

# Imaging and Modelling of Flows of Gold Paste Tailings During Deposition — Laboratory Study and Field Scale Predictions

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## Abstract

*One important challenge in managing surface deposition of paste or thickened tailings is to predict the evolving geometry of the tailings stack during deposition. In this work, paste gold tailings are dynamically imaged during multilayer deposition in a flume and in conical flows. Each layer is left to self-weight consolidate before the next layer is placed. It is found that both the steady-state profiles and the dynamic flows can be modelled as a Bingham fluid using equations developed from lubrication theory. Yield stress and viscosity were determined using rheometry and slump tests. It is shown that the yield stress obtained from the slump test may overestimate the yield stress of significance to flow deposition; namely that yield stress that characterises when the tailings stop flowing. The lubrication theory equations show that the overall angle of a tailings deposit at steady-state is dependent on the scale of the flow, which may explain the discrepancy between laboratory flume angles and field angles noted in practice. The lubrication theory model is extended to three dimensions and compared to field profiles at one paste tailings site.*

## 1 Introduction

While the behaviour of thickened or paste tailings in the pipeline have been studied to a significant degree (Nguyen and Boger, 1998; Pullum et al., 2006; Sofra and Boger, 2002), the relation between rheology and deposition geometry has received less attention. Most studies have focused on charactering the geometry using a single angle (Kwak et al., 2005; Sofra and Boger, 2002) at the laboratory scale. However, it has widely been observed in practice that the overall angles of deposits in the field are typically less than angles measured in the laboratory (Oxenford and Lord, 2006; Engman et al., 2004). This was partially thought to be attributed to shear thinning occurring during transport (Houman et al., 2007). Simms (2007) proposed, based on non-Newtonian flow theory that had been previously applied to mud and lava flows (Liu and Mei, 1989), that the equilibrium beach profile is not characterised by a unique angle, and that the overall angle of the deposit is a function the size of the flow. Henriquez and Simms (2008) showed that lubrication theory provides very good estimates of steady state and transient flows measured in flumes and axi-symmetric flows, for a gold tailings prepared at a solid concentration of 70% and a yield stress between 30-50 Pa. Other researchers have begun tackling this problem (Fitton et al., 2008; Pirouz and Williams, 2007) using a different approach, one based on the critical deposition velocity for turbulent overland flow. This paper pursues the method proposed in Simms (2007) and Henriquez and Simms (2008), presenting some results from Henriquez and Simms (2008) as well as comparing 3D extrapolation of those results to the field.

## 2 Theory

The equilibrium profiles of yield stress fluids may be derived using ‘lubrication theory’, in which the continuity and momentum equations for fluids are simplified by assuming the slow spreading of a thin layer or film. The simplifying assumptions are:

- The ratio of thickness to horizontal extent of the flow are small.
- The velocity of the material is slow, such that terms that include the ratio of inertial to viscous forces will vanish from the Navier-Stokes equations.

From these assumptions equations describing equilibrium profiles for certain geometries can be derived (Yuhi and Mei, 2004; Liu and Mei, 1989; Balmforth et al., 2002; Coussot and Proust, 1996). For example, for two dimensional flow across an inclined bed, the distribution of shear stress may be written as:

$$\tau = (h - z) \left( \rho g \cos \theta \left( \tan \theta - \frac{\partial h}{\partial x} \right) \right) \quad (1)$$

Where  $p$  is pressure,  $\rho$  is the bulk density,  $\theta$  is the angle of the inclined surface from the horizontal,  $g$  is the acceleration due to gravity,  $\tau$  is the shear stress, the  $x$  axis is the direction of the inclined plane, and the  $z$  axis is perpendicular to the inclined plane. Setting  $z=0$  and  $\tau$  to the yield stress  $\tau_y$ , we can find an equilibrium profile as a function of  $x$  for a given  $\theta$ . For example, if  $\theta=0$ , we can find the equilibrium profile explicitly:

$$h^2 - h_0^2 = \frac{2\tau_y}{\rho g} (x - x_0) \quad (2)$$

where  $h_0$  is the height at  $x_0$ .

