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Uncertainty in Rock Mass Jointing Characterisation

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Abstract

The analysis of jointing in a rock mass is a critical step in any rock engineering project as the extent of fracturing in the rock mass is the dominant factor controlling the rock mass strength. Uncertainties in the characterisation of a rock mass are very rarely considered.

Historically, the assessment of jointing has been a largely empirical process and a project may proceed when in the opinion of the geotechnical staff 'enough' data has been collected. For example, the number of fractures in a length of core is used to calculate the RQD or the fracture frequency, but the direction of the core axis may be ignored even though it is known that the fracture frequency is a function of direction when the jointing is not isotropic and thus the fracture frequency may be incorrectly assessed.

The orientation distribution of joint sets in a rock mass can be analysed quite easily as conventional scanline mapping of the rock mass provides a reasonably precise estimate of the dip and dip direction of every joint logged. However, to make a three-dimensional estimate of the extent of fracturing in a rock mass, it is necessary to determine joint persistence in three dimensions. To do this requires the assumption of a model of the geometry of the joints and flat disks are usually chosen as this is the simplest possible two-dimensional shape, requiring the least amount of information to describe the joints.

The estimation of the size distribution of the joints in a joint set is a statistical estimation process that is in principle well defined, once a model for the joints has been adopted. If the estimation process has a sound mathematical basis, it is possible to estimate the uncertainty in the parameters. These estimates of the uncertainties in the nature of the joints can be propagated through to arrive at a range of values for the rock mass indices of interest rather than a single value of unknown reliability. Once these ranges are known, an objective decision can be made regarding the state of knowledge of the rock mass.

This paper introduces the new paradigm of quantification of uncertainty in the assessment of the extent of fracturing in jointed rock masses and provides some examples of how estimates of rock mass indices are affected by the quantity of data available for analysis.

1 Introduction

One of the most important questions to arise in rock engineering is: How 'well' must a rock mass be characterised to meet a given design objective? Whether the design objective is ensuring that an excavation is stable or that a blast design is effective the characterisation of the rock mass is a key input in ensuring that the design objective is 'achievable'.

It is not physically or practically possible to make all the measurements required to fully characterise a rock mass and, therefore, any description of a rock mass that we might develop is subject to some degree of uncertainty. The uncertainty in the characterisation of a rock mass imposes limits on our confidence in predictions of the behaviour of the rock mass and thus in the suitability of a rock engineering design and may be a critical limit in achieving design objectives.

The control of the uncertainty and the understanding of the results of uncertainty in predicting the behaviour of a rock mass may therefore be critical requirements in the characterisation of a rock mass and the modelling of the behaviour of the rock mass.

The characterisation of the jointing in a rock mass involves the aggregation of a set of measurements to obtain parameters that can be used to describe the behaviour of the rock mass under certain, usually limited, conditions. The measurements that are made in characterising a rock mass provide a set of samples of the parameters used to describe the rock mass. Since these measurements are aggregated to form a description of the characteristics of the rock mass the parameters derived from them are therefore, as with any sample set, subject to uncertainty and should be modelled statistically.

A valid statistical description of any rock mass is an ideal requirement for any attempt to describe its behaviour. Therefore, quantifying and understanding the uncertainty in our description of a rock mass and the effect that the uncertainty may have on our ability to model the behaviour of the rock mass is critical.

The errors that can occur with any estimate of a parameter include bias in the estimate and the problems associated with estimation of parameters include the risk that the bias is larger than predicted or that when estimating a parameter that describes a property of a set of measurements the estimated parameter differs from the true value by an unacceptable extent. For example in estimating a parameter such as the variance of the orientation of a joint set, if the estimate is too small predicting the behaviour of the rock mass may be prone to significant risks.

In addition to the requirement for a valid statistical model of the parameters describing a rock mass there is the ever present problem of the existence of unknown geological conditions. Such unknowns add to the uncertainty associated with the prediction of rock mass behaviour. However, the discussion presented here will focus on the characterisation of a rock mass from the surface exposure of the mass. Such an exposure is itself a sampling process. The complex nature of the sampling that the exposure of rock at anything other than a perfectly flat face entails precludes any mathematical model of the sampling imposed by the exposure. Methods of dealing with sampling at an exposure such as scan line mapping have been developed to enable practitioners to effectively utilise measurements that may be taken at an exposure.

New methods of measuring the spatial characteristics of the joints exposed at a rock face raise the problem of dealing with the statistics of the sampling process that these measurement methods implement and with these methods new aspects of uncertainty in the characterisation of a rock mass arise.

2 The characterisation of rock mass jointing

Prior to the development of remote sensing measurement techniques such as laser scanning systems and modern photogrammetric systems the limitations of non-contact or remote sensing techniques meant that the majority of the data collected for the characterisation of the jointing of a rock mass was obtained through measurements that required some form of direct access to the rock mass.

The measurements of jointing that could be made were generally restricted to those that could be made with measuring instruments requiring operator control of the physical contact with the surface of the rock mass. This restricted the range or scope of the measurements that could be made thus imposing a sampling bias on the data. Methods were developed to deal with the sampling bias, most notably scan-line mapping and the associated statistical characterisation of the data. However, the problem of the time and cost of data acquisition remained.

