Prediction of Non-Segregating Thickened Tailings Beach Slope — A New Method

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ABSTRACT

The experimental data from full-scale flume tests at Peak Gold Mines NSW, Australia on channelised tailings flow in equilibrium slope condition have been used to develop a method for the prediction of non-segregating tailings beach slope. The developed method for prediction of beach slope is based on the theory that on a large scale, the slope of the overall tailings beach is determined by the slope of self-formed tailings channels (Williams, 2001; Pirouz et al., 2005). The flow in the channel is "steady state total transport uniform turbulent flow". An empirical relation has been established between the zone-settling velocity of non-segregating tailings samples and the Reynolds number (level of turbulence) in channelised tailings flow in equilibrium slope condition. The theory of "stable channel section" has been applied to determine the geometrical characteristics of the self-formed tailings channels on the beach. Some of the friction loss formulae for slurry transport in pipes have been modified for the calculation of the energy gradient in open channel flow and to determine the equilibrium slope of the channel.

1 INTRODUCTION

Prediction of the beach slope is the basis for design of stacked tailings disposal schemes but unfortunately despite all the efforts of different researchers, there is no satisfactory theoretical expression presently available for the prediction of beach slope. After the "master profile" concept was introduced by Melent'ev in 1973, many other researchers have applied it to work of their own. What the "master profile" does not achieve is prediction of the overall beach slope. Much of the previous work has been concerned with segregating tailings and has been as much pre-occupied with forecasting hydraulic sorting as it has beach slope. For the special case of non-segregating tailings there is a) by definition, no hydraulic sorting, and b) a planar beach slope, i.e. no concavity (Williams, 2001).

Salvas (1989) pointed out that "an analytical relation has not yet been obtained for such beach profile. However, it is known that the finer the tailings the less the thickening required to obtain a given slope, and vice versa." Winterwerp et al. (1990) has concluded that: "a thorough understanding of the mechanisms involved in the transport and deposition of hyperconcentrated sand water mixture flow could not be gained from literature readily available. Therefore, more basic knowledge and accurate experimental data had to be gained about the characteristic behaviour of hyperconcentrated sand water mixture flows and the subsequent settling of the sand."From the analysis of the equations for overall slope presented in the literature, Kupper (1991) has concluded that "There is still no adequate method to predict the beach overall slope at the design stage."

Blight (1994) stated that "At the present there is no rational adequate empirical method of predicting the average gradient for a beach."After reviewing the results of several research works on tailings beach slope, Pinto and Barrera (2002) stated that: "None of the research works establishes a comprehensive theory or formula to forecast the slope of any tailings hydraulically discharged."

On studying the theories and mechanisms put forward by the various researchers and authors in the field, it is also apparent that there is no uniformity of view on the mechanism that governs tailings deposition and beach formation. There have from time to time been different research campaigns supporting different ideas about how the tailings beach forms and develops. Most of the work has focussed on the behaviour of sheets and fan flow in the laminar regime and concentrated on the estimation of the equilibrium stability slope for these sheets and fans once stationary. By contrast, there are others (Winterwerp et al., 1990; Williams, 2001;

Pirouz et al., 2005) who believe in a more complicated system of sheet and fan flow working together with turbulent channelised flow to create and develop the beach. The influencing parameters and phenomena that are claimed to determine the overall beach slope are different for each supporting theory.

The present research has adopted the Williams and Meynink (1986) model, restated in Williams (2001) - i.e. has adopted the premise that the critical gradient of self-formed channels is the overall beach slope - and is dedicated to trying to develop flow equations that describe self-formed channel flow and the critical gradient of such flow for a non-segregating slurry.

2 PRINCIPLES

When continuous flow of non-segregating tailings with any flowrate occurs over a flat area from a central point the spread of tailings slurry does not occur uniformly as an ever-expanding sheet over the full 360°. Instead it is a matter of common observational experience that whilst the flow initially spreads itself out in the form of sheets and fans which provide a bed of fresh tailings, very soon flow then channelizes and confines itself to a relatively wide channel of laminar flow. As further time passes and the tailings flow continues, and as soon as a bed layer of sufficient thickness is created from newly deposited tailings, a narrow self-formed channel quite suddenly appears within the new, now stationary laminar sheet, and the flow depth and velocity increase sharply within this new channel such that the flow is turbulent. This is the mechanism by which flow passes trough the transition regime and changes the regime from laminar to turbulent flow (Pirouz et al., 2005). Figure 1 shows turbulent channels, some active, some relic, formed on the tailings stack of Peak Gold Mines in NSW, Australia.

Figure 1 Turbulent channels formed on the tailings stack of Peak Gold Mines in Cobar, NSW, Australia

Zone-settling, in which all particles settle together in zones, encountered in non-segregating thickened slurry mixtures is recognized as the main mechanism for density profile formation in laminar flow of non-segregating slurries. Laminar flow cannot persist over long distances since zone-settling within the flowing layer will lead to deposition after a finite period of time. Turbulence can overcome zone-settling. Thus for long distance travel the tailings confines itself into a self-formed concentrated turbulent channel. It is postulated that the slope of the channel will be steep enough for the generated turbulence to just overcome zone-settling. The turbulence in the flow must simply be just enough to overcome and counterbalance the zone-settling of the tailings particles and to prevent the density gradient with depth forming. There are experimental measured data and evidence that confirm the idea that the flow in the self-formed channels is "steady state total transport uniform turbulent flow" (Pirouz et al., 2005).

