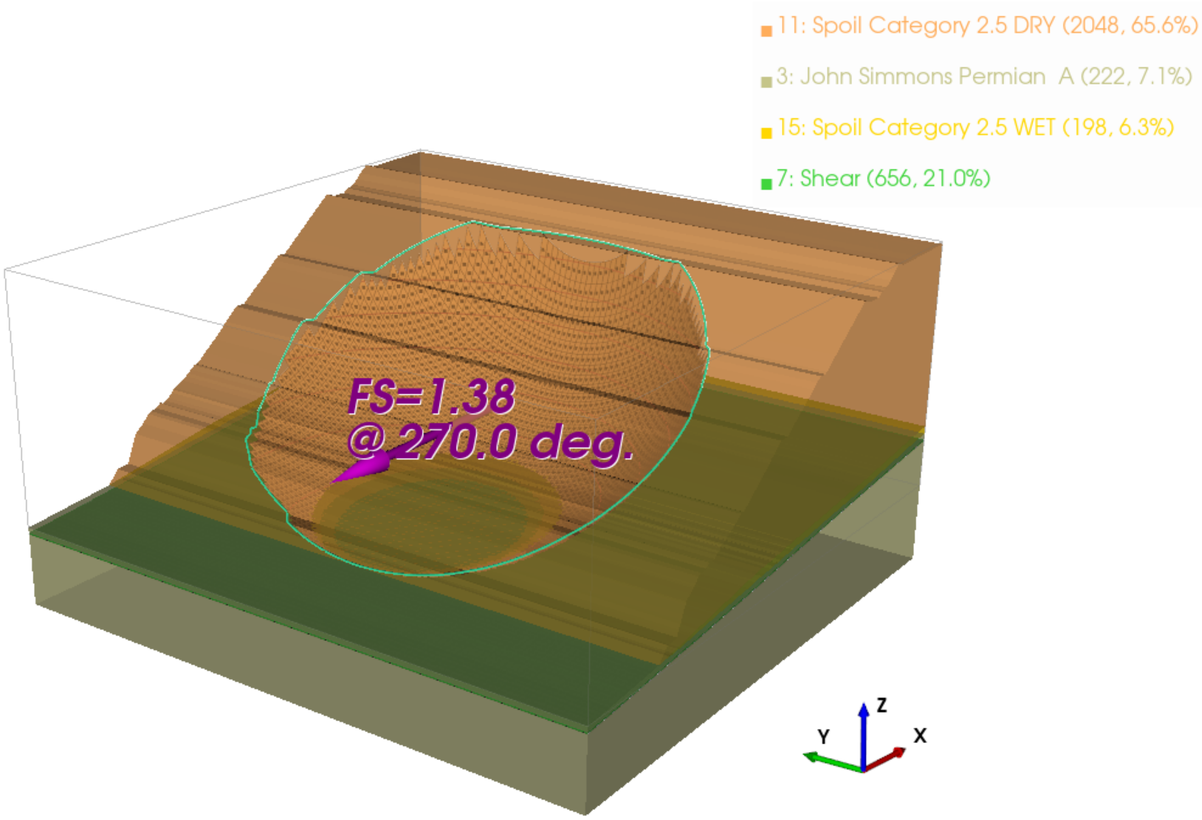

Figure 10 3D slope case with slip surface composite of sphere and weak layer

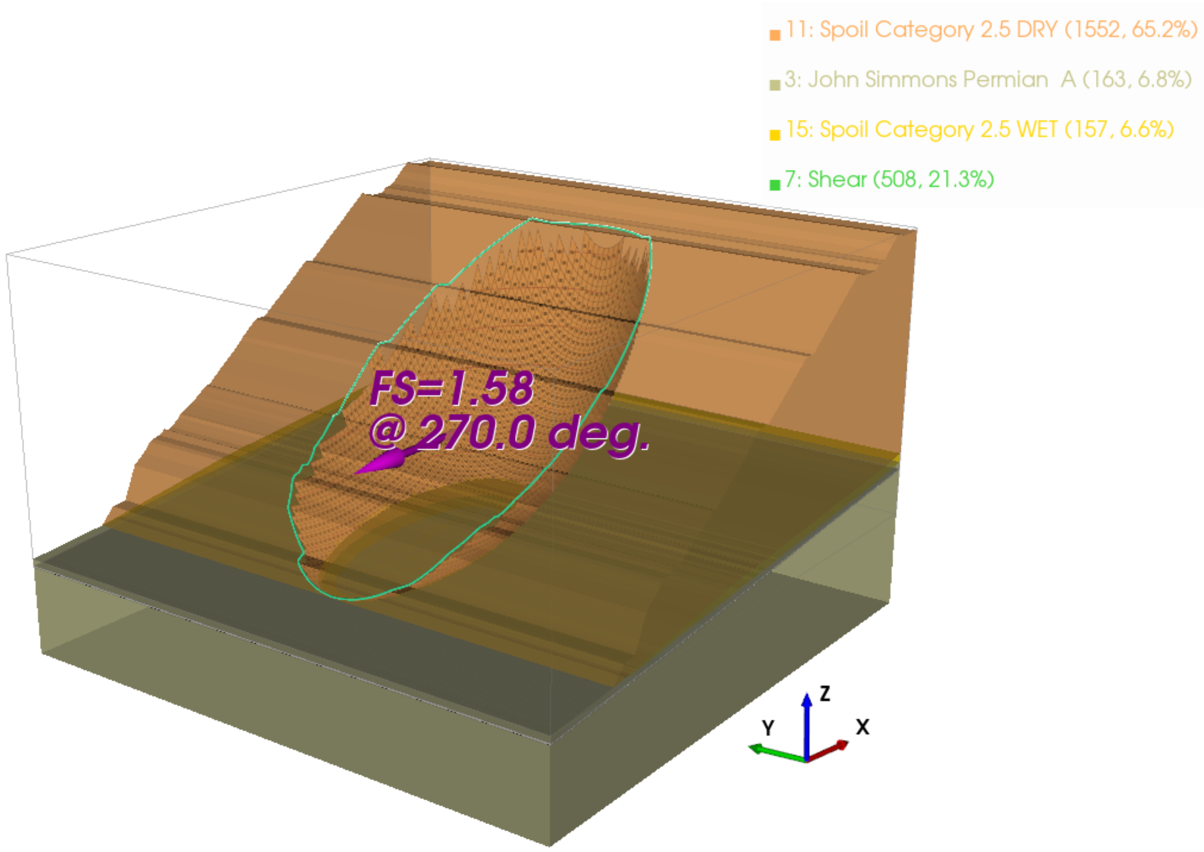

Figure 11 3D slope case with slip surface composite of ellipsoid and weak layer

5 Reliability of 3D limit equilibrium analyses

We have used the same section heading as Read (2021) and discuss the points that were raised in this section of his paper. There were three main themes to Dr Read's discussion, as follows.

5.1 Verification and benchmarking

This has been a challenge through the development of 3D LE analysis software. Following the long history of the use of 2D methods there are many published studies that provide useful verification and benchmarking. We have used these extensively to check our 2D LE slope cases.

However, there are few well-documented 3D LE analysis verification models available. Some are closed-form analytical solutions that by nature are simplified compared with real-world problems. Another class of problem are 3D wedge failures where well-documented analytical solutions are available, such as presented by Wyllie (2017). We recommend that users of 3D LE software check how well the program is able to replicate published wedge solutions.

We are not aware of any benchmarking studies that have been carried out for 3D LE software.

5.2 Discretisation using vertical columns

As discussed, LE analyses require the discretisation of the ground above the potential slip surface. Vertical slices for 2D analyses have long been accepted and vertical columns are the logical 3D extension.

An important part of the discretisation is the location in space of the slip surface at the base of each column. The nature of the ground above the slip surface, apart from unit weight as that directly affects the normal stress on the slip surface, is generally irrelevant. The strength distribution within the slope mass may affect how the