On the Relation Between Laboratory Flume Tests and Deposition Angles of High Density Tailings

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ABSTRACT

Experience with large scale surface deposition has shown that field slope angles of fresh paste are often flatter than angles achieved in small-scale flume experiments. A review of theoretical predictions of non-Newtonian flow may show us why. Predictions of slow spreading of a Bingham fluid over a horizontal plane show that the overall angle of the deposit at equilibrium is dependent on the scale of the flow. The deposition angle is sharp near the toe, but flattens out as one travels away from the toe towards the deposition point. The theoretical predictions compare very well with flume geometries of two different tailings. The theory has the potential to better extrapolate laboratory flow experiments to the field.

1 INTRODUCTION

At present, the prediction of deposition geometry of fresh thickened or paste tailings in the field remains difficult. This is in part due to changes in rheology that occur during pipe transport, due to shear thinning, temperature effects, air entrainment, and other phenomena (Shuttleworth et al. 2005; Crowder, 2004; Pullum et al., 2006). However, this is insufficient to explain the common observation that field deposition geometry does not resemble that of laboratory scale experiments (Crowder, 2004; Engman et al., 2004; Fourie and Gawu, 2004). Indeed, flume tests showing a deposition angle from 10 to 20 % often achieve only slopes of 4% or less in the field. This is not unique to tailings: Engman et al. (2004) refers to a study on fine sands (CUR, 1992), where the deposition angle was found to be a function of scale. The present paper will show that characterizing the geometry in terms of a single deposition angle may be misleading. An alternative way to characterize thickened tailings geometry and relate laboratory scale tests to field geometry will be expounded, and compared to laboratory flume tests and one field result.

2 THEORY

Equations for the equilibrium profiles of yield stress fluids may be derived using "Lubrication Theory", in which the continuity and momentum equations of fluid mechanics are simplified by assuming the slow spreading of a thin layer or film. The simplifying assumptions are:

- The ratio of thickness to horizontal extent of the flow is small.
- The velocity of the material is slow, such that terms that include the ratio of inertial to viscous forces will vanish from the momentum equation. In other words, the Reynold's number is small.

These simplified momentum and continuity equations have been solved analytically for given special geometries and special conditions by several authors in the fluid mechanics literature, looking at applications such as mud or lava flow (Yuhi and Mei, 2004; Liu and Mei, 1989; Balmforth et al., 2002; Coussot and Proust, 1996). For example, after the above assumptions are applied, if one considers flow along an inclined plane, the momentum equation in the direction of flow would reduce to (Liu and Mei, 1989):

$$\frac{\partial p}{\partial x} = \rho g \sin\theta + \frac{\partial \tau}{\partial z} \tag{1}$$

Where p is pressure, ρ is the density, θ is the angle of the inclined surface from the horizontal, g is the acceleration due to gravity, τ is the shear stress, the x axis is the direction of the inclined plane, and the z axis is perpendicular to the inclined plane. If we assume the pressure distribution to be hydrostatic, then:

$$p = \rho g (h - z)\cos\theta \tag{2}$$

Where h is the height of the free surface or thickness of the flow at a particular x, h is measured perpendicular to x. Then differentiating (2) to substitute it into the left-hand side of (1), and solving for τ one obtains an expression in terms of depth z:

$$\tau = (h-z)\left(\rho g \cos\theta\left(\tan\theta - \frac{\partial h}{\partial x}\right)\right) \tag{3}$$

Now, setting z=0, one may obtain an expression for the steady state profile of a Bingham fluid, given the condition $\tau < \tau_y$, the yield stress. To examine the equation for a flat bed, we can set θ=0 and solve for h to obtain:

$$h^2 - h_0^{\ 2} = \frac{2\tau_y}{\rho g}(x - x_0) \tag{4}$$

Where h_0 is the height at x_0. This formula has been used to describe lava flows as early as Hulme (1974).

3 APPLICATION TO LABORATORY FLUME TESTS AND CONSEQUENCES FOR THE FIELD SCALE

Equation 4 can be fitted to laboratory flume tests by setting x_0 to be the runout distance, and therefore h_o becomes zero. Figures 1 and 2 compare profiles of flume tests performed on gold tailings and Kaolinite, with profiles predicted using Equation 4. The measured data for Figure 1 was obtained by manually interpreting photographs of flume tests on gold tailings taken by Crowder (2004), while the measured data for Figure 2 was obtained from photographs in Kwak et al. (2005). The other parameter, density, was calculated from reported water contents, specific gravities, and assuming the degree of saturation was 1.