The overall beach slope of the stack is therefore determined by the limiting equilibrium channel slope which is required to create the required steady state turbulent total transport flow condition in the self-formed channel for a particular tailings.

Thus, the problem of prediction of the overall beach slope centres on the estimating and forecasting of the minimum required slope for the channel to create steady-state total transport uniform turbulent flow. If the slope becomes steeper than this limiting slope, erosion will occur in the channel. On the other hand if the slope becomes flatter than this equilibrium slope, deposition will occur. With this limiting slope, the tailings slurry can flow to any distance without deposition or scour occurring.

Based on visual observations at Peak a descriptive model was developed for the process of thickened tailings beach formation and development. The validity of the model was examined and confirmed by special experimental tests.

3 EXPERIMENTAL TESTS AND RESULTS

To better understand the characteristics and behaviour of channelised tailings flow and also determine the regime of the flow at limiting equilibrium slope conditions, a series of tests were carried out in a full-scale flow-through experimental flume installed at the Peak mine site.

What distinguishes these tests from the flume tests of previous researchers are a) the size of the flume, and b) its ability to take up to the full tailings output stream from the plant. The intention was to "capture" a length of naturally formed turbulent channel to permit detailed observations of its dimensions and flow charactertics. To avoid any scaling effects, all tests were carried out in a large straight, metal framed flume with continuous slurry flow from the effluent tailings pipeline which carries the tailings from the thickener underflow to the tailings storage area. The details of the experimental setup and the test program are explained in Pirouz et al. (2005).

In the first stage of the experimental tests the flume was set to a horizontal position and the valves opened to let slurry of a specific percent solids and discharge rate enter the flume from the upstream inlet box. The slurry was allowed to build up its own beach and channel of certain shape and slope (by depositing solids materials on the bed). After the stabilization of the beach and flow in the flume, the self-formed channel dimensions and other flow parameters were measured. Figure 2 shows some of the self-formed tailings channel sections measured in the flume tests with different flowrates.

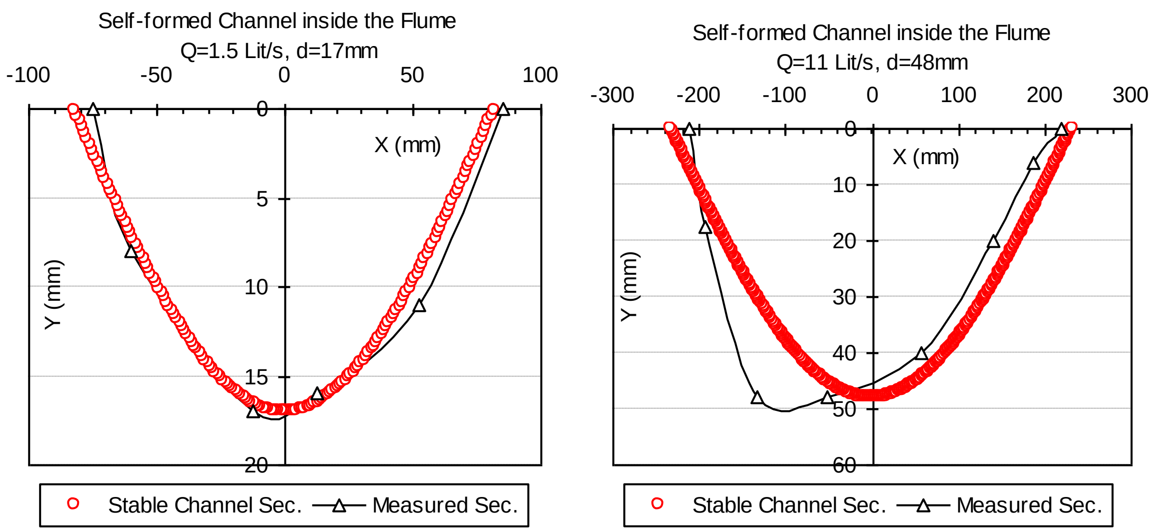

Figure 2 Stable channel profile fitted to the measured self-formed tailings channels

The fit of "stable channel profile" developed by U.S. Bureau of Reclamation (1951) for non-cohesive soils, has also been shown in the figure. The theory of "stable alluvial channel cross section" which is defined with the following equations can be found in many textbooks and research work (Simons et al., 1992).

$$y = d_{\max}.\cos\left(\frac{\tan\phi}{d_{\max}}x\right) \quad , \quad 0 \le x \le \frac{\pi}{2}.\frac{d_{\max}}{\tan\phi} \qquad (1)$$

where: ϕ = angle of repose of the material, $d_{\max}$ = maximum depth at the centreline of the channel, y = depth of the flow in any other point, x = horizontal distance form the centreline of the channel

It is observed that the geometrical characteristics (i.e. the top width, wetted perimeter, cross section area and the general shape) of the U.S.B.R. stable channel profile (with $\phi = 18^{\circ}$) have reasonably good and acceptable fit to the measured data of the self-formed tailings channel sections measured in the flume tests and on the real stack at Peak Gold Mines.