The development of remote sensing techniques has enabled the collection of large amounts of spatial data from extended areas of a rock mass that are not accessible to measurement that requires physical contact with the rock mass. The remote sensing techniques that are easily deployable are generally those that generate some form of spatial data from which the position, orientation and spacing of some of the exposed discontinuities in a rock mass can be estimated.

Laser scanning and photogrammetry systems acquire ‘point clouds’, collections of measurements of points in 3D space that can be ‘seen’ by the relevant sensor. With some data sets, estimates of the roughness or shape of exposed surfaces can also be made.

Since a rock mass is a finite collection of discontinuities, no discontinuity set can be fully characterised unless the parameters of each discontinuity can be accessed and measured. The feasibility of doing this is questionable and so there is generally some degree of ‘unknown’ structure in the rock mass. The ‘difference’ between the descriptions of the jointing that can be derived from the measurements that may be made and knowledge of the parameters for every member of the set, is the uncertainty with which we must work.

Effective use of estimates of parameters that make up the characterisation of the jointing of a rock mass can only be made when the uncertainty in the estimates is known and the importance of the uncertainty in controlling any engineering decisions that are made is known and understood.

Figure 1 illustrates bias in the estimation of a mean value and error in the estimation of the variance of the parameter. Both sources of error may be significant but while methods for the determination of confidence limits for the estimated mean of a parameter are known and software packages may provide tools for such analysis the determination of the error in the estimation of variance of a population is not as easily handled (Kulatilake et al., 1990; Webster and Oliver, 2007). Generally, we cannot detect bias or correct for it in our sampling without a 'ground-truthing' exercise.

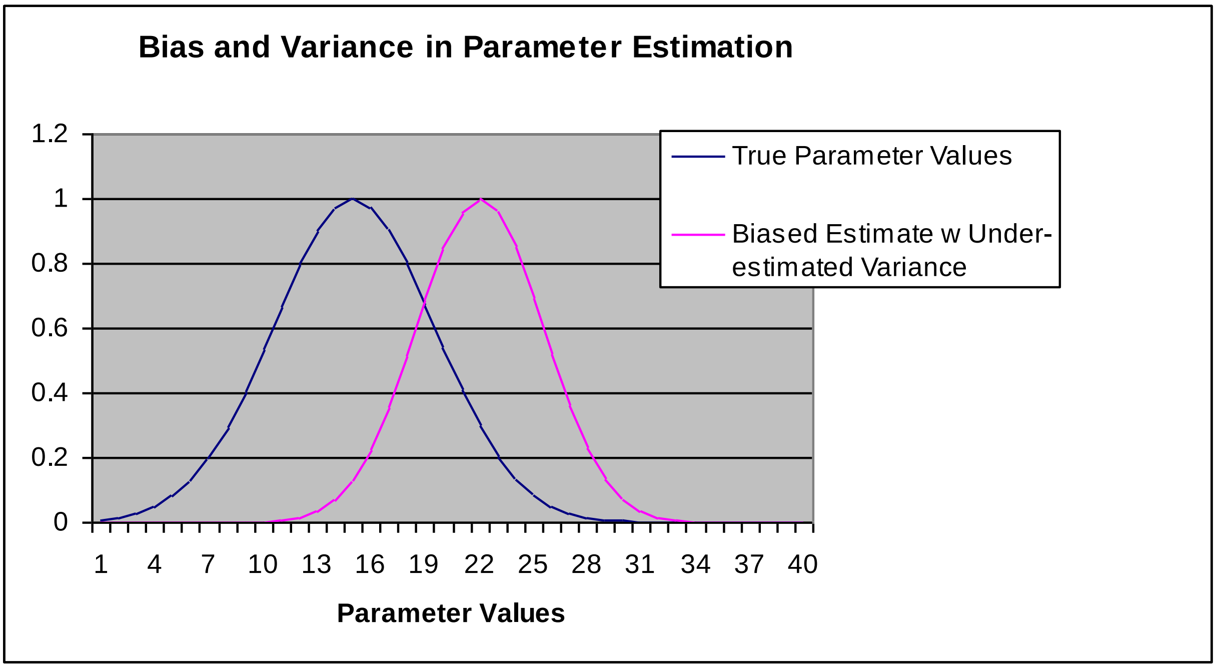


Figure 1 Illustration of bias in the estimation of a mean value and error in the estimation of the variance of a parameter

3 Uncertainty versus variability

The term ‘uncertainty’ is commonly used to refer to our lack of knowledge, or lack of precision in many circumstances. In application to the characterisation of a rock mass it is usually used to describe variability in relation to the parameters used to describe the rock mass. However, ‘uncertainty’ addresses two components that limit our knowledge, stochastic variability and the absence of knowledge. The ability to predict the behaviour of a rock mass is limited by both aspects of uncertainty. Stochastic variability is usually addressed using probabilistic models; the absence of knowledge may be difficult to quantify or model. In many applications these are combined as ‘total uncertainty’. However, they represent fundamentally different aspects of the limitations of our knowledge of the behaviour of the rock mass.