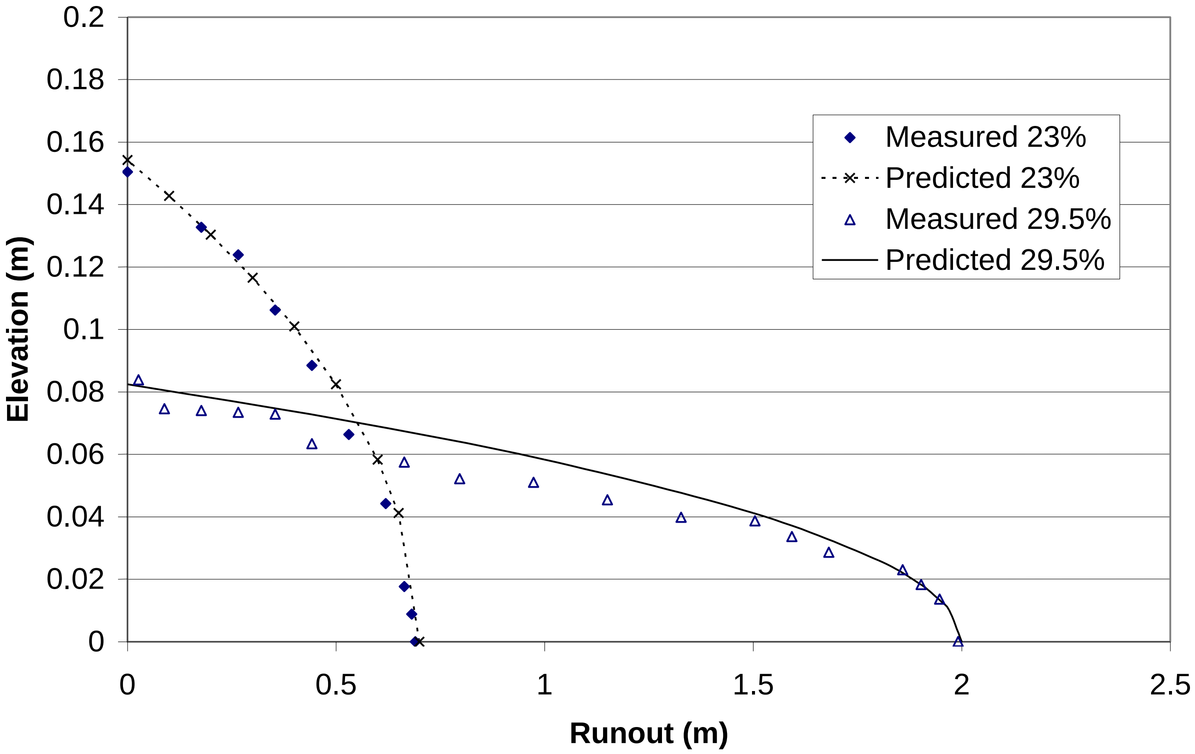

Figure 1 **Measured and predicted flume profiles of gold tailings with flocculent added, assuming yield stresses of 30 and 300 Pa. Density was calculated using a specific gravity of 2.98 g/cm³ and assuming S=1**