A depth integrated transient solution based on lubrication theory may be similarly derived (Yuhi and Mei, 2004):

$$\frac{\partial h}{\partial t} = \frac{g\rho}{\mu} \frac{dh}{dx} \frac{1}{6} (3h - h_y - 2H)(h_y - H)^2 \quad (3)$$

Where  $\mu$  is the dynamic viscosity,  $h$  is the vertical height of flow,  $h_y$  is the depth above which plug flow occurs, and  $H$  is the bed elevation. Equation (3) is based on the assumption of laminar flow in sheared zones and plug flow in regions where the yield stress is not exceeded.

### 3 Materials and methods

The tailings used were non-plastic (plastic limit of 20) gold tailings from the Bulyanhulu mine in Tanzania. These tailings are predominantly silt-sized, and have a specific gravity of 2.9. The tailings are prepared at a gravimetric water content (GWC) of 40%, corresponding to the pumping water content in the field. Tailings were shipped at the pumping water content, but arrived at the laboratory in a compacted state with bleed water on top. The tailings were prepared to the pumping water content using this bleed water and a mechanical mixer. The mechanical mixer is applied to the sample for at least ten minutes. Though tests were conducted over the period of a year, pH measurements ranged from 6.5 to 7 showing no trend with time. Tailings were stored underwater to minimise oxidation. Pore-water analyses were conducted at different times and no trend with time was observed. The tailings pore-water is dominated by iron, calcium, magnesium, and sulphate ions. More information on these tailings is available in Simms et al. (2007).

For this study flume tests were performed on horizontal planes and on successive layers. A high speed camera, Model IN250 from High Speed Imaging was used to record the tests. The camera has a capture rate of 50 frames per second. For multilayer tests, each layer was left for about 24 hours to allow it to reach the post-bleed GWC of 30%, before the next layer was added. This water content was confirmed by sampling. Pore-pressures were monitored using a tensiometer to ensure no significant suction developed. At this water content the older layers remain saturated and no cracking develops. Flume tests were performed by pouring the tailings through a funnel at one end of the flume. The funnel had an opening diameter of 2.4 cm. Once prepared at the appropriate water content, the paste was placed in a bucket and mixed with a stirring machine for approximately 10 minutes prior to being deposited in the flume. The flume is made of see-through acrylic and is 30 cm tall, 15 cm wide, and 2.5 m long.

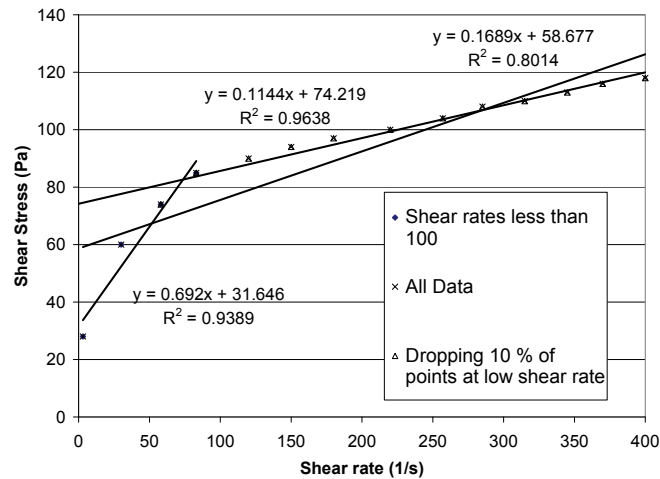
Initial tests were performed to examine the effect of width of the flume and lubrication of the walls. A hydrophobic grease was used as a lubricant, and the walls were narrowed from 15 to 10 cm. It was found that neither greasing nor changing the flume width had any significant effect on the final equilibrium profile of the flows (using the same volume of material per unit width). The parameters of interest are the yield stress, the viscosity-shear stress relationship (flow curve), and the bulk density. Yield stress and viscosity were obtained from flow curves measurements at different water contents using a Brookfield RS rheometer using a cone and plate fixture. Flow curves were obtained both by increasing and then by decreasing the

strain rate to check for wall depletion: no significant difference was found between the curves. Yield stresses were also determined based on slump tests, using the interpretation of Palshias et al. (1996).

## 4 Results

### 4.1 Rheological testing

In this paper we only report the flow curves measured at the pumping water content, shown in Figure 1.

