

On the Need for Polyhedral Representation of Blocky Rock Masses

M. Elmouttie *CSIRO Exploration and Mining, Australia*

G.V. Poropat *CSIRO Exploration and Mining, Australia*

A. Guest *De Beers Group Services, South Africa*

Abstract

Modelling rock mass structure accurately requires the determination of what degree of accuracy is needed for the rock mass model. Structural modelling requires approximations with respect to the geometry and other characteristics of the discontinuities or fractures present. Not only are there uncertainties with respect to the fracture characteristics as measured in the field, but the method of representation of these structures in a model is also subject to judgement.

Model simplification must be guided by the principle of retaining the critical characteristics of the rock mass being investigated. Of all the structural modelling techniques currently available, the use of polyhedral modellers has recently received considerable attention. This paper will discuss the benefits of modelling rock using a polyhedral representation based on finite persistence discontinuities. In particular, the predictions of such a model with respect to stability analysis, rock mass characterisation and excavation analysis will be compared to the more traditional techniques that have been used.

1 Introduction

The determination of the stability of a rock mass is a complex process that is dependent on understanding the structure and composition of the rock mass. The presence of discontinuities inevitably leads to the existence of discrete blocks and points or regions of potential weakness. A rock mass is thus a three dimensional distribution of spatial entities and any effective model of the behaviour requires a three-dimensional (3D) model that is able, at a level appropriate to the analysis to be performed, to represent the structure of the rock mass.

The simplest three-dimensional structure that can be represented with a surface consisting entirely of intersecting planes is a tetrahedron. While a tetrahedral block is convex, any general polyhedral block may contain re-entrant sections and such concavities result in a collection of polyhedral blocks having interlocking structure that may influence the stability of the ensemble of blocks. Further, decomposition of a general polyhedron into tetrahedral components is not only computationally expensive but it is also not guaranteed (Ruppert and Seidel, 1992).

Any block that forms part of an existing rock face or a planned rock face has the potential to fall and, in so doing, expose other, previously unexposed blocks. This process can continue and allow the progressive failure of a wall. Therefore, the analysis of even the simplest aspects of the kinematic behaviour of a rock mass requires some form of model of the block structure of the rock.

Ideally, the modelling of a complex matrix such as a rock mass structure permeated by many discontinuities would be performed numerically at the particle level. All aspects of rock mass and discontinuity behaviour could then be incorporated and work is being undertaken to utilise just such a synthetic rock mass for the analysis of large volumes of rock mass (Pierce et al., 2007; Cundall et al., 1996).

This paper concentrates on the use of polyhedral rock mass representations for the study of blocky rock structures, which can be generated quickly and can capture enough characteristics of the rock mass for use in certain analyses. Although polyhedral models can be used in the study of rock block deformation, this paper concentrates on analysis techniques that assume rigid body mechanics (e.g. limit equilibrium or key-block analyses).

2 The requirement for polyhedral block models

Since the analysis in the kinematic stability of a rock mass or more general behaviour of a rock mass requires some form of model of the structure of the rock mass shape and, since the structure is determined by distribution of the polyhedral blocks that form the rock mass, a structural model must encompass the possible shape of all polyhedral blocks that can be formed.

In addition, the polyhedral blocks that form a rock mass may contain discontinuities that are completely enclosed within the block or are partially through going. The structural model must therefore have the capability to describe the existence of polyhedral blocks as well as the existence of joints or other fractures that do not form blocks.

Modelling of the rock mass structure is, at best, an approximation of a complex assemblage of interlinked discontinuities separated by heterogeneous material with non-linear behaviour under load. Approximations are achieved by some form of simplification of the model and such simplification must be guided by the principle of retaining the critical characteristics of the rock mass being investigated.

Block theory, developed by Goodman et al. (1968) was developed as a set of geometrical and analytical procedures to assess the stability of a block structured rock mass. Some early attempts to model rock mass structure in three dimensions used pseudo-analytical models of the distribution of discontinuities in order to create manageable structural models of rock masses. These models were, at best, limited in their scope of application. Goodman and Shi (1985) developed a method of investigating the kinematic feasibility of the polyhedral blocks that can be formed by the orientation of four or more planes. The method requires the assumption that at least one of the planes is a free face. Extensions of the method have been developed (Warburton, 1981; Lin et al., 1987). These methods are essentially based on a form of combinatorial analysis. This paper discusses the application of polyhedral block modelling to rock masses based on creating simulations of the structure of an extended rock mass from the measured positions, orientations and extent of discontinuities combined with statistical distributions of the sets of distributions known to exist within the rock mass.

The use of Poisson discs as the representation of joint structures was first outlined by Baecher and Einstein (1977). An approach for generating block structures using Poisson planes was described by Dershowitz and Einstein (Dershowitz and Einstein, 1988). Lin et al. (1987) described an algorithm that considered convex or concave blocks as well as finite or infinite blocks. It was asserted that the algorithm described by them was the first to consider such polyhedral blocks. Jing (2000) described a boundary representation of block structures which, it was claimed, overcame the limitations of previous models because the blocks formed were of arbitrary shape from joints of finite size without the need for artificial fractures of defined shapes and sizes. Lu (2002) further developed these ideas.

In discussing the creation and use of polyhedral models care must be exercised since a polyhedral model must be a universal model capable of handling any general polyhedral solid. A suitable data structure must be defined in order to handle the complex shapes that may be created by intersection joints where the intersections may truncate one or both of the joints.