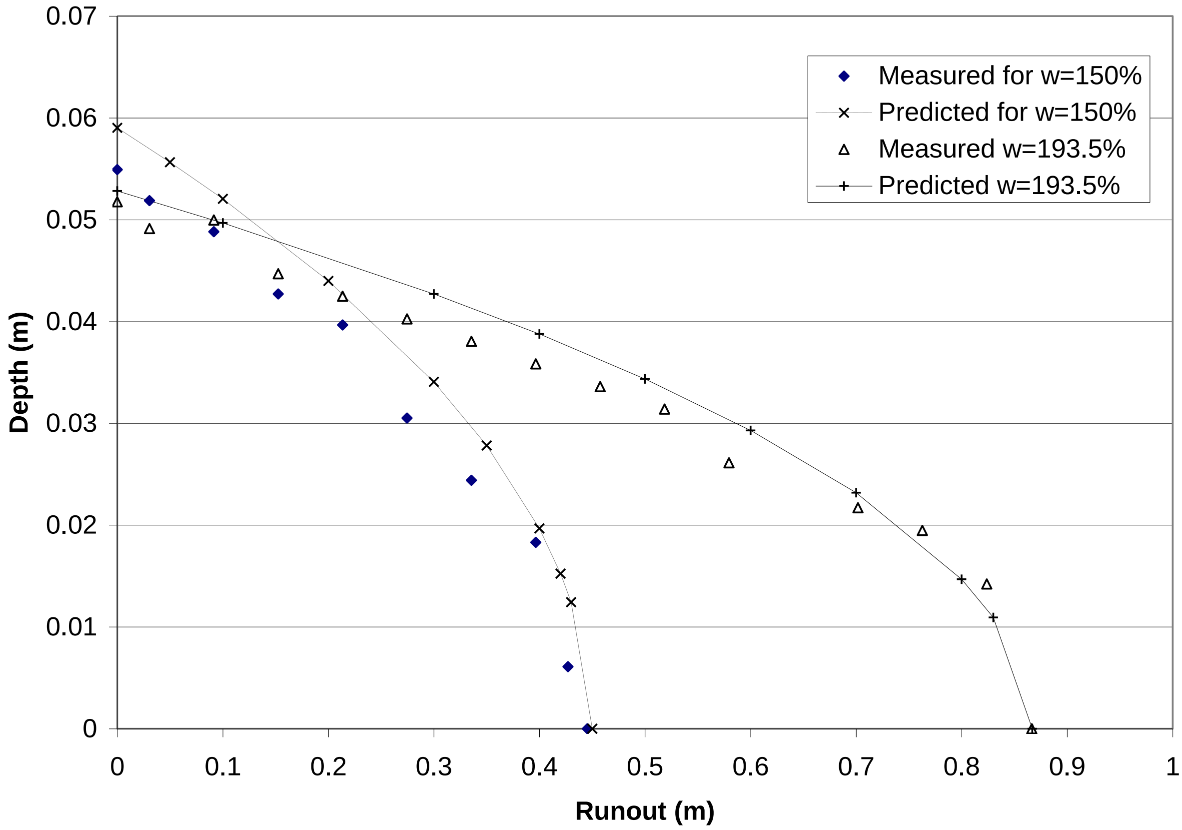

Figure 2 **Flume profiles of kaolinite fitted with theoretical predictions using independently measured yield stresses of 50 and 20 Pa. Density was calculated using a specific gravity of 2.62 and assuming S=1**

Equation 4 describes the steady-state profiles extremely well for both these two materials. Note that for Figure 1 the curves are generated using best-fit yield stresses, while the predictions in Figure 2 employ independently measured yield stresses. In both cases, the shape of the geometry is well-captured by Equation 4.

Now, if Equation 4 is then used to predict the geometry of larger flows, as shown in Figure 3, one sees the influence of scale of the average slope of the deposit. If Equation 4 holds, a material which exhibits a "slope" of over 20% in the laboratory can end up with a slope less than 5% at the field scale at the same water content and density.