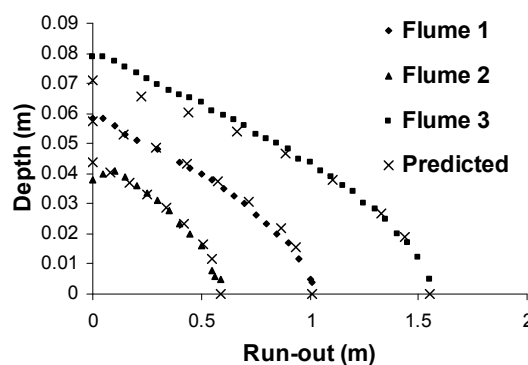


**Figure 1** Flow curve at 40% GWC

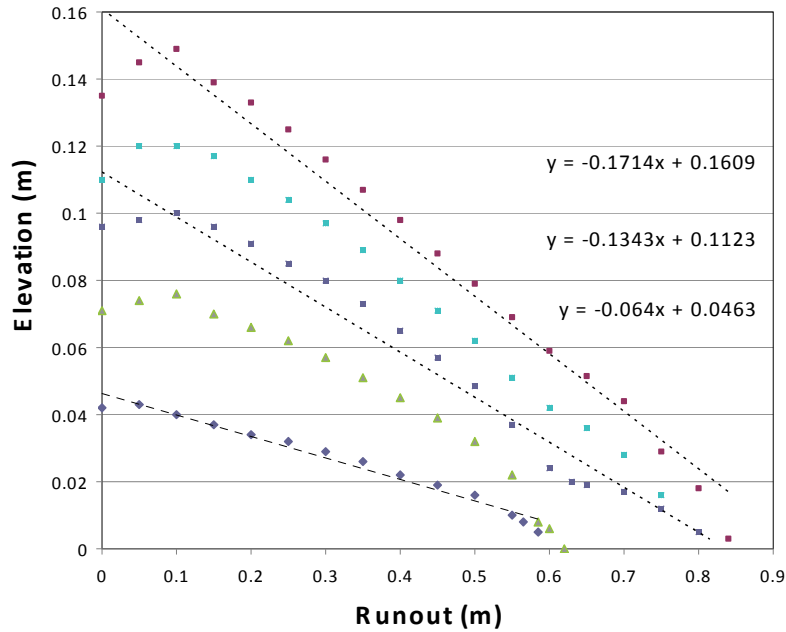
Figure 1 shows the measured flow curve fitted with three possible fits with the Bingham model (the Bingham assumes a constant viscosity and therefore a constant slope to the flow curve). It will be shown that the best fit to the data at the low shear rate range gives the best agreement with theoretical predictions. However, yield stresses determined from the slump test at the pumping water content ranged between 70 and 100 Pa, which agree better with yield stresses determined using a Bingham fit to the whole flow curve. It should be noted that quite similar flow curves were found using a TA Instruments model AR 2000 rheometer using a vane fixture on older tailings from the same mine (Kwak et al., 2005).

### 4.2 Flume test static profiles

Figure 2 presents three equilibrium profiles for flows of different volumes. This figure shows that the overall angle of the deposit is scale-dependent, the angle decreasing as the deposit gets larger. Each of these flows can be fitted using Equation (2) using the same density and yield stress. This result is important as it explains why direct extrapolation of flume angles to field scale deposition has not worked.