A second stage of the experimental tests was arranged using two different size PVC pipes (D = 360 mm, D = 424 mm) as artificial channels to transport the slurry mixture. These pipes were cut in half (length wise) and assembled inside the flume. The slope of the flume was set initially to a relatively steep slope that caused supercritical accelerating flow to occur. Then the slope was reduced gradually until sedimentation along the half-pipe just started to take place and a steady uniform flow developed within the pipe (with no further deposition on the bed). The flow characteristics for the semi-circular channel tests in equilibrium slope condition are presented in Table 1. It is observed that the Reynolds numbers are within the turbulent regime (Re >2000). From the flow data in Table 1 it can also be seen that the Froude number values for all of the tests are greater than 1 (Fr >1) and vary between 1.68 and 2.77 indicating the existence of supercritical flow condition in such thickened slurries and homogeneous fine tailings.

Table 1 Flow characteristics in semi-circular channel tests (equilibrium slope condition)

Q (L/s)	Slope %	Solids Percent (%)	Slurry Density (kg/m^3)	Depth (mm)	Hydraulic Radius (mm)	Average Velocity (m/s)	Bingham Plastic Viscosity (Pa.s)	Yield Stress (Pa)	Froude Number Fr	Reynolds Number Re	Hedstrom Number He
4.0	3.72	55.2	1547	30	19.21	0.99	0.0148	2.336	2.21	1983	6090
7.2	2.93	53.7	1523	36	22.87	1.36	0.0108	1.761	2.77	4384	12023
8.0	3	57.7	1587	43	27.26	1.08	0.0204	3.704	2.01	2290	10494
11.5	2.6	55.1	1545	54	33.78	1.11	0.0144	2.290	1.85	4033	19465
14.5	2.53	56.1	1560	57	35.52	1.30	0.0161	2.733	2.09	4464	20751
15.5	2	56.1	1560	57	36.67	1.39	0.0161	2.733	2.24	4772	22122
16.1	2.31	58.2	1595	67	40.70	1.23	0.0226	4.062	1.82	3537	21011
19.0	1.68	55.5	1552	73	44.58	1.19	0.0150	2.478	1.68	5478	33967

The definitions for Reynolds number, Froude number and Hedstrom number (which are all dimensionless numbers) are given below. Reynolds number is the most commonly used parameter to describe the level of turbulence in the flow.

$$\mathrm{Re} = \frac{\rho_m V R}{\eta}, \qquad \mathrm{Fr} = \frac{V}{\sqrt{gR}}, \qquad \mathrm{He} = \frac{R^2 \rho_m \tau_0}{\eta^2} \tag{2}$$

where: R is the hydraulic radius, ρ_m is the density of the slurry mixture, V is the average flow velocity, τ_0 is the yield stress of the slurry and η is the coefficient of rigidity of the slurry. It should be noted that in calculation of Froude number, Reynolds number and Hedstrom number for open channel flow R = D has been used instead of D = 4R (R is the hydraulic radius of the channel in open channel flow and D is the diameter of the pipe in pressure pipe flow). Figure 3 shows the equilibrium slope as a function of Flowrate, Reynolds number, Hedstrom number and Froude number.

It is noteworthy that, even in the turbulent regime, the equilibrium slope is a function of Reynolds number. It is also observed that the equilibrium slope appears to be a function of Froude number as well although there is a lot of scatter. Variation of Hedstrom number with equilibrium slope for the channelised tailings flow, shows an increase in equilibrium slope as the Hedstrom number decreases. There is less scatter for the Hedstrom number than the Froude number. It is also observed that the equilibrium slope decreases as the flowrate increases.