3 Polyhedral models

A polyhedral rock mass model is one in which the rock mass volume is partitioned into individual 3D sections and in which the surface of each section is a closed polyhedron. As such, each polyhedron is described by a set of faces or polygons. Each polygon consists of a set of edges and each edge is described by a pair of vertices. A polyhedral model does not intrinsically handle truncated joints. The ability to handle truncated joints is provided by the data structures and algorithms that pre-determine the intersection precedence between different sets of finite persistence discontinuities. Therefore, polyhedral models can be built with infinite persistence representations of discontinuities (yielding convex shaped blocks) or finite persistence discontinuities (yielding concave faces and therefore concave polyhedra or blocks).

Once the polyhedral model has been constructed, the blocky structure of the simulated rock mass can be interrogated in a number of ways. For stability analyses, the limit equilibrium analysis can be used to evaluate the presence of potential failure prone blocks. Providing the geometry of the rock wall being studied has been captured accurately, the rock mass model can then be evaluated and a number of geometries

assessed, all utilising the same realisation of the rock block model. Finally, many realisations of discontinuities can be generated resulting in multiple discrete fracture networks (DFN), each yielding a different realisation of the rock block model.

4 Comparative study

In order to highlight the value of polyhedral modelling and the importance of realistic structure representation within such models, some simulations have been performed. Three sets of block models have been created. All sets represent three benches of an open pit mine with one set incorporating finite persistence structures (joints) and the other two using infinite persistence. The choice of a multiple-bench model geometry was deliberate so as to ensure the study of block structures extending beyond the traditional domain of study (in this context, a single bench analysis) could be analysed. The discontinuity parameters were chosen so as to generate a significant number of daylighting failures (i.e. movable blocks at the free surface of the model). The DFN were generated using a Baecher joint model (Baecher and Einstein, 1977) which assumes log-normally distributed radii, Poisson distributed joint centres and normally distributed orientations. The joint set parameters are shown in Table 1.

Table 1 Discontinuity information. For IP models, the joint radii generated using these parameters were rescaled to ensure they became fully through-going

Dip (°)	Dip Direction (°)	Expectation Trace Length (m)	Spacing (m)	Friction Angle (°)
45±5	45±5	10	random	30
70±5	135±5	10	10	30
85±5	265±5	10	random	30

As previously stated, three sets of 3D block models have been generated using this discontinuity data. The first set utilises 100 realisations of the finite persistent joints (FP), therefore yielding 100 block models. The second set utilises 100 realisations based on the same joint set data except that the persistence of the joints is now infinite (IP). More accurately, the joints have been rescaled so that they are fully through-going with respect to the three-bench geometry. This clearly alters the intensity of discontinuities as determined using, say, a scan-line technique to determine the number of discontinuity traces per unit length or the so-called P_{11} fracture frequency (Dershowitz and Herda, 1992). Therefore, the IP models have been used to generate a third set of models, however now with reduced joint numbers so as to keep the fracture frequency constant. These models will be referred to as IPff. An example model from each of these sets and sourced from the same DFN realisation is shown in Figure 1. The geometry used for the construction of the benches is shown in Table 2.

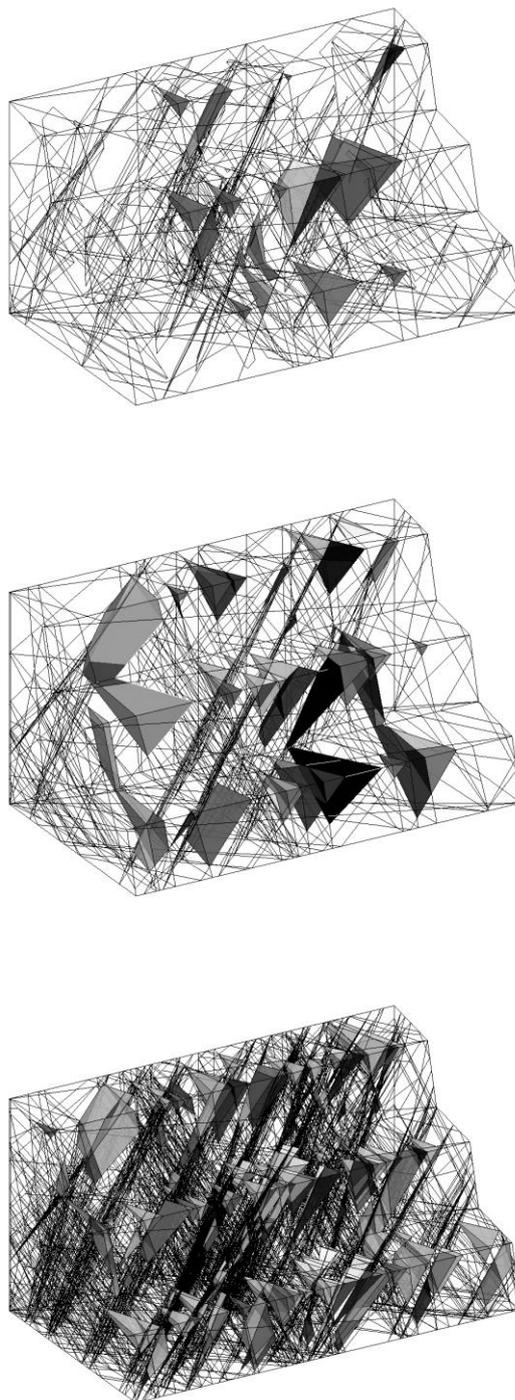


Figure 1 Examples of the three bench models built using the same DFN realisation and incorporating discontinuities of type FP (top), IPff (middle) and IP (bottom). Only removable blocks are shown as solids

A number of analysis techniques have been used to assess the three model types. Further, for some data, the more traditional plane and wedge failure models have also been assessed. Note that the stability of blocks has been classified using the Goodman and Shi schema (Goodman and Shi, 1985) whereby black blocks indicate Type I (unstable), dark gray blocks indicate Type II (stable given the assumed surface properties) and light gray blocks indicate Type III (geometrically stable).