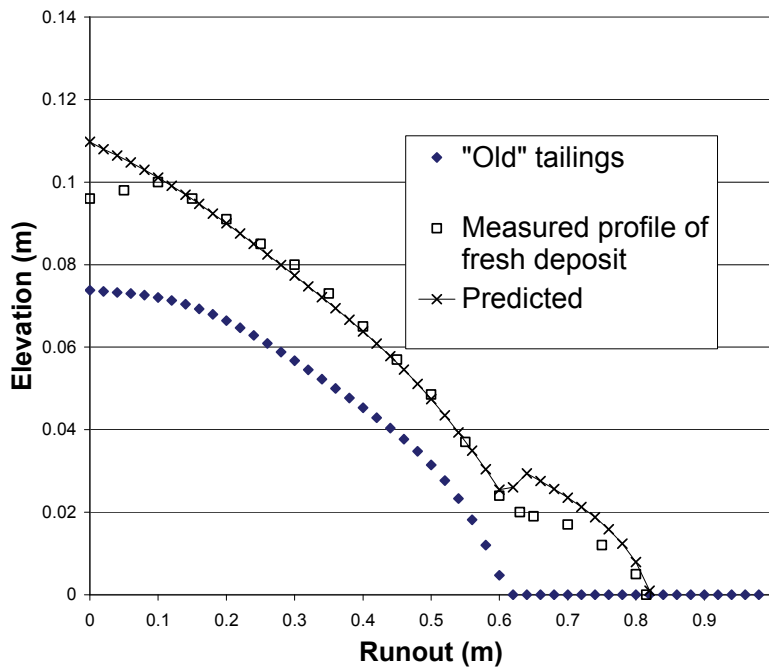
**Figure 2** Equilibrium profiles of three flows of different volumes, all fitted using Equation (2), employing a yield stress of 36 Pa



**Figure 3 Sequential deposition, each layer initially placed at 40% GWC**

Figure 3 presents equilibrium profiles for multilayer deposition. It can be seen that with each layer the layer thickness decreases and the overall angle of the slope increases.

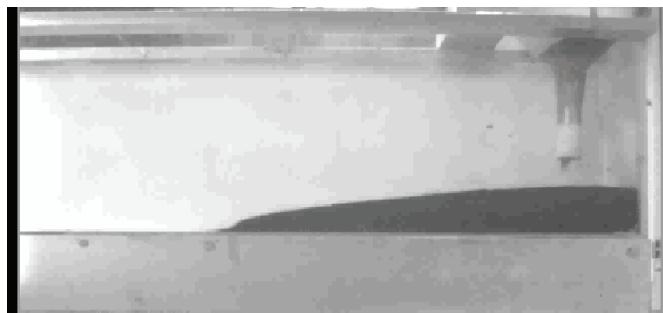
It was found that the multilayer static profiles could be iteratively determined using Equation (1) by calculating  $dh/dx$  in discrete steps back from toe, considering that  $\theta=\theta(x)$ . An example of the prediction from this test is shown in Figure 4.



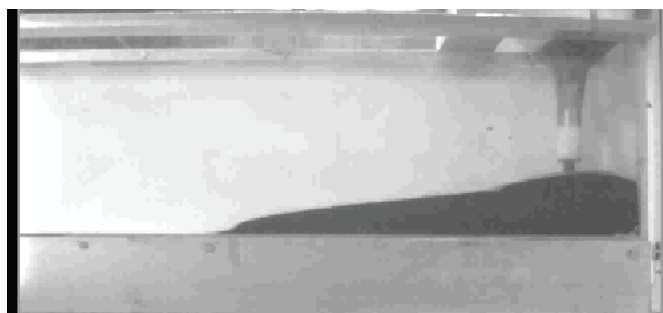
**Figure 4 Predicted and measured equilibrium profiles for multilayer deposition**

### 4.3 Dynamic flow profiles

Figure 5 shows an example of dynamic imaging from a flume test.



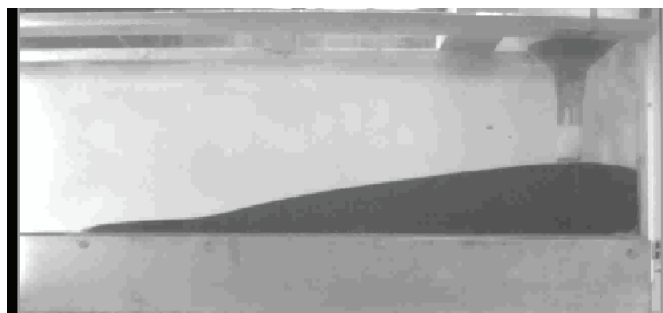
a)



b)



c)