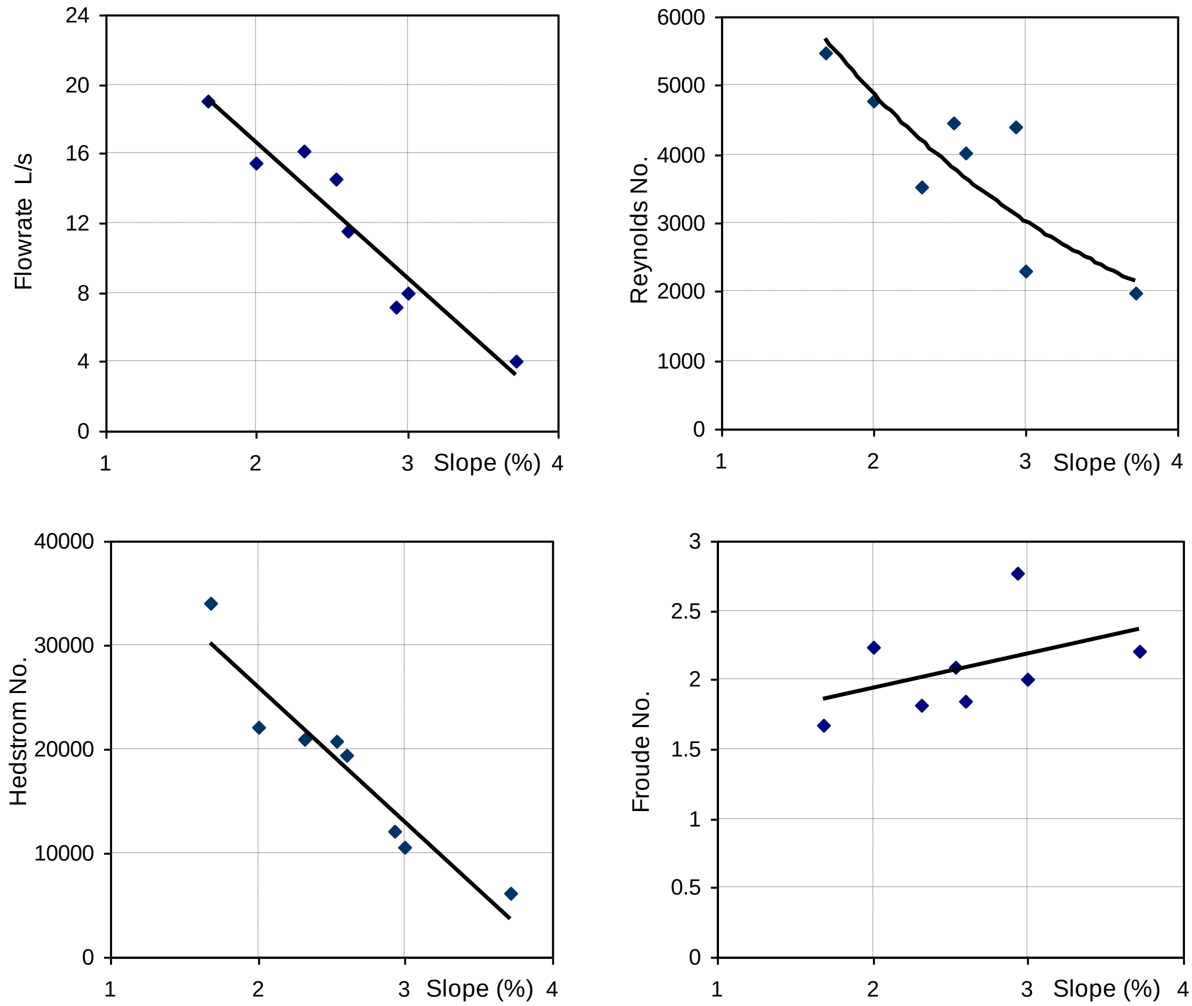

Figure 3 **Equilibrium slope as a function of the flowrate, Reynolds number, Hedstrom number and Froude number**

These values of Froude number are comparable to the suggested values for critical Froude number by different researchers for the design of slurry transport flumes. Some of the recommended values for the critical Froude number needed to establish non-deposition condition in slurry flumes are Fr = 1.25 by Faddick (1986), Fr >1.2 by Wilson (1991) and Fr >1.5 by Abulnaga (2002). The experimentally obtained data in Table 1 clearly show that it is not possible to define a constant specific value for a critical Froude number that can guarantee the non-deposition condition in the channel. It appears that the critical Froude number is a function of not only the flowrate but also the solids percent of the slurry. So, it can be concluded that the Froude number alone is not a reliable design criterion to ensure that a particular channel achieves total transport and non-deposition.

The results obtained from the full-scale flume tests clearly show that the equilibrium slope for channelised tailings flow (which determines the beach slope) is a function of Reynolds number, Froude number and the Hedstrom number. This is a very important point to be aware of if an attempt is made to use scale models to study tailings beach formation and its characteristics. The results also indicate that even in the turbulent regime (if achievable in laboratory scale tests) any decrease in flowrate from the full-scale value will have a significant effect on the beach slope.

Rheology testing was also carried out at the Rheology and Materials Processing Centre of RMIT University on the tailings samples taken during flume tests using a Contraves Rheomat 155 rheometer to obtain the rheological parameters of the Peak tailings. All of the samples, despite the different solids percents, exhibited a Yield Stress Pseudoplastic behaviour which can be well defined by the Herschel-Bulkley model:

$$\tau = \tau_y + K\dot{\gamma}^n \qquad (3)$$

where: τ is the shear stress (Pa), τ_y is the yield stress (Pa), $\dot{\gamma}$ is the shear rate (s^{-1}), K is the power law consistency factor ($Pa.s^n$), and n is the power law behaviour index. Table 2 shows the Rheogical parameters for the tailings samples from Peak Gold Mines.

Table 2 Bingham plastic viscosity and the Herschel-Bulkley model parameters for the Peak tailings samples

	Solids Percent (%)	Herschel-Buckley Model Fit Parameters			Bingham Plastic Viscosity (Pa.s)
		n	K	τ_y	
Semi-Circular Channel Tests	55.2	0.8086	0.0523	2.336	0.0148
	53.7	0.8086	0.0412	1.761	0.0108
	57.7	0.8086	0.0770	3.704	0.0204
	55.1	0.8086	0.0514	2.290	0.0144
	56.1	0.8086	0.0597	2.733	0.0161
	56.1	0.8086	0.0597	2.733	0.0161
	58.2	0.8086	0.0832	4.062	0.0226
	55.5	0.8086	0.0550	2.478	0.0150

The data and results from the channelised tailings flow tests which are presented and discussed above, have been analysed further to develop a methodology for evaluation and prediction of the "Tailings Beach Slope".

4 PROPOSED METHODOLOGY FOR BEACH SLOPE PREDICTION

As stated earlier the overall beach slope of the stack is determined by the limiting equilibrium channel slope which is required for a particular tailings, to create the steady state turbulent total transport flow condition. Thus, the problem of prediction of the overall beach slope centres on the estimating and forecasting of the minimum required slope for the channel to create steady-state total transport uniform turbulent flow.