Table 2 The bench geometry

Number of benches	3
Bench length	100 m
Bench strike	270°
Face angle	75°
Face height	15 m
Berm width	15 m
Berm angle	10°

4.1 Rock block morphology

As discussed earlier, only the representation of joints as finite persistence structures can replicate the complex, concave rock block structures seen in nature. The use of infinite persistent structures yields convex rock blocks. Figure 2 shows an example of a rock block formed from the finite persistent joints and the equivalent volume of rock mass formed from the infinite persistence counterparts to those very same joints. Clearly, aside from the morphological difference, there are implications to the estimates block numbers, volumes and behaviours that result from this discrepancy and these are discussed later. Further, the investigation of block-internal fracture density or frequency is impossible if one adopts a model based on infinite persistence joints.

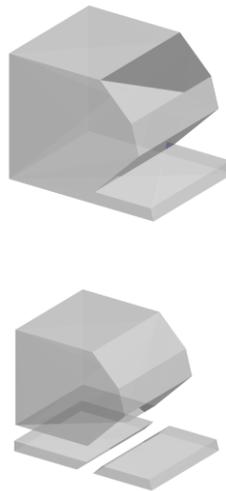


Figure 2 A concave rock block (top) from an FP model decomposed into its corresponding convex components as constructed in the IPff or IP model (bottom)

Figure 3 shows the histograms of block numbers for the various model types. The distribution for the FP model is centred around 500 blocks, for the IPff it is around 800 blocks and for the IP it is 12,000 blocks. The use of infinite persistence discontinuities changes both the peak and distribution shape associated with the number of blocks.

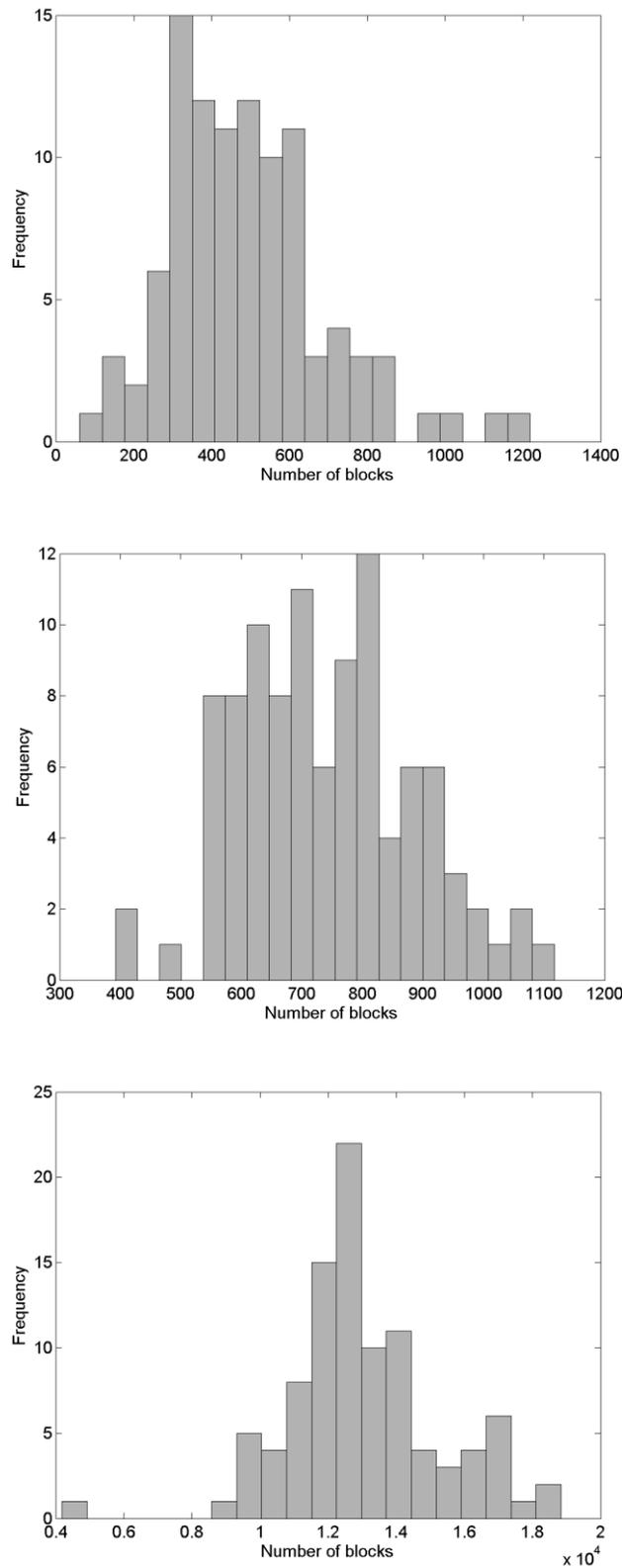


Figure 3 Histograms comparing the number of blocks generated by the FP (top), the IPff (middle) and the IP models (bottom)