In the assessment of risk, the separation of these components of total uncertainty has been recognised as a requirement in the development of mathematically correct models of risk (Vose, 2000). This paper will focus on variability as a component of total uncertainty. In keeping with common terminology we will use the term ‘uncertainty’ in reference to variability, however, it should be kept in mind that total uncertainty includes ‘lack of knowledge’ of the parameters driving a model as well as variability of those parameters.

3.1 Issues

Some proprietary software packages require discontinuity orientation data to be defined as two distributions, one for dip-direction and one for dip-angle (Meyers and Priest, 2000). This restriction imposes an assumption on the data that is not strictly valid. Unless it can be proven that the dip-angle and dip-direction are controlled by processes that are not coupled, an unwarranted assumption in most cases, the use of two distributions to describe the orientation of a discontinuity when modelled as a single plane is not valid.

4 Uncertainty in the characterisation of rock mass jointing

Understanding the uncertainty in the characterisation of jointing in a rock mass is essentially a problem in geostatistics with some complications that arise from the nature of jointing in a rock mass. As discussed previously uncertainty arises from the stochastic nature of the structure in a rock mass as well as the sampling process that is almost always applied to the measurement process. In addition measurement errors may, and probably will, add to the uncertainty associated with estimates of parameters. This last source of uncertainty is often not dealt with in any detail in rock engineering.

Good practice requires that a design includes evaluation of the effect of the uncertainty of the parameters used in the design on the design objectives. To do so requires estimation of the confidence limits of the parameters and evaluation of the design at the confidence limits of the parameters.

4.1 What does uncertainty in the characterisation of rock mass jointing mean?

The parameters used in rock mass engineering encapsulate knowledge of the rock mass in a form that can be used in engineering design. Essentially the uncertainty in our knowledge of these parameters means that no estimated parameter can be assumed to encapsulate knowledge of the rock mass characteristic it is used to describe with complete accuracy. Therefore, any design based on these parameters is subject to some degree of unpredictability.

4.2 Can I trust my data?

Uncertainty is a measure of the degree of accuracy with which the parameters of a data set have been estimated. If it were possible to completely characterise every parameter of every discontinuity in a finite set of discontinuities those parameters could be used to predict the behaviour of every member of the set under the conditions that the parameters described. Since this is not technically or practically feasible the conditions under which engineering designs can be assumed to incorporate reliable descriptions of the behaviour of a rock mass must be quantified. That means that the limits of applicability of a design must be known and thus the uncertainty in the parameters quantified and the impact of uncertainty controlled. When this is achieved it can be assumed that the data can be trusted.

4.3 How do we measure uncertainty?

The most common means of measuring the uncertainty of an estimated parameter is the estimation of confidence limits associated with the parameter. These are usually expressed in terms of the probability that a parameter lies within a specified range. Measures of uncertainty are based on determining the confidence limits that apply to the estimation of parameters. These limits are sometimes based on assumptions regarding the underlying probability distributions of the parameters that are assumed to describe the stochastic nature of the parameters.

4.4 How is uncertainty controlled?

> *"As has been observed joint set orientations are not described by a one dimensional normal distribution and thus the estimation of joint set parameters requires more appropriate methods. In addition, with small sample sets the variance of a sample is a poor estimate of the variance of the population from which the sample is drawn."* (Webster and Oliver, 2007).

The only feasible method of controlling uncertainty at the input to a design is to establish reliable sampling processes than can be used with mathematical models to determine the confidence limits applicable to the parameters estimated from the measurements.

4.5 How are the effects of uncertainty quantified?

Uncertainty can be quantified during the design phase of a rock engineering project if the variability of input parameters is captured and formulated in mathematical terms. By so doing the uncertainty can be propagated through analysis or simulation based on numerical computations and the effects of uncertainty analysed.

5 Statistical models of uncertainty

5.1 What are the applicable statistical models?

The statistical models that are most readily applicable to rock engineering designs are those that enable us to make the best possible estimate of the characteristics of the rock mass required for a particular design or analysis task and estimate the limits at which we can have confidence in the parameters used in the design at some level of probability. The definition of 'best' will vary with the task and the assumptions used in choosing and applying the appropriate models will vary considerably depending on the problem being considered.

5.1.1 Circular distributions

In the characterisation of the jointing in a rock mass the dip angles and dip directions of joints can be treated as being independent variables under the assumption that there is no correlation between the dip and dip direction. If this assumption were correct it would support the analysis of the orientations to be undertaken using different probability distributions for the between the dip and dip direction. The appropriate distribution would then be one of the set of circular probability distributions, however, the assumption that the dip angle and dip direction are uncorrelated is difficult to support.

The normal distribution is sometimes used in analysis of orientation data but the use of distributions that are not truncated such as the normal distribution can lead to significant error in the estimation of confidence limits if the scatter of the measurements is significant. Therefore the normal distribution can only be used as an approximation when the deviations from the mean are such that the probability of the deviation estimated using the normal distribution is effectively zero (Fisher et al., 1993). The normal distribution can be modified to handle angular data in the range 0 to 2π radians by modifying the distribution so that it wraps cyclically (Fisher et al., 1993). However, this is a distribution that can be used to model circular data rather than spherical data. The wrapped normal and von Mises distribution can be used as probability models for directions in the plane (Fisher et al., 1993).