column side forces are distributed and this may affect how the normal stress on the assumed failure surface is distributed, thus affecting the FoS. However, this needs to be investigated further. We have reviewed models with intricate details of interbedded and folded structure that only inform the normal stress at the base of each column.

The geological and structural components of the geotechnical model that concerned Dr Read must be taken into account with the definition of the slip surface, or surfaces. We understand his objection was a result of considering 3D LE results that were derived from a search function in the program where the slip surface is unlikely to be correctly modelled by a predefined geometric shape such as an ellipse.

5.3 Slope displacement

Cheng & Yip (2007) were cited by Read (2021) to support his contention that using square-shaped vertical columns in 3D LE analysis is illogical. However, no matter how a body is discretised, the discretised parts must still satisfy force and moment equilibrium. The discretisation into square vertical columns does not create real columns: they are 'virtual' columns where the forces acting on each column must keep that 'virtual' column in equilibrium. This is done in finite-element analyses (FEA) and other analyses, and the shape of the discretisation should not affect the laws of physics, although a well-shaped element is desired for numerical accuracy. Using a discretisation that accounts for faults and joints may be preferable for FEA when those faults or joints are being modelled, but for LE this is generally not possible. The use of square vertical 'virtual' columns is still valid; the issue is how to define the side forces and forces on the base of the column so that it is in equilibrium. The presence of joints may well affect the effective side-force angle for the slope and thus the resultant side-force magnitude and location.