4.2 Accommodation of complex free surfaces

Traditional planar and wedge failure analysis has been limited to single bench analysis. This is because the techniques used in such an analysis to determine the presence of unstable rock blocks are limited to the use of idealised infinitely persistent structures and to the consideration of two or perhaps three intersecting discontinuities with a simple free surface. The modelling of the rock mass associated with arbitrary numbers of interacting joints and a complex free surface necessitates the use of a polyhedral modelling technique (FP, IPff or IP) which can incorporate the geometry of the local surface in any stability analysis. This is certainly the case when modelling the open-pit environment (Figure 4). Unless one limits an analysis to single bench mechanisms and failures, concave rock blocks which form at the free surface need to be accommodated. Failure to do this will render an analysis insensitive to multiple bench phenomena. This is discussed again in Section 4.5.

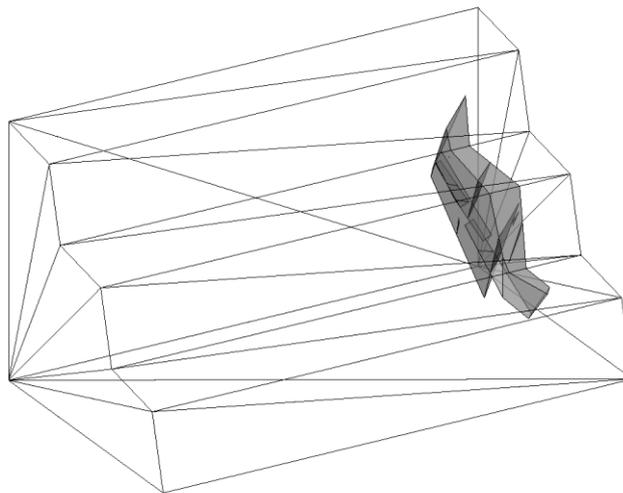


Figure 4 An example of a failure daylighting across multiple benches

4.3 Potential failures of non-daylighting structures

Another limitation of traditional techniques is that they are restricted to studying the structures that intersect with the free surface, whereas predictions of the internal rock mass structure are invaluable for:

- Determining how the model predicts the interaction of stochastically generated structures with deterministic structures (e.g. beds and faults).
- Assessing how ‘progressive failures’ may develop – that is, failures of previously stable and non-daylighting structures.

Only by the construction of the full polyhedral model (FP, IPff or IP) can those interactions be properly investigated. Figure 5 shows the identification of further potential failures (for example, the grey and black blocks towards the right side of the model) when one considers the failure of blocks now deemed removable after the initial failure has taken place. Such analysis can be performed using an iterative procedure involving the removal of unstable blocks at the free surface, recomputing a new free surface and removing the newly exposed unstable blocks. This can continue until no further instabilities are found.

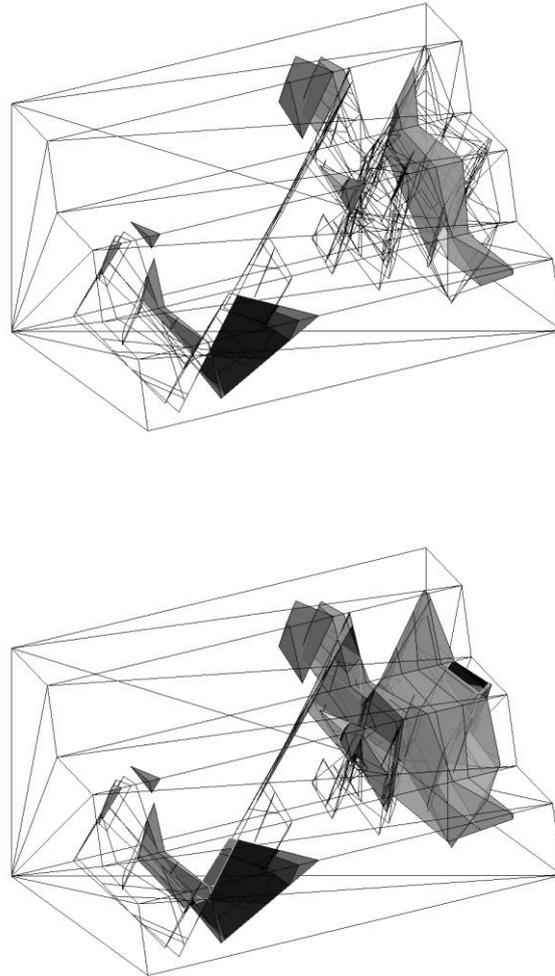


Figure 5 Comparison of identified instabilities assessing only daylighting failures (top) versus assessing all potential failures (bottom)

4.4 In situ block size estimation

Various methods have been outlined for estimating the in situ block size distribution within a rock mass including the use of sieve analysis (Jern, 2004) and traditional equation methods modified to incorporate finite persistence and several joint parameter distribution types (Lu and Latham, 1999; Latham et al., 2006). The value of polyhedral models to alleviate the need to over-simplify the statistics and geometry associated with joint interactivity has been previously noted (Maerz and Germain, 1996).

The prediction of block size distributions is strongly correlated with the persistence of discontinuities. Figure 6 clearly indicates that, as one would expect, large block volumes are severely under-represented (or absent) from the IP models (less so in the IPff models). Further, the scatter in distributions associated with the FP models indicates that the degree of uncertainty associated with such predictions is actually much larger than the both IP model types would suggest. Thus the use of infinite persistence can lead to an under-estimation of parameter variance even when a multiple realisation study is performed.