Any methodology for estimation of the equilibrium slope for the channel should include proper answers to the following questions:

- How much turbulence is needed to overcome the zone-settling of the tailings particles and to break up the density gradient (with depth) in the flow?
- How much bed slope (energy gradient) is needed to create that certain level of turbulence to overcome all friction losses in the channel in order to keep the flow going?

The review of previous research revealed that there are no easy or simple answers to the above mentioned questions. There is not even a worldwide accepted methodology for friction loss calculation or the design of artificial channels to carry homogenous slurry flow.

In order to find a proper answer to the first question the value of zone-settling velocity for the tailings sample most be known. The zone-settling velocity of a particular tailings sample is a function of initial percent solids, rheology of the sample (yield stress, viscosity) and the initial depth of the sample. For a particular depth of flow it is possible to measure and determine the zone-settling velocity of the slurry mixture in the laboratory. A special laboratory technique and procedure has been developed in the technical laboratory of Australian Tailings Consultants (Ennis, 2001) for the measurement of the zone-settling velocity of the tailings samples. The zone-settling velocity values for the tailings samples of Peak Gold Mines with different solids percent and initial depth are presented in Table 3. The selected solids contents and initial depths are the same as those encountered during the flume tests with equilibrium slope condition at Peak.

In the turbulent flow of homogenous non-segregating tailings, to keep the solid particles within the flow, the amount of the turbulence must be enough to counterbalance this zone-settling which means that the vertical component of the turbulent velocity v' ($v = \bar{v} + v'$) should be equal to the zone-settling velocity of the slurry mixture.

Table 3 Zone-settling velocity values for the tailings samples

	Q (L/s)	Equilibrium Slope (%)	Initial Percent Solids (%)	Depth (mm)	Area (mm^2)	Average Flow Velocity (m/s)	Zone-Settling Velocity (mm/hour)
Semi-Circular Channel Tests	4.0	3.72	55.2	30	4051.40	0.987	4.209
	7.2	2.93	53.7	36	5297.44	1.359	4.958
	8.0	3.00	57.7	43	7406.68	1.080	3.456
	11.5	2.60	55.1	54	10333.78	1.113	4.798
	14.5	2.53	56.1	57	11180.09	1.297	4.349
	15.5	2.00	56.1	59	11180.09	1.319	4.385
	16.1	2.31	58.2	67	13071.58	1.232	3.672
	19.0	1.68	55.5	73	15995.49	1.188	4.995

Review of turbulence modelling theory reveals that turbulence modelling is an extremely difficult topic and despite all of the efforts that have been made during past years to increase the amount of knowledge about the subject, the field of turbulent flow has still remained the least understood area of fluid mechanics. Thus at the moment there is no satisfactory theoretical expression or method available to determine or quantify the components of the turbulent velocity, either in the direction of flow or in the direction normal to the flow direction (Munson et al., 2002). Developing a theoretical relationship between the zone-settling velocity of the solid particles of a slurry mixture and the amount of turbulence in the flow is not possible with the level of knowledge presently available about the turbulence flow structure. It is concluded that in the open channel flow of non-segregating slurries such a relationship between the zone-settling velocity and the level of turbulence in the flow, should be developed empirically and from the experimentally obtained data.

From the hydraulics of turbulent flow it is known that the fluctuating components of the turbulent velocity in the direction of motion u' and in the vertical direction v' (normal to the direction of motion) are linked together by the shear stress in the direction of motion τ_{xy} (or τ_{yx}) which is defined as below. The term $\sqrt{\overline{u'v'}}$ has the dimension of velocity and is called "friction velocity" (U_*):

$$U_* = \sqrt{\frac{\tau_{xy}}{\rho}} = \sqrt{\left|\overline{u'v'}\right|} \tag{4}$$

The "degree or the intensity of turbulence" is defined as:

$$\frac{\sqrt{\frac{1}{3}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)}}{\bar{u}} \tag{5}$$

It is seen that there is a link between the fluctuating components of turbulent velocity, and any increase in the intensity of the turbulence of flow will increase the turbulence fluctuation in all three directions (u', v' and w'). Knowing that the most commonly used parameter to describe the level of turbulence in the flow is the Reynolds number, it should be possible then to find a relationship between the zone-settling velocity of a non-segregating tailings sample and the Reynolds number of the channelised tailings flow in equilibrium slope condition. As the main parameters that directly affect the zone-settling velocity of the tailings are the yield stress and viscosity (rheology of the tailings) it should also be possible to develop the same sort of relation, between the Reynolds number and the Hedstrom number which is a dimensionless number that indicates the effect of both yield stress and viscosity. Figure 4 shows the relationships between the Reynolds numbers and the zone-settling velocities and also the Hedstrom number for the full-scale flume tests of the present research in equilibrium slope condition.