5.1.2 Spherical distributions

The use of a spherical distribution such as the Fisher distribution is common when analysis of joint orientations is undertaken (Kemeny et al., 2002; Priest 1993). However, the Fisher distribution is a symmetric distribution and can only be used as an approximation for anisotropic data. It has been argued that currently available probability distributions are not adequate to represent all joint orientation distributions (Kulatilake et al., 1990).

The rock mechanics literature universally applies a spherical Fisher distribution to the orientation data for joints. Since joint data is defined only on the hemisphere (it is axial not true directional data) this is a mistake. Fisher's original work (Fisher, 1953) dealt with directionally unbiased sampling of palaeomagnetic data which was true directional data. This work has been incorrectly adapted to joint data.

Analysis of orientations of a sample from a unimodal distribution which is rotationally symmetric about its mean is practically feasible. However, analysis of multimodal or anisotropic distributions is not a practical reality (Fisher et al., 1993).

Other distributions may be used to analyse orientation data (Fisher et al., 1993). However, the tools required to use these distributions (distributions on a sphere) are not commonly available in a form suitable for day to day use in rock engineering.

The estimate of the mean orientation of the set obtained as the resultant vector of a set of observations is the best estimate of the (unknown) mean orientation of the joint set (Fisher et al., 1993), but the estimate of the mean orientation of a joint set does not characterise the joint set sufficiently.

5.1.3 Hemispherical distributions

The orientation data that is collected from the rock mass is a class of data known as axial data. Axial data provide a direction but no sense of direction. The assumption that the vector normal to a joint plane always points downwards is simply an assumption. It is equally possible to assume that they always point upwards. Thus the normal vector provides a direction but no evidence of pointing up or down and the distribution of axial data is defined on a hemisphere and not on a full sphere.

There are a number of other distributions that can be devised to describe axial data (Fisher et al., 1993), however, only the hemispherical Fisher distribution will be considered here. The hemispherical Fisher distribution is symmetric about the mean direction of the joints. Although this means that it is not really suited to the modelling of anisotropically distributed joint directions, it often happens that the data set is not large enough to conclusively prove that the orientation distribution is not symmetric.

5.1.4 Probability distributions for other parameters

To fully characterise the jointing in a rock mass, it is also necessary to estimate the persistence and spacing of the rock joints as well as the surface characteristics such as roughness. Again, a statistical model is assumed as a starting point for the analysis. The common assumption treats the joints as discs having randomly located centres, a size distribution and a Fisher orientation distribution. Estimation of persistence is equivalent to the estimation of the distribution of the diameters of the discs. The size and orientation distributions are assumed independent.

In the discussion presented, attention will be focussed on the orientation distribution as this case is sufficient to illustrate the critical issues to be put forward. Estimates of joint spacing and size are required and these are dealt with in detail in the literature, for example (Priest, 1993).

5.2 What do the statistical models deliver?

All estimates are subject to error. The error in an estimate of a population statistic can be eliminated only if we can sample every member of a population. The statistical models enable us to estimate bounds on the errors when the probability distribution describing the measured parameters is known.

Mathematical models of probability distributions enable us to define the probability that a realisation of a stochastic process will lie within certain limits. Typically, this is expressed in terms such as ‘the value of a normally distributed random variable will lie within one sigma of the mean with a probability of 67%’. The mathematical model of the normal distribution of a random variable cannot support an assessment of when, that is in which particular trial, the value of a random variable will fall within those limits or outside those limits.

For example, if a random variable, x, is known to be normally distributed with mean μ and standard deviation σ we estimate the probability that a realisation of the random variable lies with a given range, a to b, using:

$$p = \int_a^b \frac{1}{\sqrt{2 \cdot \pi} \cdot \sigma} \cdot e^{\frac{-(x-\mu)^2}{2 \cdot \sigma^2}} \cdot dx \tag{1}$$

5.3 What do estimation methods deliver?

The estimation of the mean direction of the joint set can be obtained from the sample mean by applying a number of methods. The simplest, although erroneous, estimation process is simply to average the dip and dip direction. For reasons discussed previously for variance, this is incorrect.

There are many estimation methods that are used in data analysis including Bayesian estimation (which requires a prior knowledge of the underlying probability distribution) and Maximum Likelihood (ML) estimation which is a special case of Bayesian estimation under the assumption that the a priori probability density is constant, best unbiased estimators and least squares estimators (ASPRS, 2004). Under certain assumptions these estimators can be equivalent. For example best linear unbiased estimates can be interpreted as ML estimates with normally distributed observations.

If no a priori information is available the maximum likelihood estimator can be used and in this case is equivalent to estimation using Bayesian methods (ASPRS, 2004).

Traditionally several assumptions are made in the application of maximum likelihood estimation. If it is assumed that the error is normally distributed the maximum likelihood estimation corresponds to the solution of a least-squares problem (ASPRS 2004). Maximum likelihood estimation is used extensively in applications such as photogrammetry and surveying where the estimation of parameters must be supported by estimates of the associated errors.

If the measurements from a population are normally distributed, the maximum likelihood estimator is normally distributed and can be tested to verify the confidence limits on the estimate of the mean.

The calculation of the mean is a good example of an estimation method. Under the assumption that a random variable is normally distributed and that each measurement is mutually independent the mean of a sample is the maximum likelihood estimate of the mean of the population from which the sample was drawn (Mikhail, 1983).