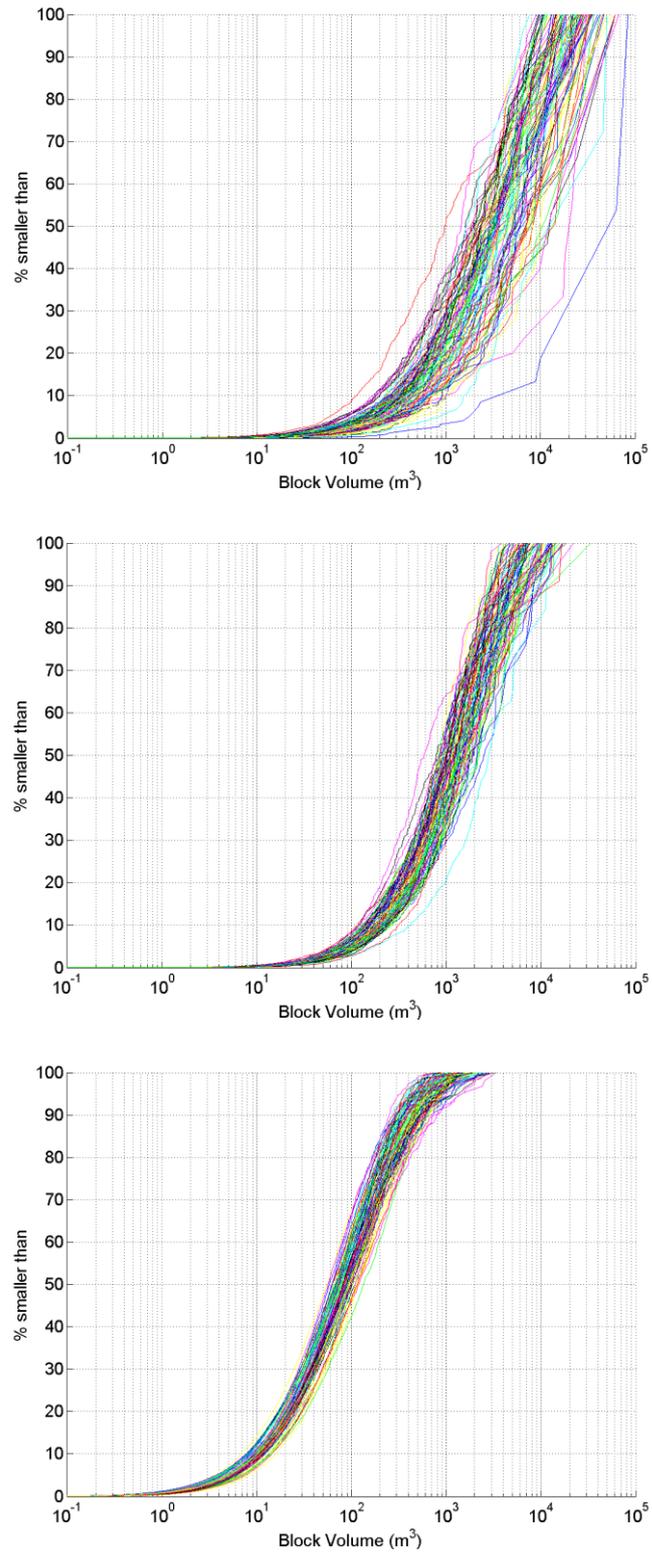


Figure 6 Block size distributions for the 100 realisations of the FP (top), IPff (middle) and IP (bottom) models. Only data for block sizes greater than 0.1 m^3 are shown

4.5 Backbreak analysis

Analysis of the predicted backbreak or crest damage of an open pit bench can serve to refine the design of bench dimensions and catch bench locations (Ryan and Pryor, 2000). A comparison of the results obtained when using the three model types is shown in Figure 7. In all cases, the analyses focus on failures involving bench two of this three-bench geometry. For further comparison, backbreak curves have been generated using the standard single bench analysis technique similar to that outlined by Ryan and Pryor (2000) and these are shown in Figure 8.