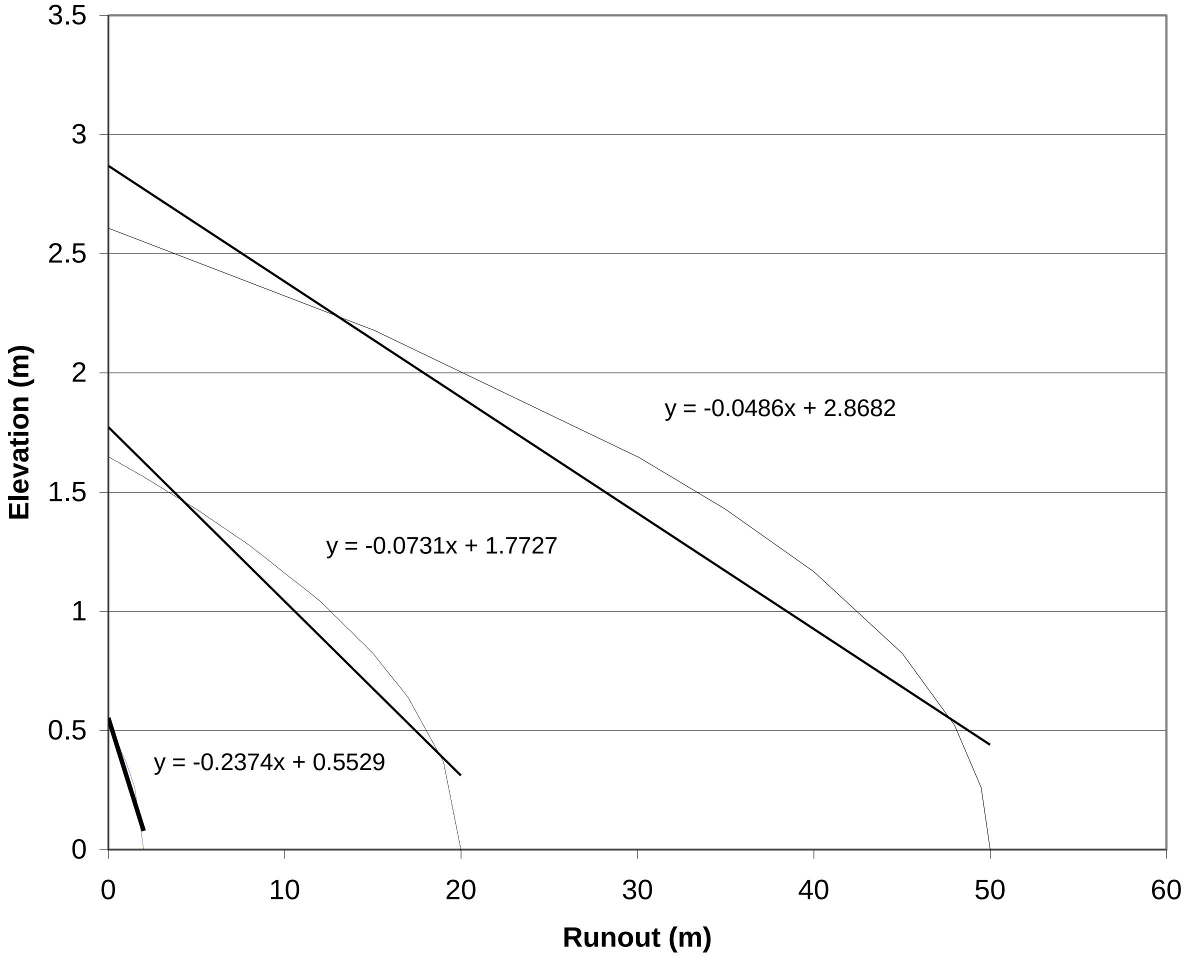

Figure 3 **Profiles of theoretical predictions of run out along a horizontal plane, changing only the volume of paste. The overall slope of the deposit is dependent on scale**

3.1 Effect of Topography

In the same vein as the derivation of Equation 4, an equation for the steady-state profile of flow from the top of an inclined hill may be derived (Yuhi and Mei, 2004):

$$h' - h_0' + \ln(1 - h') = x' - x_0' \tag{5}$$

Where h' and x' are normalized variables, such that $h = h'$ [τ_y / ($\rho g \sin\theta$)] and x = x' cot θ [τ_y / ($\rho g \sin\theta$)]. Figure 4 shows the influence of the slope of the topography underlying the flow on the final geometry.