6 Analysis of orientation data

In characterising the orientation of joints in a rock mass, we require an estimate of the mean direction of the joints within a particular joint set and an estimate of the spread of these joint directions about the mean direction. We may, and should, also require an estimate of the probability that the mean orientation of the joint set lies with a specified range where that estimate is obtained from the mean orientation of our sample of measurements.

An alternative to the assumption of a known underlying probability distribution and the estimation of confidence limits based on the assumed distribution is the application of maximum likelihood estimation.

The value of the maximum likelihood (ML) method is that it can provide confidence limits for the estimates of parameters being solved. The alternative approach of estimating bounds on K, the concentration parameter, could be used and tests for a specified concentration parameter K have been developed (Fisher et al., 1993). However, ML estimation methods provide a more direct method of estimation and a discussion of the tests for a specified concentration parameter is beyond the scope of this paper.

The rock mechanics literature universally applies a spherical Fisher distribution to the orientation data for joints (Priest, 1993; Potsch et al., 2007). Since joint data is defined only on the hemisphere (it is axial not true directional data) this approach is incorrect. We will review the use of the Fisher distribution briefly to provide background for the discussion that follows.

The probability density for the hemispherical Fisher density is:

$$\phi_{hF}\left(\mathbf{n}_j,\boldsymbol{\mu},K\right)d\delta d\theta=\frac{K}{2\pi\sinh K}\cosh\left[K\left|\hat{\mathbf{n}}_j\cdot\boldsymbol{\mu}\right|\right]\sin\delta\, d\delta d\theta \tag{2}$$

where $\hat{\mathbf{n}}_j$ is the unit normal to the j^{th} joint, $\hat{\boldsymbol{\mu}}$ is a unit vector in the mean direction of the joints in the set and K is the Fisher dispersion constant which is analogous to the standard deviation of the distribution. The angles δ and θ describe the orientation of the vector.

$$\mathbf{n}_j=\sin\delta\sin\theta\,\hat{\mathbf{i}}+\sin\delta\cos\theta\,\hat{\mathbf{j}}+\cos\delta\,\hat{\mathbf{k}} \qquad \begin{array}{c}-\pi\le\theta\le\pi\\ 0\le\delta\le\pi\end{array} \tag{3}$$

Using this distribution, and making allowance for the fact that scan line mapping has a bias against joints whose normal are perpendicular to the scan line, it is possible to develop a maximum likelihood (ML) estimation procedure to estimate $\boldsymbol{\mu}$ and K. In the general case, for data collected from a number N_S of scanlines, with M_k joints hitting the k^{th} scanline, the log-likelihood is written as:

$$\Lambda\left(K,\boldsymbol{\mu} \mid \underline{\underline{\mathbf{n}}}\right)=\sum_{k=1}^{N_S}\left\{M_k \ln A\left(\delta_{\mu_k},K\right)+\sum_{j=1}^{M_k}\ln\left[\cosh\left[K\left|\mathbf{n}_{jk}\cdot\boldsymbol{\mu}\right|\right]\right]+\sum_{j=1}^{M_k}\ln\left(\left|\cos\delta_{jk}\right|\sin\delta_{jk}\right)\right\} \tag{4}$$

Where:

$$\left|\cos\delta_{jk}\right|=\left|\mathbf{n}_{jk}\cdot\mathbf{s}_k\right| \tag{5}$$

and $\mathbf{n}_{jk}$ is the j^{th} joint normal in the set of joints intercepted by the k^{th} scanline, $\mathbf{s}_k$. δ_{μ_k} is the angle between the mean joint set direction and $\mathbf{s}_k$, which is parallel to the k^{th} scanline.

A major advantage of the ML method is that the negative of the inverse matrix of the second derivatives of the log likelihood with respect to the parameters of the distribution being estimated, evaluated at the ML estimates of the parameters can be shown to be asymptotically equal to the covariance matrix of the parameter estimates (the asymptotic result is reached in the limit of a large data set). This result and the fact that the parameter estimates are asymptotically normally distributed can be used to find confidence intervals or regions for the parameter estimates. ML estimates of parameter also carry the property of minimum variance; they are the estimators with the smallest possible variance out of all the possible estimators that one might develop.

Thus the formulation of the estimation of the orientation distribution parameters as a ML problem provides both the best estimators of the parameters in a minimum variance sense and provide the basis on which to estimate the uncertainties in the parameters as a function of the quality and quantity of the data. ML methods can also be used to provide an estimate of goodness of fit of the model to the observed data.

6.1 Confidence limits on the mean orientation of a set

6.1.1 Confidence limits based on an assumed distribution

Fisher (1953) derived a relationship for estimating the confidence limits on the mean of a sample set of orientations, assuming that the sample is symmetrically distributed. This relationship expressed the probability that the mean of a sample set of orientations, r, makes an angle less than θ with the true mean of the population as:

$$P(r<\theta)=1-\left(\frac{M-|r|}{M-|r|\cdot\cos\theta}\right) \tag{6}$$

where M is the sample size.