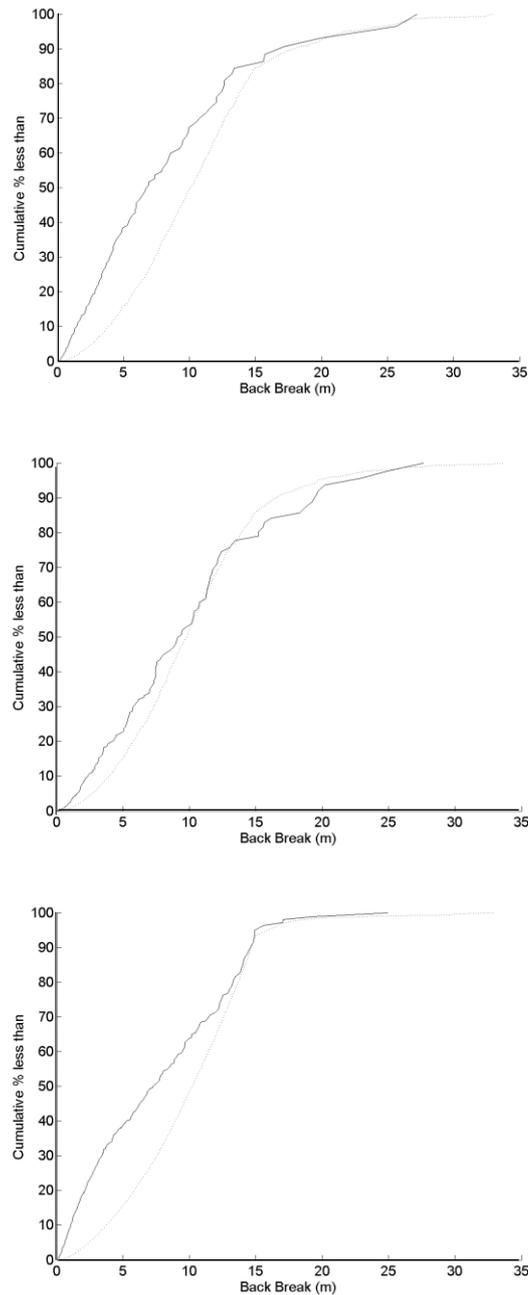


Figure 7 Average backbreak predictions for the 100 FP (top), IPff (middle) and IP (bottom) models. The dotted curves indicate all removable blocks (Type I, II and III), the solid only Type I

The polyhedral models predict much greater backbreaks to occur than the traditional analysis, with the FP models deviating the most. This can be explained as follows:

- The traditional analysis will underestimate the formation of large failures, as it is limited to single bench analysis and will therefore miss failures that daylight across multiple benches (Figure 4).
- The traditional analysis is also limited to the study of simple tetrahedral wedges and trapezoidal, laterally persistent blocks (i.e. along the bench strike). It will therefore underestimate the formation of smaller, more complex shaped failures that rely on multiple discontinuities to interact.

Examination of the curves shows that the IP models predict 5% of backbreaks to be multiple bench (> 15 m) events, whereas the FP model predicts 15%. Interestingly, the IPff models agree with the FP models and also predict around 15% of failures to be multiple bench events. At least for the geometry used in these simulations, the fracture frequency adjustment has captured the presence of the larger, multiple bench blocks.

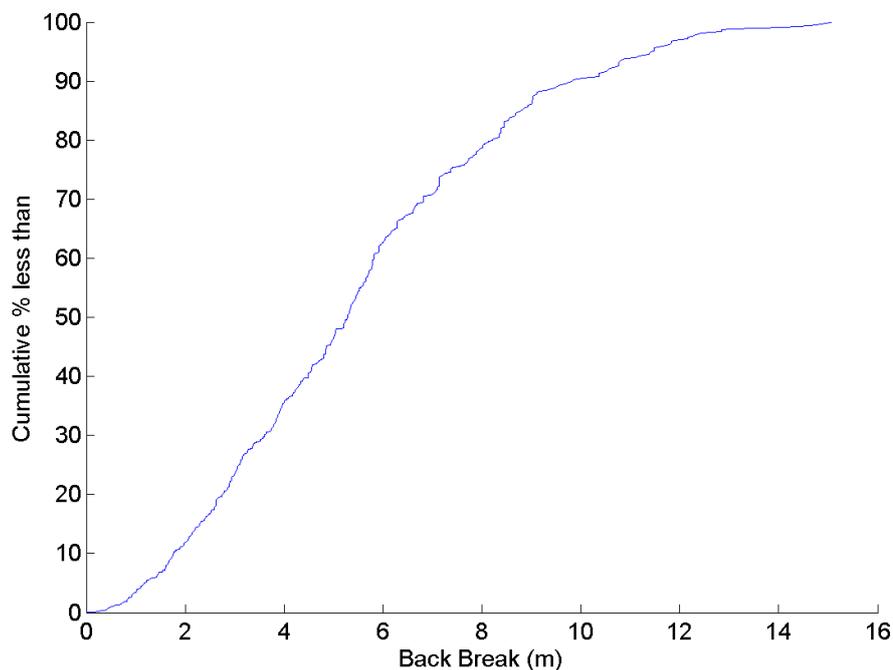


Figure 8 Back break estimates using the standard single bench analysis

4.6 Bench retention

The polyhedral models can be used to assess the retention of unstable blocks. Such analyses rely on assumptions regarding the behaviour of the failed rock mass in a catch bench scenario (Gibson et al., 2006). The data shown in Figure 9 clearly shows the marked differences between the FP and the IP model. The IP model predicts much more failure (of lower volume material), as expected. Therefore, derived quantities such as the spill widths predicted by the FP model will also differ. Note that once again, the IPff model shows some similarity to the FP model although the predicted variation in failure numbers is not seen.