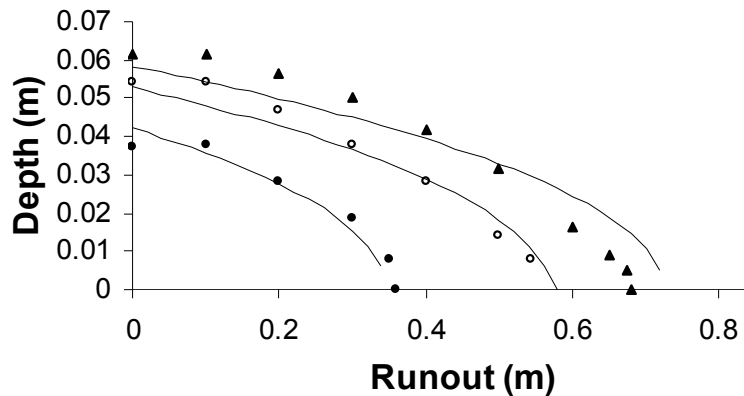


d)

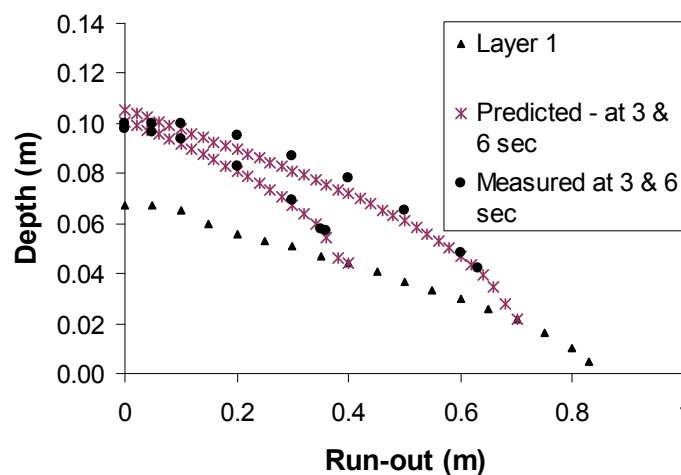
**Figure 5** Example of a dynamic visualisation a) 0 s; b) 2 s; c) 4 s; d) Stopped at 18 s

A 1-D finite difference solution of Equation (3) was employed to simulate the dynamic flows. A spatial step of 1 cm and a time step of 0.0001 s were employed. A constant viscosity of 0.65 Pa, corresponding to the fit of the flow curve in the low shear rate zone, was used.

Further details can be found in Henriquez and Simms (2008). Predictions of the transient model are shown in Figure 6 for a single layer and Figure 7 for multilayer flow. In general, the model tended to predict faster flows than were observed. This may be due to the assumption of constant viscosity.



**Figure 6** Predicted and measured flow profiles at 3, 6, and 9 s during single layer deposition



**Figure 7** Predicted and measured flow profiles of a fresh layer flowing on top of an older layer

## 5 Discussion

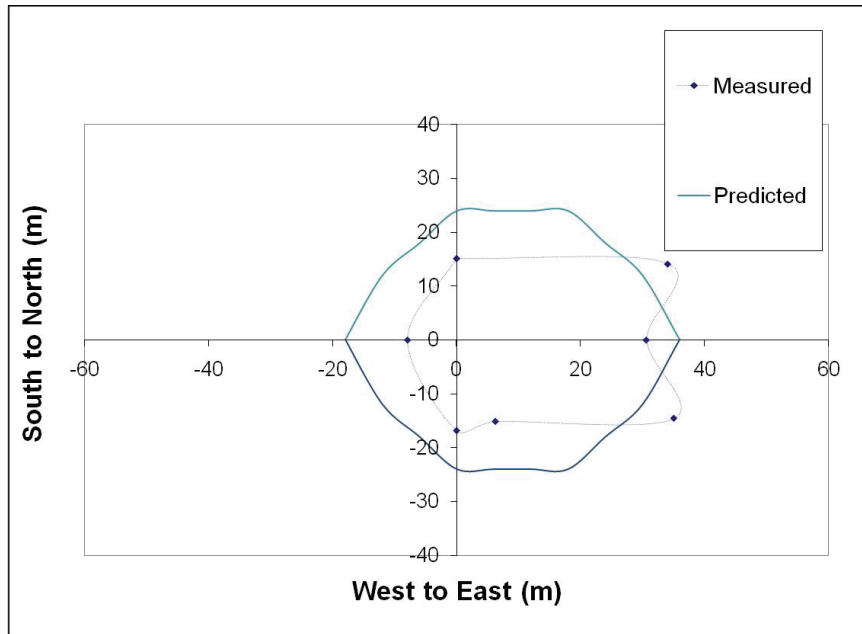
### 5.1 Significance of the ‘best fit’ yield stress