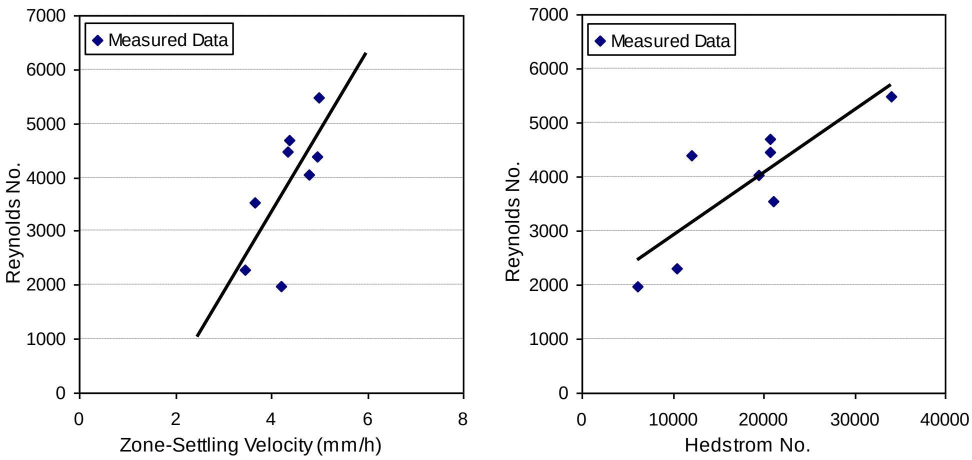

Figure 4 **Reynolds number versus zone-settling velocity and Hedstrom number**

It is observed that although the data are scattered they show a well-defined trend. Unfortunately the set of data obtained in the present research is the only available set and there are no other data of this sort available from previous research to be added to this figure. Many more data of this type from different tailings are needed to verify the graphs in Figure 4.

The relationship between the Reynolds number and the zone-settling velocity or the Hedstrom number may not be a linear relationship and may be more complicated. But as the effect of all of the influencing parameters (i.e. the effect of the mineralogy, the effect of the size of the solid particles and the solids concentration which are both hidden in the zone-settling velocity and the rheology of tailings, and the effect of the density of the slurry mixture) are included in the graph, it is expected that graphs like the one shown in Figure 4 for the tailings of Peak Gold Mines (NSW-Australia) should be independent of the tailings type as long as the tailings slurries are kept in the range of non-segregating slurries.

Graphs like the ones in Figure 4 provide a proper and satisfactory answer to the first question of "How much turbulence is needed to overcome the zone-settling of the tailings particles and break up the density gradient (with depth) in the flow?" but the second, and still important, question of "How much bed slope (energy gradient) is needed to create that certain level of turbulence, to overcome all friction losses in the channel to keep the flow going?" remains unanswered.

A review of the resistance to flow equations for open channel flow reveals that the data and formulae available in the references such as fluids mechanics and open channel hydraulics textbooks for friction factor or roughness coefficients are only applicable for clear (or dirty) water, or very dilute mixtures, but do not constitute a solution for real slurries. The suggested values and equations for estimating the friction coefficient are not correct for slurry transport in open channels (Abulnaga, 2002). The most commonly used parameters for describing the resistance to steady uniform flow are the Darcy-Weisbach friction factor f_D and the Fanning friction factor f_n. Since the Darcy-Weisbach friction factor f_D is usually accepted as four times the Fanning friction coefficient f_n ($f_D = 4 \times f_n$), the friction loss formula may be written in terms of Fanning factor as:

$$S_E = \frac{h_f}{L} = \frac{f_n \times U^2}{2gR} \tag{6}$$

where h_f is the friction loss associated with flow in the channel, L is the length of the channel, R is the hydraulic radius of the channel, U is the mean velocity of flow, $S_E = h_f / L$ is the energy gradient (bed slope) and g is the acceleration of gravity. The main parameter that needs to be determined is f_n the Fanning friction coefficient.

There are special friction loss formulae developed by researchers for the transport of slurry in pipes. Some of these equations which are more commonly used in the flow of homogeneous slurries are reviewed by Abulnaga (2002). Because there are no formulae or relations specifically developed for the energy loss calculation in open channel flow of homogenous slurries, some of these friction loss formulae for slurry transport in pipes have been modified for the calculation of the energy gradient in open channel flow. The results obtained from the methods of Darby et al. (1992), Tomita (1959) and Irvine (1988), with some modification, showed very good correlation and fit to the measured values of channel slope in the experimental tests.

The Darby et al. (1992) method has been modified as below:

$$f_{NL} = \frac{16}{\mathrm{Re}}\left[1+\frac{He}{6\,\mathrm{Re}}\right], \qquad f_{NT} = 10^{a}\,\mathrm{Re}^{b} \tag{7}$$

$$a = -1.27\times\left[1+0.146\exp\left(-2.9\times10^{-5}\,He\right)\right], \qquad b = -0.193 \tag{8}$$

$$f_{N} = \left(f_{NL}^{\,m} + f_{NT}^{\,m}\right)^{(1/m)}, \qquad m = 1.7 + 40000/\mathrm{Re} \tag{9}$$

Where: Re is the Reynolds number, He is the Hedstrom number, f_{NL} is the Fanning friction factor for laminar flow, f_{NT} is the Fanning friction factor for turbulent flow and f_N is the combine laminar and turbulent Fanning friction factor (this f_N has been used in the calculation of channel slope).

In the Darby et al. (1992) method to calculate the Fanning friction coefficient for Bingham plastics, Equation 2 were used to calculate the Reynolds number and Hedstrom number. As stated by Abulnaga (2002), the values of the parameters "a" and "b" in this method are based on empirical data for closed conduits. It was found that the above values for a and b have better fit to the measured data.