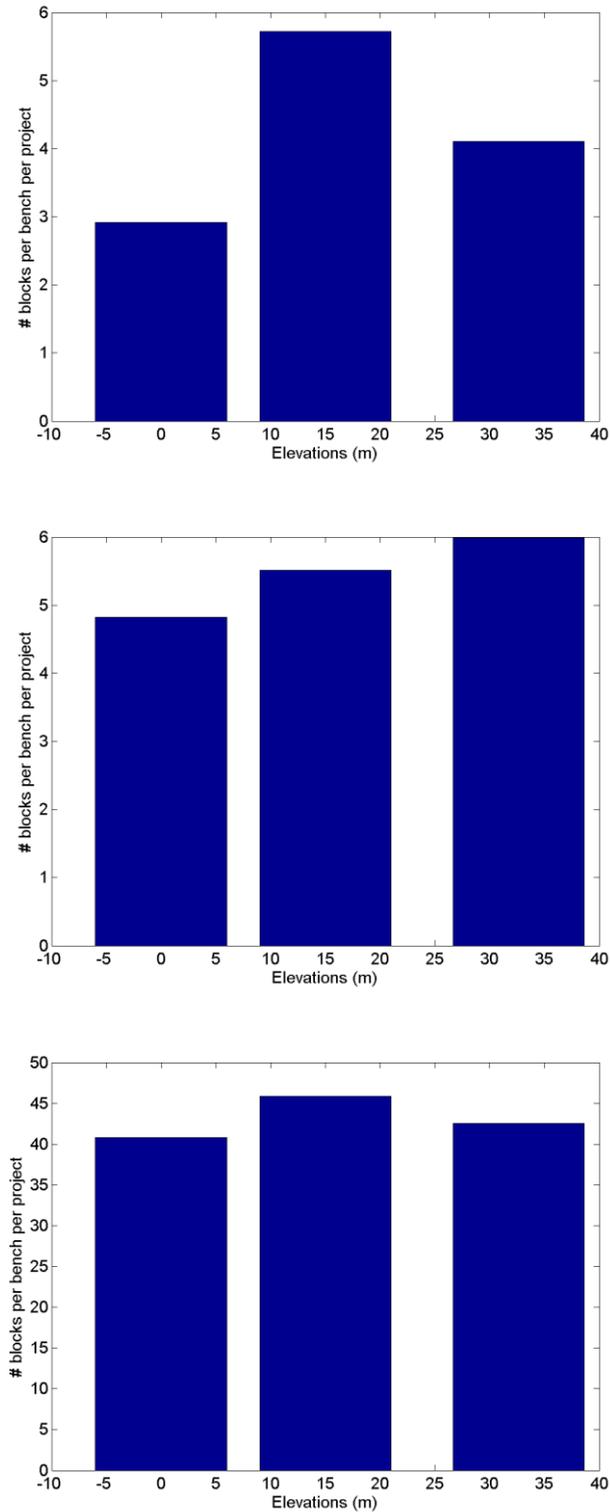


Figure 9 Average bench retention data for 100 realisations using the FP (top), the IPff (middle) and IP models (bottom)

4.7 Stability analysis

Stability analysis can be performed utilising the technique of stability isoplethogram (Windsor and Thompson, 1996). This is a contour diagram representing the stability of a rock mass assessed along multiple excavation planes with differing orientations. This analysis has been performed for each type of model and

utilises data from 10 realisations for each model type. The sampling intervals were 15 degrees in dip and 90 degrees in dip direction, and the excavation planes were anchored at the top crest. The isoplethograms show failures to occur predominantly for excavation planes parallel to these benches. This is due to the use of joint set parameters which maximise failures for the model bench orientations (strikes). The IPff and IP models consist of more densely spaced, smaller volume blocks and hence larger numbers of failures. More significantly, however, the IP models predict much more failure to occur and the range of excavation planes orientations for which this occurs is also greater. Therefore, the IP models yield an overly pessimistic stability scenario.

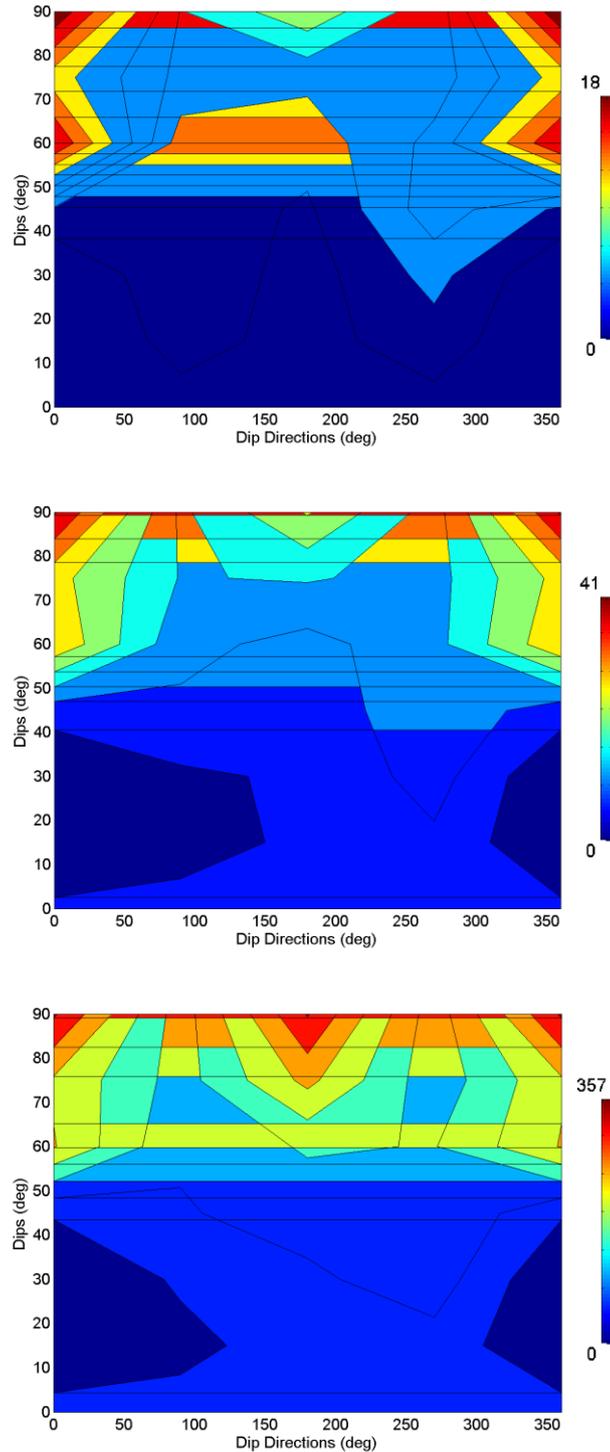


Figure 10 Stability isoplethograms for the FP (top) and IPff (middle) and IP models

5 Conclusion

This paper has outlined the importance of the polyhedral representation of rock mass structure over traditional techniques. The use of finite persistence discontinuity structures has also been discussed. Several analysis techniques have been used to assess the various model types and most have shown marked differences in predicted failure mechanisms and frequencies, thus supporting the argument that a polyhedral model incorporating finite persistence structures should be used whenever possible.