When M is large this can be approximated as:

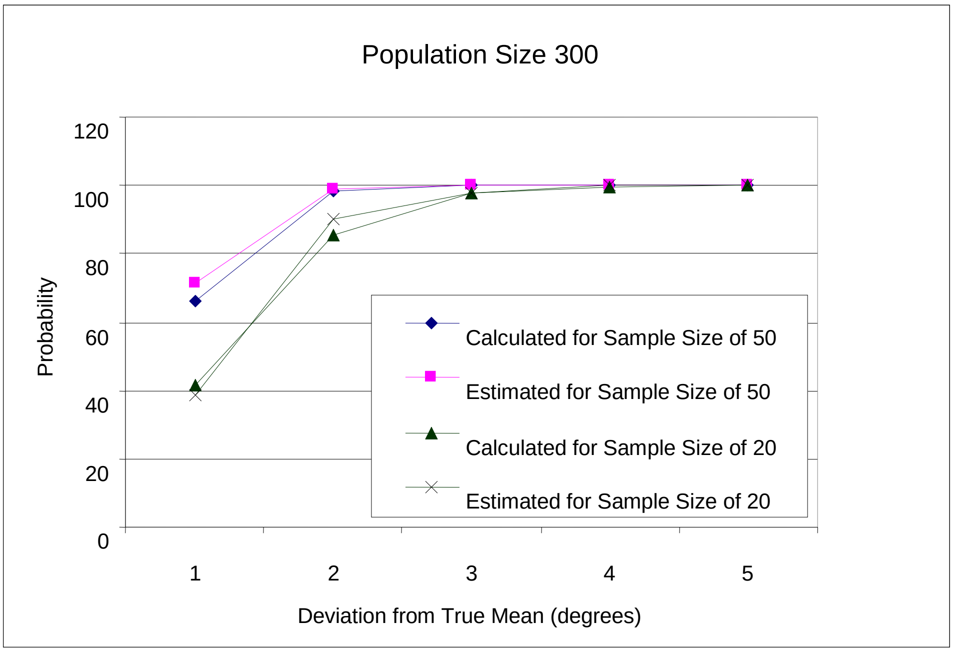
$$P(r<\theta)=1-e^{K\cdot|r|\cdot(\cos\theta-1)} \tag{7}$$

where K is the Fisher's constant for the population and is a measure of the degree of clustering of the population. This relationship can be inverted (Priest, 1993) to yield:

$$\cos\theta=1+\frac{\ln\left(1-P(r<\theta)\right)}{K\cdot|r|} \tag{8}$$

which enables us to calculate the angular confidence limit for a given probability subject to the assumption that the distribution is symmetric.

To demonstrate the effect of sample size we ran some simulations on an isotropically distributed set of orientation vectors and compared the results of the simulations with the probabilities predicted using these formulae. The results are shown in Figure 2. The effect of the size of the population from which a sample is drawn, and the sample sizes are immediately obvious from the data. The variability seen in the data are the result of the use of a finite number of simulations, however, the trend is clear; as either the population size decreases or the sample size decreases the confidence limits on the mean orientation estimated from the as an estimator of the true mean of the population widen.



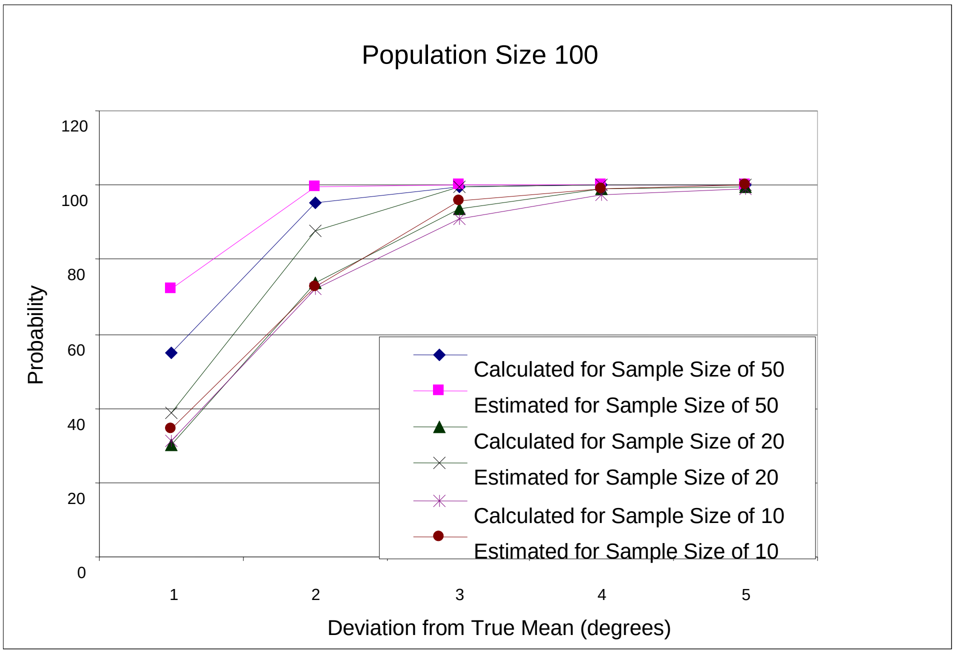


Figure 2 Variation of the probability that the estimated mean is within a defined angular range of the true mean with population size and sample size

6.1.2 Confidence limits based on maximum likelihood estimation

The following discussion illustrates the application of ML estimation to the problem of determining the confidence limits that apply to an estimate of a parameter, in this case, orientation.