Tomita (1959) extended his laminar flow model to turbulent flows in smooth pipes by applying Prandtl's mixing length concept, and developed an implicit equation for Fanning friction factor in turbulent pipe flow (f_{PLT}):

$$\frac{1}{\sqrt{f_{PLT}}} = 4\log_{10}\left(\mathrm{Re}_{PL}\sqrt{f_{PLT}}\right) - 0.4 \tag{10}$$

$$\mathrm{Re}_{PL} = \frac{6}{2^{n}}\times\left(\frac{\left[1/n+3\right]^{1-n}}{1/n+2}\right)\times\left(\frac{D_i^{n}V^{2-n}\rho_m}{K}\right) \tag{11}$$

$$\frac{\Delta P}{L} = \frac{3(1+3n)\times\rho_m\times V^{2}\times f_{PLT}}{2D_i(1+2n)} \tag{12}$$

where: K is the power law consistency factor, n is the power law behaviour index, D_i is the pipe diameter, L is the pipe length and V is the mean velocity of flow. It should be noted that the calculated ΔP in Equation 12 has the pressure dimension and for using the Tomita method in open channels, ΔP should be converted to the slurry head knowing that $P = \rho gh$.

A third method was derived for the calculation of Fanning friction factor by combining the modified Reynolds number proposed by Heywood (1991) for pseudoplastics fluids and the Irvine (1988) equations for Fanning friction factor. The same set of equations has been used by Abulnaga (2002) to calculate the friction loss in pipe flow:

$$\mathrm{Re}_{\mathrm{mod}} = \frac{\rho_m\times V\times D_i}{K}\left(\frac{4n}{1+3n}\right)^{n}\left(\frac{D_i}{8V}\right)^{n-1} \tag{13}$$

$$f_n = \frac{F'(n)}{\mathrm{Re}_{\mathrm{mod}}^{[1/(3n+1)]}} \tag{14}$$

$$F'(n) = \frac{2}{8^{n-1}}\left(\frac{8n^n}{7^{7n}(1+3n)^n}\right)^{1/(3n+1)} \tag{15}$$

where: ρ_m is the density of the slurry mixture, K is the power law consistency factor, n is the power law behaviour index, D_i is the pipe diameter, V is the mean velocity of flow.

It should be noted that for the calculation of Re and He in Darby method and also to calculate Re_{PL} in Tomita using the value $D_i = R$ (R is the hydraulic radius of the channel) showed a better fit to the measured data but in Irvine method for the calculation of $\mathrm{Re}_{\mathrm{mod}}$ the value of $D_i = 4R$ was used to get better fit to the measured data. The value of $D_i = 4R$ has been used for the friction loss calculation in Equation 6 for Darby and Irvine methods and in Equation 12 for Tomita method.

Figure 5 shows the equilibrium slope values for the experimental data calculated by the three different methods. The values of measured equilibrium slope have also been shown in the figure for comparison. As can be observed, the Darby, Tomita and Irvine methods all have reasonably good fit to the measured slope values.

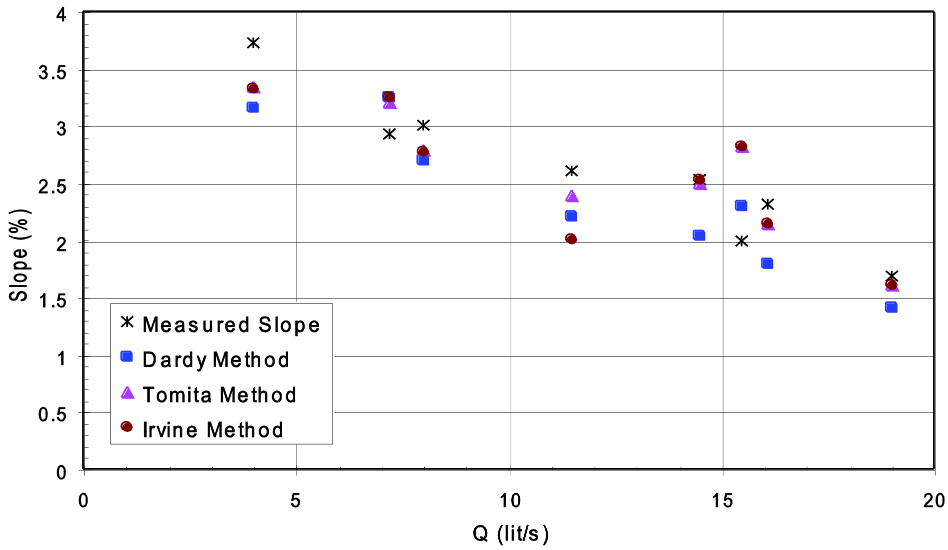

Figure 5 **Calculated channel slope for different flow rates (Darby, Tomita and Irvine Methods)**

5 STEP BY STEP PROCEDURE FOR BEACH SLOPE PREDICTION

Based on the above mentioned strategy, the following model is developed for the estimation of overall beach slope formed by non-segregating tailings.

5.1 Inputs to the Model

- Slurry density "ρ_m" and solids percent $C\%$ by weight.
- Angle of repose for tailings particles "ϕ".
- Zone-Settling velocities for different depths and solids percents "V_Z".