Acknowledgements

The authors wish to acknowledge the support of the Australian Coal Association Research Program (ACARP) who funded elements of the research by CSIRO Exploration and Mining personnel into the computational geometry required to develop the polyhedral rock models described in this paper. The sponsors of the Large Open Pit Slope Stability Project (LOP) are also acknowledged for their support in developing the application of these algorithms towards the analysis of bench and inter-ramp slope stability in the open pit mining environment. Finally, the review and comments of Dr Cedric Lambert are also appreciated.

References

- Baecher, G.B. and Einstein, H.H. (1977) Statistical description of rock properties and sampling. 18th US Symposium on Rock Mechanics, Colorado School of Mines, Golden CO, Johnson Publishing Company.
- Cundall, P.A., Potyondy, D.O. and Lee, C. (1996) Modeling Rock Using Bonded Assemblies of Circular Particles. Rock Mechanics Tools and Techniques. Proceedings Second North American Rock Mechanics Symposium, Montréal, June 1996, pp. 1937–1944.
- Dershowitz, W.S. and Einstein, H.H. (1988) Characterizing Rock Joint Geometry with Joint System Models. *Rock Mechanics and Rock Engineering*, 21, pp. 21–51.
- Dershowitz, W.S. and Herda, H.H. (1992) Interpretation of fracture spacing and intensity. 33rd US Symposium on Rock Mechanics, Rotterdam, Balkema, pp. 757–766.
- Gibson W.H., de Bruyn, I.A. and Walker, D.J.H. (2006) Considerations in the optimisation of bench face angle and berm width geometries for open pit mines. Stability of rock slopes in open pit mining and civil engineering situations, SAIMM Symposium Series S44, 3–6 April, pp. 557–578.
- Goodman, R.E. and Shi, G. (1985) *Block Theory and its Application to Rock Engineering*, Prentice Hall New Jersey.
- Goodman, R.E., Taylor, R.L. and Brekke, T. (1968) A model for mechanics of jointed rock. *Journal of Soil Mechanics and Foundations Division ASCE* 94(SME3), pp. 637–659.
- Jern, M. (2004) Determination of the In Situ block size distribution in fractured rock, an approach for comparing in-situ rock with rock sieve analysis. *Rock. Mech. Rock. Eng.*, Vol. 37 (5), pp. 391–401.
- Jing, L. (2000) Block system construction for three-dimensional discrete element models of fractured rocks. *Int. J. Rock Mech. Min. Sci.*, 37, pp. 645–659.
- Latham, J-P., Van Meulen, J. and Dupray, S. (2006) Prediction of in-situ block size distributions with reference to armourstone for breakwaters. *Eng. Geology*, pp. 18–36.
- Lin, D., Fairhurst, C. and Starfield, A.M. (1987) Geometrical identification of three dimensional rock block systems using topological techniques. *International Journal Rock Mechanics Mining Sciences and Geomechanics Abstracts.*, 24, No. 6, pp. 331–338.
- Lu, J. (2002) Systematic identification of polyhedral rock blocks with arbitrary joints and faults, *Computers and Geotechnics*, 29, pp. 49–72.
- Lu, P. and Latham J-P. (1999) Developments in the Assessment of In-situ Block Size Distributions of Rock Masses. *Rock Mechanics Rock Engineering*, 32(1), pp. 29–49.
- Maerz, N.H. and Germain, P. (1996) Block size determination around underground openings using simulations. Proceedings FRAGBLAST 5 Workshop on Measurement of Blast Fragmentation, Montréal, Quebec, Canada 23–24 August, pp. 215–223.
- Pierce, M., Cundall, P., Potyondy, D. and Mas Ivars, D. (2007) A synthetic rock mass model for jointed rock. *Rock Mechanics: Meeting Societies Challenges and Demands*, 1st Canada-US Rock Mech. Symposium, Vancouver, pp. 341–349.
- Ruppert, J. and Seidel, R. (1992) On the difficulty of triangulating three-dimensional non-convex polyhedra. *Discrete Computational Geometry*, 7, pp. 227–253.
- Ryan, T.M. and Pryor, P.R. (2000) Designing Catch Benches and Interramp Slopes. Slope stability in surface mining W.A. Hustrulid, M.K. McCarter and D.J.A. Van Zyl (editors), pp. 27–38.
- Warburton, P.M. (1981) Vector stability analysis of an arbitrary polyhedral rock block with any number of free faces. *International Journal Rock Mechanics Mining Sciences and Geomechanics Abstracts*, 18, pp. 415–427.
- Windsor, C.R. and Thompson, A.G. (1996) Block theory and excavation engineering. Proceedings Second North American Rock Mechanics Symposium, Montreal, Volume 2, pp. 1753–1760, Balkema, Rotterdam.