It was observed that the flow curve in the low shear rate range ( $\sim 32$  Pa) agreed with the yield stress that gave the best fit of the lubrication theory predictions to the measured data (36 Pa). This is a reasonable result, as the Bingham model fit to the data closest to the origin would most likely give the best approximation of the actual yield stress. Unfortunately, the low shear rate region is also the most susceptible to measurement error, and indeed is often neglected in interpreting flow curves to derive model parameters for pipe flow analysis. However, flume tests themselves may serve to obtain the yield stress pertinent to deposition modelling though their interpretation using Equation (2).

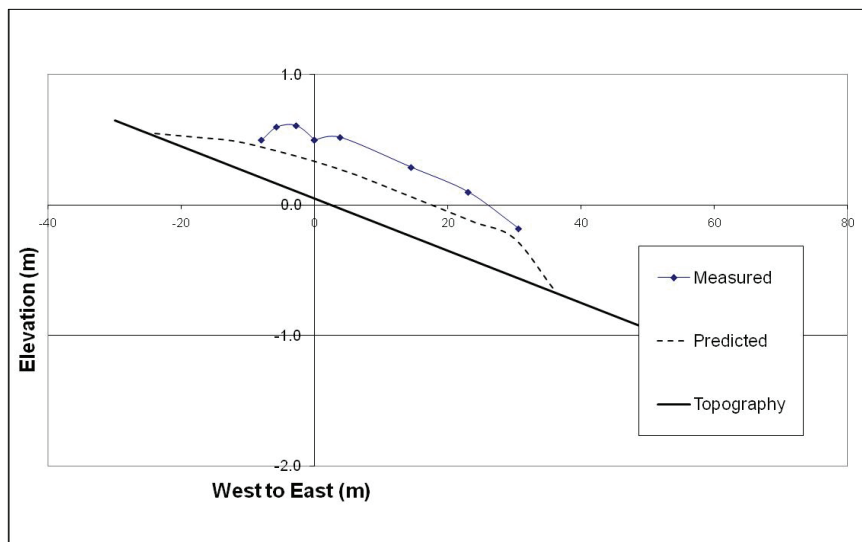
It was found that the yield stress predicted from the slump tests (70–100 Pa) conformed to the best fit of the Bingham model to the flow curve for shear rates greater than  $50 \text{ s}^{-1}$ . This suggests that this yield stress is more appropriate for determining friction losses in pipe flow, rather than for characterising the stress at which the tailings stop flowing during deposition.

## 5.2 Application of theory to the field scale

The authors have so far had only limited access to field scale data of only crude precision. The following figures compare the geometry of the first layer poured from one of the towers at the Bulyanhulu site to predictions using the transient model extended to three dimensions. Figure 8 shows a plan view and Figure 9 compares the east-west cross section. It is believed that the original surface slopes down about 2% going West, and this was the base topography assumed in the numerical prediction. It was also assumed that the pumping water content was 40% gravimetric, and the associated yield stress and density 36 Pa and 1900 kg/m<sup>3</sup> respectively. The mesh size is 3 x 3 m.



**Figure 8** Prediction of field scale deposition using transient model, plan view



**Figure 9** Field scale prediction, east-west profile through tower at (0, 0)

The field data was estimated using stakes placed prior to deposition and a laser finder. One can see that the predicted flow is somewhat flatter and more spread out than the real flow. However, the numerical prediction seems to capture the general shape, including the tendency of the tailings stack to be biased along the slope of the underlying topography. Field data of better quality, including accurate characterisation of the topography, will be required for a proper evaluation of the numerical method described in this paper.

## 6 Conclusions

Equilibrium and transient predictions of lubrication theory compare very well to laboratory visualisations of deposition of a gold paste tailings. The theory explains the scale dependence of beach angles so often noted in practice. It is observed that the yield stress that best characterises when tailings stop flowing is not the same yield stress derived from current interpretations of the slump test. Field scale predictions show promise though more accurate field data is needed for a proper validation or calibration of the lubrication theory based methods.

## Acknowledgements

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