Read (2021) Figure 3a shows a unique sliding direction for all columns and that is something that should be calculated by a 3D LE program. Such a sliding direction provides the lowest FoS for the slip surface and a check that the solution is guided by the shape of the slip surface. However, the result of a 3D LE analysis is only relevant to shear failure along a slip surface at the base of the columns. It cannot be expected to represent any of the other likely mechanisms of slope failure or model slope displacement. If that is an objective of slope stability analysis then a numerical method such as FEA or discrete element should be used.

The use of the same direction of failure for all columns is a simplifying assumption that makes the analysis possible. The calculated direction for 3D LE may not necessarily be aligned to the expected kinematic direction of failure as the LE does not consider displacement in its analysis.

6 Conclusion

Filling of the Puente Hills landfill was completed in 2013 more or less as shown in the 1989 design, without any slope stability problems. The low strength liner made it difficult to show acceptable stability in the 2D case, however, the 3D effects of the sloping sides of the canyon were able to be more realistically modelled using a 3D analysis.

With advances in geological modelling software, and new techniques for acquiring 3D data and modelling surfaces, 3D LE software users have been able to more efficiently access the data required to build a 3D slope model. Therefore it has become relatively easy to transfer the relevant surfaces from the mine geological and/or survey system into a 3D LE model. The slope stability model should therefore directly reflect the geological model, and any uncertainties are associated with the interpretations that were necessary to derive that model. The 3D LE software should not alter the underlying geological model as part of the analysis process or require unrealistic simplifications to the geological model in order to carry out the analysis.

Fundamental to a valid slope stability analysis is the correct location in the 3D space of the slip surface. We are not aware of an algorithm that can reliably check a rock volume for potentially valid slip surfaces in a region of complex geological structure. Any simple 3D search is likely to provide an unreliable result.

The slope discussed in Section 4.2 indicates how FoS can vary as the ellipse axes change. We have shown in a number of examples that the FoS will decrease as the length of the axis normal to the direction of sliding is increased, until we reach the limiting case that is equivalent to the plane strain model from a 2D LE analysis.

It is not clear what the correct solution should be when we start using virtual surfaces to define a slip surface. As should be apparent to all geotechnical engineers involved in slope stability analysis, the slip surface must reflect the geological structure exposed in the slope, guided by the engineer's experience and knowledge of past slope failures in a similar setting. There does not seem to be a software system that will replicate the judgement that is required to make sure the stability problem is appropriately defined. However, we are working on it.

Acknowledgement

The development of TSLOPE and our understanding of 3D LE have been influenced by many colleagues. In particular we acknowledge Dr Robert Pyke, Dr Desmond Fitzgerald, Mike Norred and Professor Nicholas Sitar.

References

Cheng, YM & Yip, CJ 2007, 'Three-dimensional asymmetrical slope stability analysis extension of Bishop's, Janbu's and Morgenstern-Price's techniques', *Journal of Geotechnical and Geoenvironmental Engineering* vol. 133, no. 12, pp.1544–1555.

Gillon, MD, Graham, CJ & Grocott, GG 1992, 'Low level drainage works at the Brewery Creek Slide, in DH Bell (ed.), *Proceedings of the Sixth International Symposium on Landslides*, A.A. Balkema, Rotterdam, pp. 715–720.

Read, JRL 2021, 'Three-dimensional limit equilibrium slope stability analyses, method, and design acceptance criteria uncertainties', in PM Dight (ed.), *SSIM 2021: Second International Slope Stability in Mining, Australian Centre for Geomechanics*, Perth, pp. 3–10, https://doi.org/10.36487/ACG_repo/2135_0.01

Spencer, E 1967, 'A method of analysis of the stability of embankments assuming parallel inter-slice forces', *Géotechnique,* vol. 17, no. 1, pp. 11–26.

TAGAsoft 2023, *TSLOPE*, version 2.3, computer software, TAGA Engineering Software Ltd, Wellington, www.tagasoft.com

TECHBASE International, Ltd 2007, *TECHBASE® Reference Manual*, TECHBASE International, Ltd, Reno.

Wyllie, DC 2017, *Rock Slope Engineering*, CRC Press, Boca Raton.