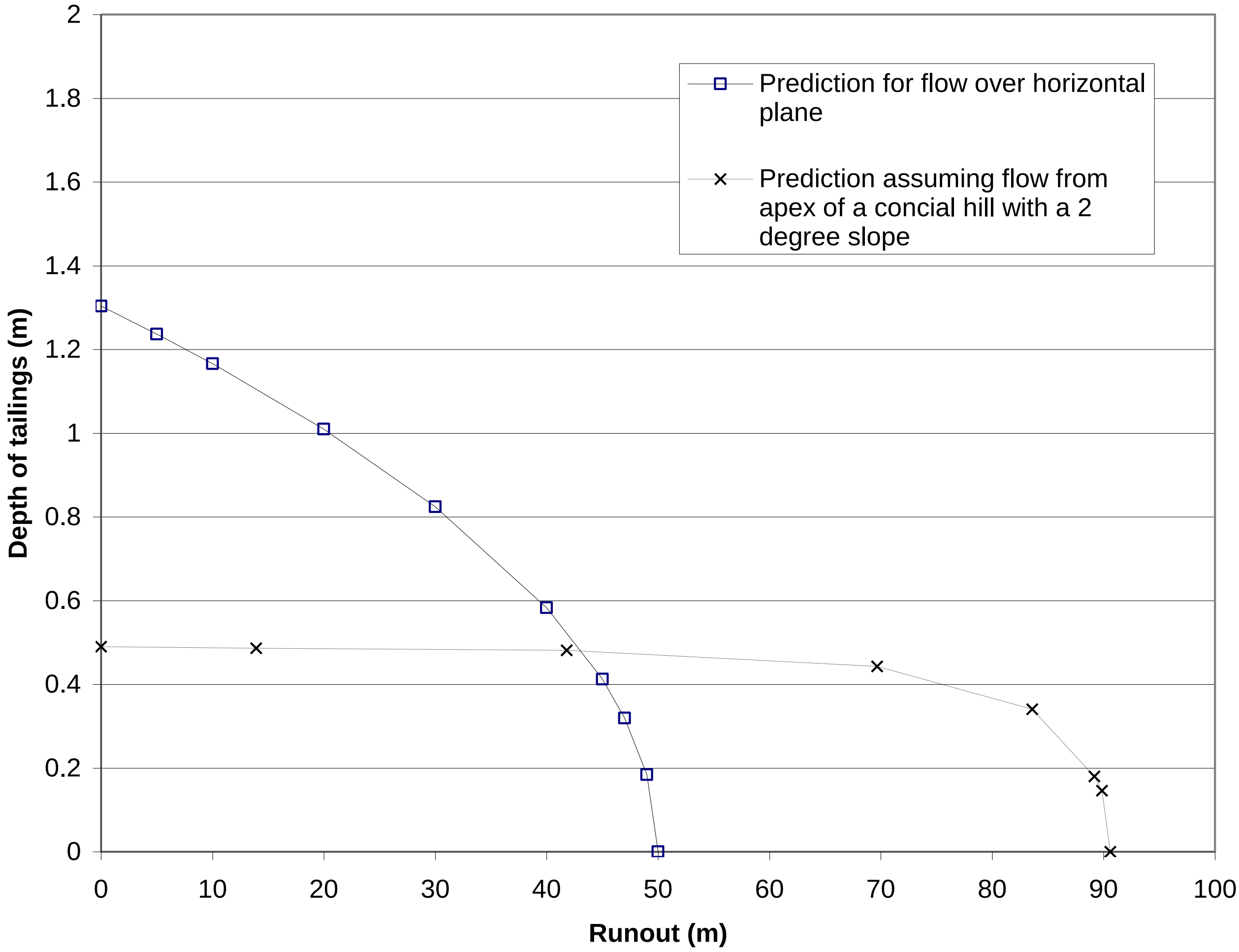

Figure 4 **Predicted geometries of deposited paste of same volume, one for flow over a horizontal plane, one for flow from apex of a conical hill. Even a 2 degree slope strongly affects the geometry**

Note that in Figure 4 the y-axis is "depth of tailings" not the elevation of the tailings from a horizontal plane. It is apparent that the slope of the tailings far away from the toe approaches the slope of the pre-existing topography.

This result is interesting as it may explain how slope angles of a tailings impoundment employing a cyclic deposition scheme evolve over time, such as at Bulyanhulu, where slopes were initially quite low, but then steepened with time (Crowder, 2004). If the first layer placed strengthens sufficiently by evaporation and self-weight consolidation, it will then represent the pre-existing topography to the next fresh layer, and so on, gradually increasing the angle of the whole deposit.

Figure 5 shows Equation 5 fitted to one horizontal transect of a tailings deposit at Bulyanhulu, obtained in the summer of 2005. This is a "best-fit", obtained by varying the yield stress of the material.