Figure 3 shows an upper hemisphere stereoplot with joint normals for joints distributed around a mean dip and dip direction of (45, 45). The red points (triangles) were logged on a N–S horizontal scanline and the blue points (diamonds) were logged on an E–W horizontal scanline. The distortion of the clouds of points due to the scanline biases are evident in the diagram. There are 100 triangle shaped points and 50 diamond shaped points.

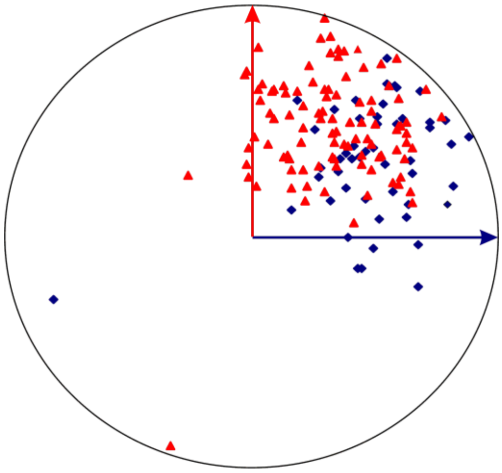


Figure 3 Upper hemisphere stereoplot of joint normals and scan line directions

Figure 4 shows the 90% confidence regions derived from the original data set (150 total points) and from a smaller data set (30 points total, same proportions). In the figure the smaller region is for the larger data set. In both cases the true values for the joint set fall inside the confidence regions and the larger confidence region for the smaller data set reflects the smaller volume of data that was available.

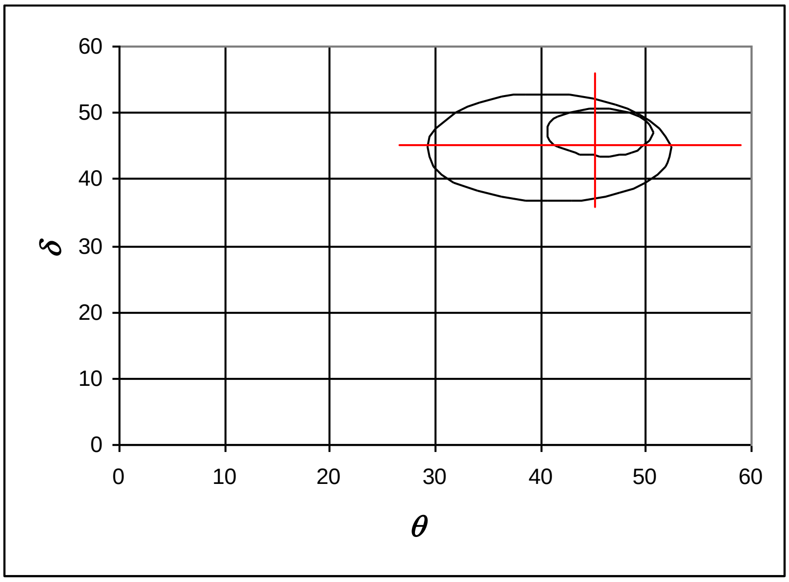


Figure 4 90% confidence region for the azimuthal (δ) and the longitudinal (θ) angles for small sample and larger sample data sets

6.2 Application to estimation of persistence and spacing

The subsequent problem in the characterisation of rock joints is estimation of other parameters characterising the jointing in a rock mass. The scope of the task includes the estimation of the number of joints per unit volume in each joint set and the persistence or size distribution of the joints in each set. With the development of block modelling techniques that utilise parameters such as the variance of the orientation of joint sets these estimates have become more important. In the setting of scanline mapping and when joints can be modelled as discs, this problem too can be solved by ML methods, with all the attendant advantages mentioned above.

The estimation of orientation distribution is largely independent of the joint size distribution problem and is usually the better-defined issue. If the orientation problem is estimated first, there are uncertainties in the orientation distribution that propagate through the joint size problem which should be formally taken into account. While the authors of this paper recognise the issue, the impact of this propagation on overall uncertainty has not been addressed.

6.3 The effect of domains on uncertainty

The effects of sample size on the confidence limits of an estimated parameter are generally recognised, however, it may not be as obvious that as the population size decreases the effect is to widen the confidence limits associated with any parameter of the population estimated from a sample of a given size. These simulations show this clearly. The consequence of this dependency is that the estimation of geotechnical parameters from domains of limited size must be done with caution if the sample size is significantly less than the population.

It can be seen from the results shown in Figure 2, Figure 3 and Figure 4 that the sample sizes recommended for estimating the mean orientation of a joint set (Priest 1993) provide control to within 2 degrees of the confidence limits for the estimated mean for joint sets with 100 members or more.

Better domaining should produce local estimates of lower uncertainty as the assumption of stationarity within the domain is more robust. However, the smaller domain has fewer data associated with the estimates, so there will be a set of optimal domain sizes for any particular prospect. Goodness of fit has a major role to play in this problem as it can be used to determine when the domain size applied is becoming too large and incompatible data from an adjacent domain are being mixed into a well-defined domain.