- Rheology of the tailings (yield stress "τ_0", plastic viscosity or coefficient of rigidity "η", the power law consistency factor "K" and power law behaviour index "n").
- Slurry flowrate "Q" for which the beach slope has to be estimated.
- Reynolds number versus zone-settling velocities graph (Figure 4).
- Reynolds number versus Hedstrom number graph (Figure 4).

5.2 Step by Step Procedure

- *Step1:* Assume a value for the depth of flow in channel y.
- *Step 2:* Having the flowrate Q, flow depth y and an adopted angle of repose of tailings solid particles, calculate the self-formed channel shape and characteristics such as cross section area (A), hydraulic radius (R) and the flow velocity ($V1$) by use of "stable channel section equations" (Equation 1).
- *Step 3:* For the assumed depth y and known solids content C%, estimate the zone-settling velocity value V_Z.
- *Step 4:* Calculate the Hedstrom number "He" for the assumed flow depth and known rheology parameters and density (Equation 2).
- *Step 5:* Having the zone-settling velocity value for the assumed depth, estimate the required Reynolds number "Re" (turbulence) needed for counterbalancing "V_Z" from the graph in Figure 4, or alternatively via the Hedstrom number "He" calculated on Step 4 using the graph in Figure 4.
- *Step 6:* Having the Reynolds number from Step 5 above, calculate the velocity needed to create that amount of turbulence (Reynolds number) in the flow "$V2 = \eta \,\mathrm{Re}/\rho R$". Compare the two velocities estimated in Step 2 ($V1$) and Step 6 ($V2$).
    - If $V1 = V2$ the assumed flow depth (y) in Step1 was correct and this value of flow depth together with other flow characteristics and slurry behaviour can be used in Step 7 to calculated the channel slope.
    - If $V1 > V2$ then the assumed value for the flow depth (y) in Step 1 was too small. Increase (y) and repeat Step1 to Step 7.
    - If $V1 < V2$ then it means that the assumed flow depth value (y) in Step 1 was too large. Reduce (y) and repeat Step1 to Step 7.
- *Step 7:* Use the Darby et al. (1992) method, Tomita (1959) method and Irvin (1988) method to calculate the channel slope. The average of the three methods will give a reasonable estimate of the equilibrium slope for the channel which is equal to the beach slope value.

6 CONCLUSIONS

The following direct conclusions can be drawn from the results obtained in the present research work and from the analyses presented in this paper:

- The experimental results of the present study and also visual observation on the full scale beach formed by non-segregating homogenous tailings flow clearly show that if flume tests with non-segregating tailings are run with continuous flow of tailings through the flume over a sufficiently long period of time, they will end up achieving a channelised supercritical turbulent flow regime. This situation is not achievable in laboratory scale facilities since they all typically use discrete batch flow of tailings.

- The overall beach slope of the stack is determined by the limiting equilibrium channel slope which is required for a particular tailings to form steady state turbulent total transport flow condition in the self-formed channel. The limiting value of this equilibrium slope which creates enough turbulence in the flow to counterbalance the zone-settling of the particles is a very important parameter in the design of open channels to carry the homogenous tailings mixture.
- In non-segregating turbulent channelised tailings flow the equilibrium channel slope (beach slope) is a function of Reynolds number, Froude number and the flowrate. Therefore, in any attempt to model the channel behaviour in the laboratory, the relationship between both the Reynolds number and Froude number that exists at full scale must be replicated.
- There are very few and limited data available on the flow characteristics and behaviour of non-segregating channelised thickened tailings and the beach formed by this type of flow. Actually there are no reliable data available on the Reynolds and Froude numbers of this type of flow in equilibrium slope conditions. This is a topic well worth further research.
- The level of knowledge presently available about turbulence modelling is not sufficient to develop a theoretical model for the amount of turbulence needed to counterbalance the zone-settling in a particular tailings sample. Such relations have to be defined in the form of experimental graphs like the ones developed in Figure 4. But many more experimental data from full-scale flume tests with different tailings are needed to make it possible to create universal design graphs. This is an extremely important topic which needs further research.
- As the effects of all influencing parameters are considered in graphs like the ones in Figure 4, it is expected that such graphs, which in this case were developed for the tailings of Peak Gold Mines (NSW, Australia), should be independent of the tailings type as long as the tailings slurries are kept in the range of non-segregating slurries.
- Although there are many models and equations available for head loss calculation in pipe flow of non-Newtonian fluids and different slurries, there is no method developed yet specifically for the calculation of resistance to flow (head loss calculation) of the open channel flow of homogeneous slurries. Further research and experimental data for developing special models and formulae for friction loss calculation in open channel flow of homogeneous non-Newtonian slurries is needed.
- The methodology developed in this paper for the estimation of the equilibrium channel slope (beach slope) gave a reasonably good and acceptable prediction for the beach slope value. This can be used as an estimated figure at the design stage of a central thickened tailings discharge scheme or down valley discharge scheme but there is no doubt that further research is still needed on special topics to convert the proposed method to a generally accepted design tool for stack design.
- The USBR method for the computation of the stable channel section which was used in the beach slope estimation model is only applicable for non-cohesive soils and can not be used for tailings with considerable amounts of clay content. The measured data show that the USBR method is valid for the tailings at Peak Gold Mines. The validity of this method for other tailings at other mine sites needs to be proven by further testing and measurement. More investigation on the topic of stable channel section shape and size (especially for the typically fine mine tailings) is needed.

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