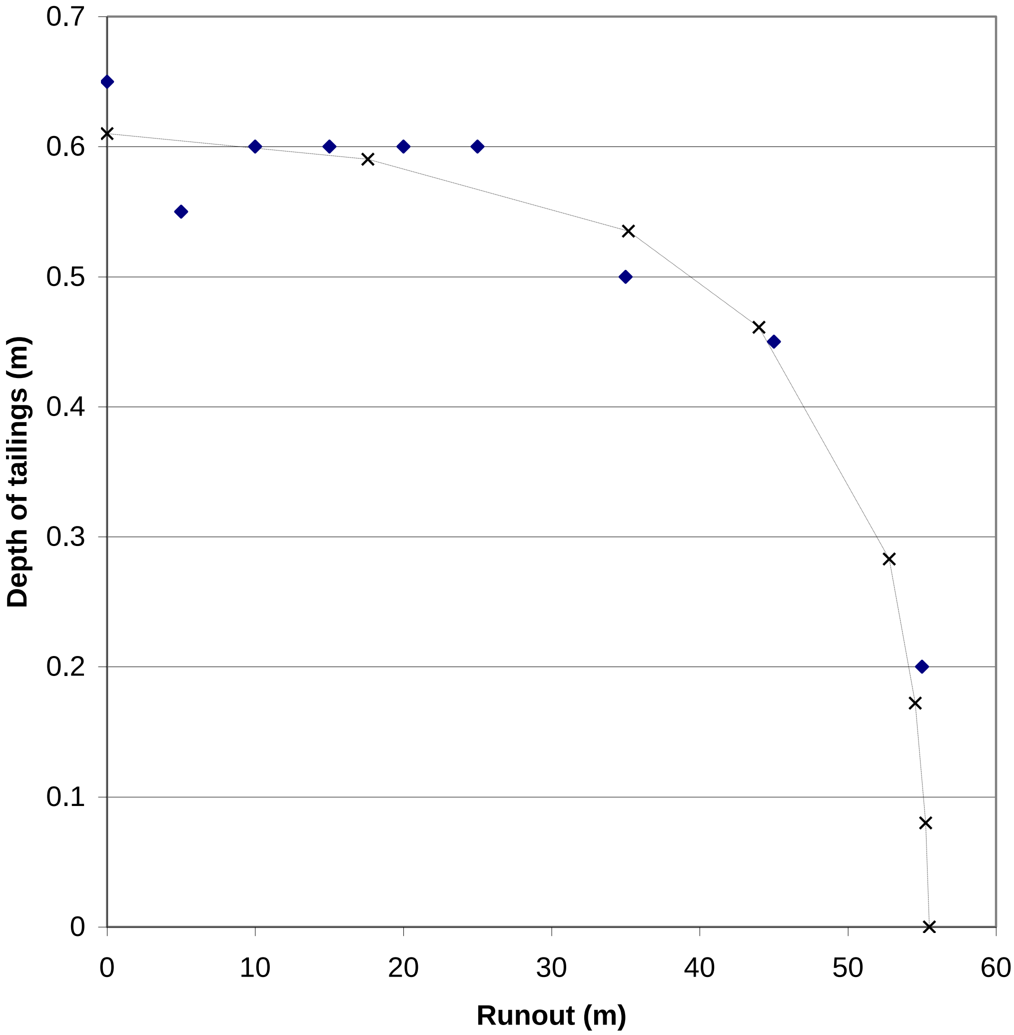

Figure 5 **Profile of Bulyanhulu tailings in the field, fitted with theory assuming deposition from apex of a conical hill of 0.5 degree slope, 100 Pa yield stress, and bulk density of 1900 kg/m^3**

4 DISCUSSION

Equations 4 and 5 may have limited applicability in practice due to the assumptions in their derivation, such as slow spreading and uniform slope of the underlying material. However, they show a way forward. They can be used to validate more sophisticated numerical methods. This author is involved in a project looking at the applicability of different kinds of models to simulate large flows of non-Newtonian materials over complex topography, and intends to use these equations as test cases.

These results stress that the increase in strength of tailings post-deposition through evaporation and consolidation are key to predicting the final geometry of the impoundment. It may be that Bulyanhulu was able to achieve its relatively steep slopes (6% compared to less than 4% at most other surface thickened tailings sites) due to its strategy of cycling deposition and the relatively high rates of evaporation.

5 CONCLUSIONS

It is not accurate to characterize the geometry of thickened tailings deposits in terms of a single angle measured in the laboratory, as the geometry is a function of the scale of the deposit. The presented equations demonstrate the influence of scale and the topography of the underlying ground surface or older tailings layer on flow geometry. The presented equations may be used to more accurately relate laboratory experiments to the geometry of field scale paste deposits. However, they are based on the assumption that the underlying material has a uniform slope. A more sophisticated model may be needed to simulate the build up of thickened tailings stacks, due to the evolving geometry. Nevertheless, the presented equations provide valuable insight into the flow behaviour of thickened tailings.

ACKNOWLEDGEMENTS

Jason Crowder photographed the flume test profiles used in Figure 1 during the course of his PhD at the University of Toronto. Akber Pabani, an undergraduate student who worked at Bulyanhulu during the summer of 2005, measured the depths of tailings appearing in Figure 5. The author wishes to thank Barrick Gold Corporation and NSERC for their ongoing support of work relating to surface tailings disposal.

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