With the development of laser systems and photogrammetric systems that generate large, spatially accurate data sets the positions of joints as they are mapped are recorded. The addition of this data supports the analysis of domains in that data can be quickly sorted on domain boundaries and analysed accordingly.

6.4 Requirements imposed by the development of new measurement techniques

The measurements delivered by remote sensing systems are fundamentally different to those obtained using scanline measurement methodologies or similar measurement approaches. The assumptions regarding the nature of the probability distributions that describe the spread of the characteristics of the joints being considered are not changed by the acquisition of data by remote sensing techniques.

However, the sampling methods and thus any biases imposed by the sampling methods are changed by the data acquisition methods. To apply ML estimation methods to jointing data acquired by remote sensing, it is necessary to modify the likelihood function to reflect the different probabilities of observing a joint of a given orientation compared to scanline mapping. Correct formulation of the problem will recognise and compensate for any sampling biases. However, as before, ML methods will provide accurate estimates of the uncertainties in the estimated parameters. In the case in which the probability of observations cannot be fully formulated, one has the option of modifying the manner in which measurements are extracted from the remotely sensed data so that the sampling biases and observation probabilities coincide with a scanline or window mapping regime for which ML solutions are available.

In practice, the data acquired using remote sensing techniques such as laser scanning or photogrammetry is measured in what is effectively a rectangular sampling window, or at least approximates a rectangular sampling window. Kulatilake et al. (1990) showed that just as sampling biases with scanline mapping are significant, so too are biases with rectangular mapping windows. Lyman (2003) has demonstrated the application of ML methods to window mapping in a simple case. Therefore, effective use of the data obtained by these methods requires the application of corrections such as those evaluated in the reported work.

7 Implementation

7.1 How do we apply models of uncertainty?

The parameters describing the jointing in a rock mass are aggregated into rock mass indices which can be used to propagate the uncertainty through the application of these indices. To test the limits of performance of a rock engineering design we must go to the boundaries of the confidence intervals and work on the best and worst case solutions that might be defined by the estimates of the parameters at the confidence limits. Otherwise, we sample the rock mass parameter estimates from the distributions of the estimates and develop a distribution of outcomes with costs and engineering implications.

7.2 Application to simulation of rock mass structure

Approaches to simulation of rock mass structure may involve the use of models of jointing distribution in a rock mass as a discrete fracture network (Dershowitz et al., 1988) or more complex polyhedral models that utilise topological descriptions of blocks formed in a rock mass using models of the distribution of joints in the rock mass (Lin et al., 1987; Elmouttie et al., 2008). The use of these techniques requires simulation of joints distributed through the volume under consideration. The effectiveness of such simulations depends on accurate characterisation of the parameters used to drive the simulation.

The effect of the uncertainties associated with the parameter estimated can be evaluated by examining the extremes in the parameter estimates and then running the simulations at the extremes and evaluating the consequences. Again, it is possible to sample from the distributions of the rock mass parameters and provide a distribution of outcomes.

7.3 Implementation of a scanline mapping analogue

To bridge the gap between current practice based on scanline or window mapping and the use of remote sensed measurements a scanline mapping analogue has been implemented. An example of the user interface is shown in Figure 5. The implementation allows the user to define the scanline parameters and either vertical or horizontal scanlines may be used. The bounding box may be defined similarly and termination characteristics etc are then simply determined from the 3D image. The use of data acquired using 3D imaging systems thus allows the user to apply scanline mapping to otherwise inaccessible areas of the rock mass and use currently available techniques to analyse the jointing data consistently.

Figure 5 Scanline mapping analogue implemented with remotely sensed 3D data

8 Conclusion

Determining the uncertainty in our knowledge of the characteristics of a rock mass is a critical issue because the outcome of reducing or controlling uncertainty in the determination of jointing in a rock mass is improved understanding of the factors related to data quality that affect rock engineering designs and, consequently, safer, more cost-effective rock engineering designs.

Remote sensing based measurement techniques are now able to provide far more data than conventional techniques of measurement of joint orientations and provide estimates of the positions of joints and other structure that can extend our parameterisation of the structure of a rock mass. The use of these measurement techniques introduces a new sampling bias in the data and thus requires the development of new methodologies to accommodate that bias or the adaption of the measurement techniques to support analysis of the data acquired by remote sensing using the analysis methods applied to mapping techniques such as scanline or window mapping.

With the development of block modelling techniques that utilise parameters such as the variance of the orientation of joint sets, estimates of the variance of the orientation of the set as well as the variances of the joint spacing and size have become more important. Estimation of these parameters has not been addressed in this paper and practical methods of making these estimates are required.

Analysis methods based on assumptions regarding the distribution of joint set parameters have provided tools that improve the characterisation of a rock mass but these methods can be improved upon. This paper has shown how maximum likelihood estimation techniques provide a means of addressing aspects of uncertainty with more rigour. The application of these methods to data acquired by remote sensing based measurement techniques requires further development.

This paper has not addressed issues such as detection of outliers in a sample measurement set. These are usually eliminated as a matter of engineering judgement. A question remains as to how much judgement is required to deal with issues such as this, that are not readily amenable to practically applicable analysis techniques.

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